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New Concepts of Neutrosophic Crisp Generalized Closed Functions

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Abstract: In this article, new forms of neutrosophic crisp closed functions are delivered which are called neutrosophic crisp generalized semi generalized (gsg) closed functions. Moreover, these forms are reconstructed to acquire new forms of neutrosophic crisp closed functions which are denoted by (gsg^{*}, gsg^{**}). Finally, the corresponding relations of these forms are studied and analyzed by proof some theorems and propositions.

Keywords: Neu_C-closed, Neu_{Cgsg}-closed, Neu_{Cgsg^*}-closed, and $Neu_{Cgsg^{**}}$-closed functions.

1. Introduction

The novel definitions of neutrosophic crisp topological spaces (Shortly, Neu_{CTS}) are shinned by Salama et al. [1]. After that, the concepts of neutrosophic crisp nearly open sets are presented by Salama [2]. Next, the thoughts of neutrosophic crisp semi-α-closed sets are conferred by Al-Hamido et al. [3]. Moreover, the ideas of weakly neutrosophic crisp functions are demonstrated by Al-Obaidi et al. [4,5]. Furthermore, in the view of neutrosophic crisp sets, the innovative sorts of weak continuity, kinds of open sets, weak separation axioms, and generalized sg-closed sets with their continuity are deliberated by Imran et al. [6-9]. Ultimately, the impressions of neutrosophic crisp topological spaces are extended to involve the ideas of generalized alpha generalized closed sets spaces and neutrosophic generalized alpha generalized separation axioms by Abdulkadhim et al. [10,11]. The impartial of this article is to define pioneering perceptions of neutrosophic crisp closed functions called neutrosophic crisp generalized semi generalized (gsg) closed functions have been provided and their relations with other concepts of neutrosophic crisp closed functions (gsg^{*}, gsg^{**}) are analyzed.

2. Preliminaries

Throughout this article, the spaces Neu_{CTSs} are specified by the following forms $(\mathcal{G},\mathcal{T}),(\mathcal{H},\mathcal{L})$ and $(\mathcal{I},\sigma)$ which can be simply written by $\mathcal{G},\mathcal{H}$ and $\mathcal{I}$, respectively. For any set $\mathcal{M}$ in a Neu_{CTS} $(\mathcal{G},\mathcal{T})$ as neutrosophic subset, then its neutrosophic crisp closure is indicated by $Neu_C cl(\mathcal{M})$, its neutrosophic crisp interior is signified by $Neu_C int(\mathcal{M})$, and its complement is signified by $\underline{\mathcal{M}} = \mathcal{G}_{Neu} - \mathcal{M}$, respectively.

Definition 2.1: [1]

For any non-empty understudy set $\mathcal{G}$ with its following three mutually exclusive subsets $\mathcal{M}_1,\mathcal{M}_2,\mathcal{M}_3$, then neutrosophic crisp set is object assigned by this form $\mathcal{M} =< \mathcal{M}_1,\mathcal{M}_2,\mathcal{M}_3 >$ and simply written by Neu_C-set.

Definition 2.2: [1]

Let $\mathcal{M} =< \mathcal{M}_1,\mathcal{M}_2,\mathcal{M}_3 >$ and $\mathcal{N} =< \mathcal{N}_1,\mathcal{N}_2,\mathcal{N}_3 >$ be Neu_C-sets of non-null set $\mathcal{G}$, then their intersection is defined as

$$\mathcal{M}\cap\mathcal{N} =< \mathcal{M}_1\cap\mathcal{N}_1,\mathcal{M}_2\cup\mathcal{N}_2,\mathcal{M}_3\cup\mathcal{N}_3 >$$

and their union is defined as

$$\mathcal{M}\cup\mathcal{N} =< \mathcal{M}_1\cup\mathcal{N}_1,\mathcal{M}_2\cap\mathcal{N}_2,\mathcal{M}_3\cap\mathcal{N}_3 >.$$

Definition 2.3:

(i) Let $\mathcal{M} =< \mathcal{M}_1,\mathcal{M}_2,\mathcal{M}_3 >$ be Neu_C-set of a non-null set $\mathcal{G}$, then its complement of the set $\mathcal{M}$ which is denoted by $\underline{\mathcal{M}}$ and defined by $\underline{\mathcal{M}} =< \underline{\mathcal{M}_1},\mathcal{M}_2,\mathcal{M}_3 >$.

(ii) Let $\varphi_{Neu} =< \varphi,\varphi,\varphi >$ and $\mathcal{G}_{Neu} =< \mathcal{G},\varphi,\varphi >$ be denoted by empty Neu_C-set and universal Neu_C-set of a non-null set $\mathcal{G}$, respectively.

Definition 2.4: [1]

Let $\mathcal{T}$ be a family of Neu_C-sets of a non-null set $\mathcal{G}$, then $\mathcal{T}$ is termed a neutrosophic crisp topology on $\mathcal{G}$ which is shortly written by Neu_{CT} if it is satisfied the succeeding postulates:

(i) $\varphi_{Neu},\mathcal{G}_{Neu} \in \mathcal{T}$,

(ii) $\mathcal{M}_1\cap\mathcal{M}_2 \in \mathcal{T}$ where $\mathcal{M}_1,\mathcal{M}_2 \in \mathcal{T}$,

(iii) $\cup\mathcal{M}_k \in \mathcal{T}$ for all families $\{\mathcal{M}_k | k \in \Delta\} \subseteq \mathcal{T}$.

Moreover, the ordered pair $(\mathcal{G},\mathcal{T})$ is entitled neutrosophic crisp topological space which is shortly written by Neu_{CTS}. Furthermore, the elements of $(\mathcal{G},\mathcal{T})$ are entitled neutrosophic crisp open sets which are simply written Neu_COSs and their complements are entitled neutrosophic crisp closed sets which are shortly written by Neu_CCSs.

Definition 2.5: [2]

Let $(\mathcal{A},\mathcal{T})$ be a Neu_{CTS}, then a Neu_C-subset $\mathcal{U}$ of $\mathcal{A}$ is termed to be

(i) a neutrosophic crisp semi-open set which is simply written by Neu_{Cs}OS if $\mathcal{U} \subseteq Neu_C cl(Neu_C int(\mathcal{U}))$.

(ii) a neutrosophic crisp semi-closed set which is simply written by Neu_{Cs}CS if $Neu_C int(Neu_C cl(\mathcal{U})) \subseteq \mathcal{U}$.

(iii) the neutrosophic crisp semi-closure of $\mathcal{U}$ of a Neu_{CTS} $(\mathcal{A},\mathcal{T})$ is the overlapping of every Neu_{Cs} CSs that include $\mathcal{U}$ and it is indicated by $Neu_{Cs}cl(\mathcal{U})$.

Definition 2.6: [9]

Let $(\mathcal{G},\mathcal{T})$ be a Neu_{CTS}, then a Neu_C-subset $\mathcal{M}$ of $\mathcal{G}$ is entitled to be

(i) a neutrosophic crisp sg-closed set which is written simply by Neu_{Csg}CS if $Neu_{Cs}cl(\mathcal{M}) \subseteq \mathcal{U}$ for any Neu_{Cs}OS $\mathcal{U}$ in $\mathcal{G}$ such that $\mathcal{M} \subseteq \mathcal{U}$. Moreover, the outside of Neu_{Csg}CS is Neu_{Csg}-open set in $\mathcal{G}$ which is written simply by Neu_{Csg}OS.

(ii) a neutrosophic crisp gs-closed set which is simply written Neu_{Cgs}CS if $Neu_{Cs}cl(\mathcal{M}) \subseteq \mathcal{U}$ for any Neu_{C}OS $\mathcal{U}$ in $\mathcal{G}$ such that $\mathcal{M} \subseteq \mathcal{U}$. Furthermore, the outside of Neu_{Cgs}CS is a Neu_{Cgs}-open set in $\mathcal{G}$ which is simply written by Neu_{Cgs}OS.

Definition 2.7: [9]

Let $(\mathcal{G}, \mathcal{T})$ be a Neu_{CTS}, then a Neu_{C}-subset $\mathcal{M}$ of $\mathcal{G}$ is entitled to be a neutrosophic crisp gsg-closed set which is simply written by Neu_{Cgsg}CS if $Neu_{C}cl(\mathcal{M}) \subseteq \mathcal{U}$ for any Neu_{Csg}OS $\mathcal{U}$ in $\mathcal{G}$ such that $\mathcal{M} \subseteq \mathcal{U}$. The family of all Neu_{Cgsg}CSs of a Neu_{CTS} $(\mathcal{G}, \mathcal{T})$ is indicated by Neu_{Cgsg}C$(\mathcal{G})$.

Proposition 2.8: [9]

In a Neu_{CTS} $(\mathcal{G}, \mathcal{T})$, the subsequent statements are sensible:

(i) Every Neu_{C}CS is a Neu_{Cgsg}CS.

(ii) Every Neu_{Cgsg}CS is a Neu_{Csg}CS.

(iii) Every Neu_{Cgsg}CS is a Neu_{Cgs}CS.

Definition 2.9: [9]

The crossing of all Neu_{Cgsg}CSs in a Neu_{CTS} $(\mathcal{G}, \mathcal{T})$ containing $\mathcal{M}$ is titled Neu_{Cgsg}-closure of $\mathcal{M}$ and is denoted by $Neu_{Cgsg}cl(\mathcal{M})$, $Neu_{Cgsg}cl(\mathcal{M}) = \bigcap\{\mathcal{N} : \mathcal{M} \subseteq \mathcal{N}, \mathcal{N}$ stands as a Neu_{Cgsg}CS$\}$.

Definition 2.10: [1]

Let $\not{f}$ be a function from $(\mathcal{G}, \mathcal{T})$ into $(\mathcal{H}, \mathcal{L})$, then it is entitled by a neutrosophic crisp closed which is simply written by Neu_{C}-closed if $\not{f}(\mathcal{M})$ is a Neu_{C}CS in $(\mathcal{H}, \mathcal{L})$ for every Neu_{C}CS $\mathcal{M}$ in $(\mathcal{G}, \mathcal{T})$.

Definition 2.11: [1]

Let $\not{f}$ be a function from $(\mathcal{G}, \mathcal{T})$ into $(\mathcal{H}, \mathcal{L})$, then it is entitled by a neutrosophic crisp continuous which is simply written by Neu_{C}-continuous if $\not{f}^{-1}(\mathcal{M})$ is a Neu_{C}CS (Neu_{C}OS) in $(\mathcal{G}, \mathcal{T})$ for every Neu_{C}CS (Neu_{C}OS) $\mathcal{M}$ in $(\mathcal{H}, \mathcal{L})$.

Definition 2.12: [9]

Let $\not{f}$ be a function from $(\mathcal{G}, \mathcal{T})$ into $(\mathcal{H}, \mathcal{L})$, then it is entitled by a neutrosophic crisp gsg-continuous which is simply written by Neu_{Cgsg}-continuous if $\not{f}^{-1}(\mathcal{M})$ is a Neu_{Cgsg}CS (Neu_{Cgsg}OS) in $(\mathcal{G}, \mathcal{T})$ for each Neu_{C}CS (Neu_{C}OS) $\mathcal{M}$ in $(\mathcal{H}, \mathcal{L})$.

Definition 2.13: [9]

Let $\not{f}$ be a function from $(\mathcal{G}, \mathcal{T})$ into $(\mathcal{H}, \mathcal{L})$, then it is entitled by a neutrosophic crisp gsg^*-continuous which is simply written by Neu_{Cgsg^*}-continuous if $\not{f}^{-1}(\mathcal{M})$ is a Neu_{C}CS (Neu_{C}OS) in $(\mathcal{G}, \mathcal{T})$ for each Neu_{Cgsg}CS (Neu_{Cgsg}OS) $\mathcal{M}$ in $(\mathcal{H}, \mathcal{L})$.

Definition 2.14: [9]

Let $\not{f}$ be a function from $(\mathcal{G}, \mathcal{T})$ into $(\mathcal{H}, \mathcal{L})$, then it is entitled by a neutrosophic crisp gsg^{**}-continuous which is simply written by $Neu_{Cgsg^{**}}$-continuous if $\not{f}^{-1}(\mathcal{M})$ is a Neu_{Cgsg}CS (Neu_{Cgsg}OS) in $(\mathcal{G}, \mathcal{T})$ for each Neu_{Cgsg}CS (Neu_{Cgsg}OS) $\mathcal{M}$ in $(\mathcal{H}, \mathcal{L})$.

Proposition 2.15: [9]

(i) All Neu_{C}-continuous are Neu_{Cgsg}-continuous.

(ii) All Neu_{Cgsg^*}-continuous are $Neu_{Cgsg^{**}}$-continuous.

(iii) All Neu_{Cgsg}-continuous are $Neu_{Cgsg^{**}}$-continuous.

3. Neutrosophic Crisp gsg-Closed Functions

This section defines three concepts of neutrosophic crisp gsg-closed functions via neutrosophic crisp gsg-closed sets, as see in the following definition:

Definition 3.1:

A function $\not{f}: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ is said to be neutrosophic crisp gsg-closed (in short Neu_{Cgsg}-closed) if $\not{f}(\mathcal{M})$ is a Neu_{Cgsg}CS in $(\mathcal{H}, \mathcal{L})$ for each Neu_CCS $\mathcal{M}$ in $(\mathcal{G}, \mathcal{T})$.

Definition 3.2:

A function $\not{f}: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ is said to be neutrosophic crisp gsg^*-closed (in brief Neu_{Cgsg^*}-closed) if $\not{f}(\mathcal{M})$ is a Neu_CCS in $(\mathcal{H}, \mathcal{L})$ for each Neu_{Cgsg}CS $\mathcal{M}$ in $(\mathcal{G}, \mathcal{T})$.

Definition 3.3:

A function $\not{f}: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ is said to be neutrosophic crisp gsg^{**}-closed (in brief $Neu_{Cgsg^{**}}$-closed) if $\not{f}(\mathcal{M})$ is a Neu_{Cgsg}CS in $(\mathcal{H}, \mathcal{L})$ for each Neu_{Cgsg}CS $\mathcal{M}$ in $(\mathcal{G}, \mathcal{T})$.

Theorem 3.4:

Every Neu_C-closed function is Neu_{Cgsg}-closed.

Proof:

Let $\not{f}: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ be a Neu_C-closed function. Let $\mathcal{M}$ be a Neu_CCS in $(\mathcal{G}, \mathcal{T})$. Meanwhile $\not{f}$ is a Neu_C-closed function, $\not{f}(\mathcal{M})$ is a Neu_CCS in $(\mathcal{H}, \mathcal{L})$. Which implies $\not{f}(\mathcal{M})$ is a Neu_{Cgsg}CS in $(\mathcal{H}, \mathcal{L})$. Hence $\not{f}$ is a Neu_{Cgsg}-closed function. ▪

The converse of above theorem need not be true as can be seen from the following example.

Example 3.5:

Suppose $\mathcal{G} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{H} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.

Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\}, \varphi, \varphi\rangle, \langle\{s_2, s_3\}, \varphi, \varphi\rangle, \langle\{s_1, s_2, s_3\}, \varphi, \varphi\rangle, \mathcal{G}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_1\}, \varphi, \varphi\rangle, \langle\{\sigma_2, \sigma_3\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_2, \sigma_3\}, \varphi, \varphi\rangle, \mathcal{H}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{G}$ and $\mathcal{H}$, respectively. Define a function $\not{f}: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ where $\not{f}(\langle\{s_1\}, \varphi, \varphi\rangle) = \langle\{\sigma_2\}, \varphi, \varphi\rangle$, $\not{f}(\langle\{s_2\}, \varphi, \varphi\rangle) = \langle\{\sigma_1\}, \varphi, \varphi\rangle$, $\not{f}(\langle\{s_3\}, \varphi, \varphi\rangle) = \langle\{\sigma_4\}, \varphi, \varphi\rangle$, $\not{f}(\langle\{s_4\}, \varphi, \varphi\rangle) = \langle\{\sigma_3\}, \varphi, \varphi\rangle$. Since the images of all sets under the function $\not{f}$ is a Neu_{Cgsg}-closed function, it follows that $\not{f}$ satisfies the definition of a Neu_C-closed function.

Theorem 3.6:

Every Neu_{Cgsg^*}-closed function is Neu_{Cgsg}-closed.

Proof:

Let $\not{f}: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ be a Neu_{Cgsg^*}-closed function. Let $\mathcal{M}$ be a Neu_CCS in $(\mathcal{G}, \mathcal{T})$, which implies $\mathcal{M}$ is Neu_{Cgsg}CS in $(\mathcal{G}, \mathcal{T})$. Since $\not{f}$ is a Neu_{Cgsg^*}-closed function, $\not{f}(\mathcal{M})$ is a Neu_CCS in $(\mathcal{H}, \mathcal{L})$. Which implies $\not{f}(\mathcal{M})$ is a Neu_{Cgsg}CS in $(\mathcal{H}, \mathcal{L})$. Hence $\not{f}$ is a Neu_{Cgsg}-closed function. ▪

The opposite of exceeding theorem need not be correct as can be realized from the next instance.

Example 3.7:

Suppose $\mathcal{G} = \{s_1, s_2, s_3\}$ and $\mathcal{H} = \{\sigma_1, \sigma_2, \sigma_3\}$.

Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_1\}, \varphi, \varphi\rangle, \langle\{s_2, s_3\}, \varphi, \varphi\rangle, \mathcal{G}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_1\}, \varphi, \varphi\rangle, \langle\{\sigma_2, \sigma_3\}, \varphi, \varphi\rangle, \mathcal{H}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{G}$ and $\mathcal{H}$, respectively. Define a function $\not{f}: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ where $\not{f}(\langle\{s_1\}, \varphi, \varphi\rangle) = \langle\{\sigma_2\}, \varphi, \varphi\rangle$, $\not{f}(\langle\{s_2\}, \varphi, \varphi\rangle) = \langle\{\sigma_1\}, \varphi, \varphi\rangle$, $\not{f}(\langle\{s_3\}, \varphi, \varphi\rangle) = \langle\{\sigma_3\}, \varphi, \varphi\rangle$. Since the images of all sets under the function $\not{f}$ is a Neu_{Cgsg}-closed function, it follows that $\not{f}$ satisfies the definition of a Neu_{Cgsg^*}-closed function.

Theorem 3.8:

Every Neu_{Cgsg^*}-closed function is $Neu_{Cgsg^{**}}$-closed.

Proof:

Let $f: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ be a Neu_{Cgsg^*}-closed function. Let $\mathcal{M}$ be a Neu_{Cgsg}CS in $(\mathcal{G}, \mathcal{T})$. Since f is a Neu_{Cgsg^*}-closed function, $f(\mathcal{M})$ is a Neu_CCS in $(\mathcal{H}, \mathcal{L})$. Since every Neu_CCS is a Neu_{Cgsg}CS. Which implies $f(\mathcal{M})$ is a Neu_{Cgsg}CS in $(\mathcal{H}, \mathcal{L})$. Hence f is a $Neu_{Cgsg^{**}}$-closed function. ▪

The opposite of exceeding theorem need not be correct as can be realized from the next instance.

Example 3.9:

Suppose $\mathcal{G} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{H} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.

Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_3\}, \varphi, \varphi\rangle, \langle\{s_1, s_4\}, \varphi, \varphi\rangle, \langle\{s_1, s_3, s_4\}, \varphi, \varphi\rangle, \mathcal{G}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_4\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_3\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_3, \sigma_4\}, \varphi, \varphi\rangle, \mathcal{H}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{G}$ and $\mathcal{H}$, respectively. Define a function $f: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ where $f(\langle\{s_1\}, \varphi, \varphi\rangle) = \langle\{\sigma_4\}, \varphi, \varphi\rangle$, $f(\langle\{s_2\}, \varphi, \varphi\rangle) = f(\langle\{s_3\}, \varphi, \varphi\rangle) = \langle\{\sigma_1\}, \varphi, \varphi\rangle, f(\langle\{s_4\}, \varphi, \varphi\rangle) = \langle\{\sigma_2\}, \varphi, \varphi\rangle$. Then f is a $Neu_{Cgsg^{**}}$-closed function, just not Neu_{Cgsg^*}-closed.

Theorem 3.10:

Every $Neu_{Cgsg^{**}}$-closed function is Neu_{Cgsg}-closed.

Proof:

Let $f: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ be a $Neu_{Cgsg^{**}}$-closed function. Let $\mathcal{M}$ be a Neu_CCS in $(\mathcal{G}, \mathcal{T})$. Since every Neu_CCS is a Neu_{Cgsg}CS, which implies $\mathcal{M}$ is Neu_{Cgsg}CS in $(\mathcal{G}, \mathcal{T})$. Since f is a $Neu_{Cgsg^{**}}$-closed function, $f(\mathcal{M})$ is a Neu_{Cgsg}CS in $(\mathcal{H}, \mathcal{L})$. Hence f is a Neu_{Cgsg}-closed function. ▪

The opposite of exceeding theorem need not be correct as can be realized from the next instance.

Example 3.11:

Suppose $\mathcal{G} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{H} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.

Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_2, s_4\}, \varphi, \varphi\rangle, \mathcal{G}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_3\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_4\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_3, \sigma_4\}, \varphi, \varphi\rangle, \mathcal{H}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{G}$ and $\mathcal{H}$, respectively. Define a function $f: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ where $f(\langle\{s_1\}, \varphi, \varphi\rangle) = f(\langle\{s_3\}, \varphi, \varphi\rangle) = \langle\{\sigma_1\}, \varphi, \varphi\rangle$, $f(\langle\{s_2\}, \varphi, \varphi\rangle) = f(\langle\{s_4\}, \varphi, \varphi\rangle) = \langle\{\sigma_3\}, \varphi, \varphi\rangle$. Then f is a Neu_{Cgsg}-closed function, just not $Neu_{Cgsg^{**}}$-closed.

Theorem 3.12:

Every Neu_{Cgsg^*}-closed function is Neu_C-closed.

Proof:

Let $f: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ be a Neu_{Cgsg^*}-closed function. Let $\mathcal{M}$ be a Neu_CCS in $(\mathcal{G}, \mathcal{T})$, which implies $\mathcal{M}$ is Neu_{Cgsg}CS in $(\mathcal{G}, \mathcal{T})$. Since f is a Neu_{Cgsg^*}-closed function, $f(\mathcal{M})$ is a Neu_CCS in $(\mathcal{H}, \mathcal{L})$. Hence f is a Neu_C-closed function. ▪

The opposite of exceeding theorem need not be correct as can be realized from the next instance.

Example 3.13:

Suppose $\mathcal{G} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{H} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.

Then $\mathcal{T} = \{\varphi_{Neu}, \langle\{s_3\}, \varphi, \varphi\rangle, \langle\{s_1, s_4\}, \varphi, \varphi\rangle, \langle\{s_1, s_3, s_4\}, \varphi, \varphi\rangle, \mathcal{G}_{Neu}\}$ and $\mathcal{L} = \{\varphi_{Neu}, \langle\{\sigma_3\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_4\}, \varphi, \varphi\rangle, \langle\{\sigma_1, \sigma_3, \sigma_4\}, \varphi, \varphi\rangle, \mathcal{H}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{G}$ and $\mathcal{H}$, respectively. Define a function $f: (\mathcal{G}, \mathcal{T}) \longrightarrow (\mathcal{H}, \mathcal{L})$ where $f(\langle\{s_1\}, \varphi, \varphi\rangle) = \langle\{\sigma_1\}, \varphi, \varphi\rangle$, $f(\langle\{s_2\}, \varphi, \varphi\rangle) =$

$\langle\{\sigma_2\},\varphi,\varphi\rangle, \not{f}(\langle\{s_3\},\varphi,\varphi\rangle) = \langle\{\sigma_3\},\varphi,\varphi\rangle, \not{f}(\langle\{s_4\},\varphi,\varphi\rangle) = \langle\{\sigma_4\},\varphi,\varphi\rangle$. Then $\not{f}$ is a Neu_C-closed function, just not Neu_{Cgsg^*}-closed.

Theorem 3.14:

Assume $\not{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{H},\mathcal{L})$ and $\not{f}_2: (\mathcal{H},\mathcal{L}) \longrightarrow (\mathcal{I},\sigma)$ are two functions, then $\not{f}_2 \circ \not{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{I},\sigma)$ is

(i) a Neu_{Cgsg}-closed if $\not{f}_1$ is a Neu_C-closed and $\not{f}_2$ is a Neu_{Cgsg}-closed.

(ii) a Neu_{Cgsg^*}-closed if $\not{f}_1$ and $\not{f}_2$ are Neu_{Cgsg^*}-closed.

(iii) a $Neu_{Cgsg^{**}}$-closed if $\not{f}_1$ and $\not{f}_2$ are $Neu_{Cgsg^{**}}$-closed.

(iv) a Neu_C-closed if $\not{f}_1$ is a Neu_C-closed and $\not{f}_2$ is a Neu_{Cgsg^*}-closed.

(v) a Neu_{Cgsg^*}-closed if $\not{f}_1$ is a Neu_{Cgsg^*}-closed and $\not{f}_2$ is a Neu_C-closed.

(vi) a Neu_{Cgsg^*}-closed if $\not{f}_1$ is a Neu_C-closed and $\not{f}_2$ is a $Neu_{Cgsg^{**}}$-closed.

Proof:

(i) Let $\mathcal{M}$ be a Neu_CCS in $(\mathcal{G},\mathcal{T})$. Since $\not{f}_1$ is aNeu_C-closed function, $\not{f}_1(\mathcal{M})$ is Neu_CCS in $(\mathcal{H},\mathcal{L})$. Since $\not{f}_2$ is a Neu_{Cgsg}-closed function, $\not{f}_2(\not{f}_1(\mathcal{M}))$ is Neu_{Cgsg}CS in $(\mathcal{I},\sigma)$. This implies $\not{f}_2 \circ \not{f}_1$ is a Neu_{Cgsg}-closed function.

(ii) Let $\mathcal{M}$ be a Neu_{Cgsg}CS in $(\mathcal{G},\mathcal{T})$. Since $\not{f}_1$ is aNeu_{Cgsg^*}-closed function, $\not{f}_1(\mathcal{M})$ is Neu_CCS in $(\mathcal{H},\mathcal{L})$, which implies $\not{f}_1(\mathcal{M})$ is Neu_{Cgsg}CS in $(\mathcal{H},\mathcal{L})$. Since $\not{f}_2$ is a Neu_{Cgsg^*}-closed function, $\not{f}_2(\not{f}_1(\mathcal{M}))$ is Neu_CCS in $(\mathcal{I},\sigma)$. This implies $\not{f}_2 \circ \not{f}_1$ is a Neu_{Cgsg^*}-closed function.

(iii) Let $\mathcal{M}$ be a Neu_{Cgsg}CS in $(\mathcal{G},\mathcal{T})$. Since $\not{f}_1$ is a$Neu_{Cgsg^{**}}$-closed function, $\not{f}_1(\mathcal{M})$is Neu_{Cgsg}CS in $(\mathcal{H},\mathcal{L})$. Since $\not{f}_2$ is a $Neu_{Cgsg^{**}}$-closed function, $\not{f}_2(\not{f}_1(\mathcal{M}))$ is Neu_{Cgsg}CS in $(\mathcal{I},\sigma)$. This implies $\not{f}_2 \circ \not{f}_1$ is a $Neu_{Cgsg^{**}}$-closed function.

(iv) Let $\mathcal{M} \subseteq \mathcal{I}$ be a Neu_C-closed set in $(\mathcal{I},\sigma)$. Since $\not{f}_2$ is Neu_{Cgsg^*}-closed, the preimage $\not{f}_2^{-1}(\mathcal{M})$ is Neu_C-closed in $(\mathcal{H},\mathcal{L})$. As $\not{f}_1$ is Neu_C-closed, the preimage $\not{f}_1^{-1}(\not{f}_2^{-1}(\mathcal{M})) = (\not{f}_2 \circ \not{f}_1)^{-1}(\mathcal{M})$ is Neu_C-closed in $(\mathcal{I},\sigma)$. Hence, $\not{f}_2 \circ \not{f}_1$ is Neu_C-closed.

(v) Let $\mathcal{M} \subseteq \mathcal{I}$ be a Neu_{Cgsg^*}-closed set in $(\mathcal{I},\sigma)$. Since $\not{f}_2$ is Neu_C-closed, the preimage $\not{f}_2^{-1}(\mathcal{M})$ is Neu_{Cgsg^*}-closed in $(\mathcal{H},\mathcal{L})$. Since $\not{f}_1$ is Neu_{Cgsg^*}-closed, the preimage $\not{f}_1^{-1}(\not{f}_2^{-1}(\mathcal{M})) = (\not{f}_2 \circ \not{f}_1)^{-1}(\mathcal{M})$ is Neu_{Cgsg^*}-closed in $(\mathcal{G},\mathcal{T})$. Therefore, $\not{f}_2 \circ \not{f}_1$ is Neu_{Cgsg^*}-closed.

(vi) Let $\mathcal{M} \subseteq \mathcal{I}$ be a Neu_{Cgsg^*}-closed set in $(\mathcal{I},\sigma)$. Since $\not{f}_2$ is $Neu_{Cgsg^{**}}$-closed, $\not{f}_2^{-1}(\mathcal{M})$ is Neu_{Cgsg^*}-closed in $(\mathcal{H},\mathcal{L})$. Then, since $\not{f}_1$ is Neu_C-closed, the preimage $\not{f}_1^{-1}(\not{f}_2^{-1}(\mathcal{M})) = (\not{f}_2 \circ \not{f}_1)^{-1}(\mathcal{M})$ is Neu_{Cgsg^*}-closed in $(\mathcal{G},\mathcal{T})$. Thus, $\not{f}_2 \circ \not{f}_1$ is Neu_{Cgsg^*}-closed. ■

Theorem 3.15:

Let $\not{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{H},\mathcal{L})$ and $\not{f}_2: (\mathcal{H},\mathcal{L}) \longrightarrow (\mathcal{I},\sigma)$ be two functions such that $\not{f}_2 \circ \not{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{I},\sigma)$ is Neu_{Cgsg}-closed. If $\not{f}_1$ is Neu_C-continuous surjection, then $\not{f}_2$ is a Neu_{Cgsg}-closed.

Proof:

Suppose that $\mathcal{M}$ is an arbitrary Neu_CCS in $(\mathcal{H},\mathcal{L})$. As $\not{f}_1$ is Neu_C-continuous, ${\not{f}_1}^{-1}(\mathcal{M})$ is Neu_CCS in $(\mathcal{G},\mathcal{T})$. Since $\not{f}_2 \circ \not{f}_1$ is Neu_{Cgsg}-closed function and $\not{f}_1$ is surjective $\not{f}_2 \circ \not{f}_1({\not{f}_1}^{-1}(\mathcal{M})) = \not{f}_2(\mathcal{M})$, which is Neu_{Cgsg}CS in $(\mathcal{I},\sigma)$. This implies that $\not{f}_2$ is a Neu_{Cgsg}-closed function. ▪

Theorem 3.16:

Let $\not{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{H},\mathcal{L})$ and $\not{f}_2: (\mathcal{H},\mathcal{L}) \longrightarrow (\mathcal{I},\sigma)$ be two functions such that $\not{f}_2 \circ \not{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{I},\sigma)$ is Neu_{Cgsg}-closed. If $\not{f}_2$ is $Neu_{Cgsg^{**}}$-continuous injective, then $\not{f}_1$ is a Neu_{Cgsg}-closed.

Proof:

Suppose $\mathcal{M}$ is any Neu_CCS in $(\mathcal{G},\mathcal{T})$. Since $\mathfrak{f}_2 \circ \mathfrak{f}_1$ is Neu_{Cgsg}-closed, $\mathfrak{f}_2 \circ \mathfrak{f}_1(\mathcal{M})$ is Neu_CCS in $(\mathcal{I},\sigma)$ and $\mathfrak{f}_2$ is $Neu_{Cgsg^{**}}$-continuous injective, then ${\mathfrak{f}_2}^{-1}(\mathfrak{f}_2 \circ \mathfrak{f}_1(\mathcal{M})) = \mathfrak{f}_1(\mathcal{M})$, which is Neu_{Cgsg}CS in $(\mathcal{H},\mathcal{L})$. Therefore $\mathfrak{f}_1$ is a Neu_{Cgsg}-closed function. ▪

Theorem 3.17:

Let $\mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{H},\mathcal{L})$ and $\mathfrak{f}_2: (\mathcal{H},\mathcal{L}) \longrightarrow (\mathcal{I},\sigma)$ be two functions such that $\mathfrak{f}_2 \circ \mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{I},\sigma)$ is Neu_{Cgsg^*}-closed. If $\mathfrak{f}_1$ is Neu_{Cgsg^*}-continuous surjection, then $\mathfrak{f}_2$ is a Neu_C-closed.

Proof:

Suppose that $\mathcal{M}$ is an arbitrary Neu_CCS in $(\mathcal{H},\mathcal{L})$. As $\mathfrak{f}_1$ is Neu_{Cgsg^*}-continuous, ${\mathfrak{f}_1}^{-1}(\mathcal{M})$ is Neu_{Cgsg}CS in $(\mathcal{G},\mathcal{T})$. Since $\mathfrak{f}_2 \circ \mathfrak{f}_1$ is Neu_{Cgsg^*}-closed function and $\mathfrak{f}_1$ is surjective, $\mathfrak{f}_2 \circ \mathfrak{f}_1({\mathfrak{f}_1}^{-1}(\mathcal{M})) = \mathfrak{f}_2(\mathcal{M})$, which is Neu_CCS in $(\mathcal{I},\sigma)$. This implies that $\mathfrak{f}_2$ is a Neu_C-closed function.▪

Theorem 3.18:

Let $\mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{H},\mathcal{L})$ and $\mathfrak{f}_2: (\mathcal{H},\mathcal{L}) \longrightarrow (\mathcal{I},\sigma)$ be two functions such that $\mathfrak{f}_2 \circ \mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{I},\sigma)$ is Neu_{Cgsg^*}-closed. If $\mathfrak{f}_2$ is Neu_{Cgsg^*}-continuous injective, then $\mathfrak{f}_1$ is a Neu_{Cgsg^*}-closed.

Proof:

Suppose $\mathcal{M}$ is any Neu_{Cgsg}CS in $(\mathcal{G},\mathcal{T})$. Since $\mathfrak{f}_2 \circ \mathfrak{f}_1$ is Neu_{Cgsg^*}-closed, $\mathfrak{f}_2 \circ \mathfrak{f}_1(\mathcal{M})$ is Neu_CCS in $(\mathcal{I},\sigma)$, since every Neu_CCS is Neu_{Cgsg}CS and $\mathfrak{f}_2$ is Neu_{Cgsg^*}-continuous injective, then ${\mathfrak{f}_2}^{-1}(\mathfrak{f}_2 \circ \mathfrak{f}_1(\mathcal{M})) = \mathfrak{f}_1(\mathcal{M})$, which is Neu_CCS in $(\mathcal{H},\mathcal{L})$. Therefore $\mathfrak{f}_1$ is a Neu_{Cgsg^*}-closed function. ▪

Theorem 3.19:

Let $\mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{H},\mathcal{L})$ and $\mathfrak{f}_2: (\mathcal{H},\mathcal{L}) \longrightarrow (\mathcal{I},\sigma)$ be two functions such that $\mathfrak{f}_2 \circ \mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{I},\sigma)$ is $Neu_{Cgsg^{**}}$-closed function. If $\mathfrak{f}_1$ is $Neu_{Cgsg^{**}}$-continuous surjection, then $\mathfrak{f}_2$ is a $Neu_{Cgsg^{**}}$-closed.

Proof:

Suppose that $\mathcal{M}$ is Neu_{Cgsg}CS in $(\mathcal{H},\mathcal{L})$. As $\mathfrak{f}_1$ is $Neu_{Cgsg^{**}}$-continuous, ${\mathfrak{f}_1}^{-1}(\mathcal{M})$ is Neu_{Cgsg}CS in $(\mathcal{G},\mathcal{T})$. Since $\mathfrak{f}_2 \circ \mathfrak{f}_1$ is $Neu_{Cgsg^{**}}$-closed function and $\mathfrak{f}_1$ is surjective, $\mathfrak{f}_2 \circ \mathfrak{f}_1({\mathfrak{f}_1}^{-1}(\mathcal{M})) = \mathfrak{f}_2(\mathcal{M})$, which is Neu_{Cgsg}CS in $(\mathcal{I},\sigma)$. This implies that $\mathfrak{f}_2$ is a $Neu_{Cgsg^{**}}$-closed function. ▪

Theorem 3.20:

Let $\mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{H},\mathcal{L})$ and $\mathfrak{f}_2: (\mathcal{H},\mathcal{L}) \longrightarrow (\mathcal{I},\sigma)$ be two functions such that $\mathfrak{f}_2 \circ \mathfrak{f}_1: (\mathcal{G},\mathcal{T}) \longrightarrow (\mathcal{I},\sigma)$ is $Neu_{Cgsg^{**}}$-closed function. If $\mathfrak{f}_2$ is $Neu_{Cgsg^{**}}$-continuous injective, then $\mathfrak{f}_1$ is a $Neu_{Cgsg^{**}}$-closed.

Proof:

Suppose $\mathcal{M}$ is any Neu_{Cgsg}CS in $(\mathcal{G},\mathcal{T})$. Since $\mathfrak{f}_2 \circ \mathfrak{f}_1$ is $Neu_{Cgsg^{**}}$-closed, $\mathfrak{f}_2 \circ \mathfrak{f}_1(\mathcal{M})$ is Neu_{Cgsg}CS in $(\mathcal{I},\sigma)$ and $\mathfrak{f}_2$ is $Neu_{Cgsg^{**}}$-continuous injective, then ${\mathfrak{f}_2}^{-1}(\mathfrak{f}_2 \circ \mathfrak{f}_1(\mathcal{M})) = \mathfrak{f}_1(\mathcal{M})$, which is Neu_{Cgsg}CS in $(\mathcal{H},\mathcal{L})$. This shows that $\mathfrak{f}_1$ is a $Neu_{Cgsg^{**}}$-closed function. ▪

Remark 3.21:

The subsequent diagram exposes the relationship occupying the various kinds of Neu_C-closed functions:

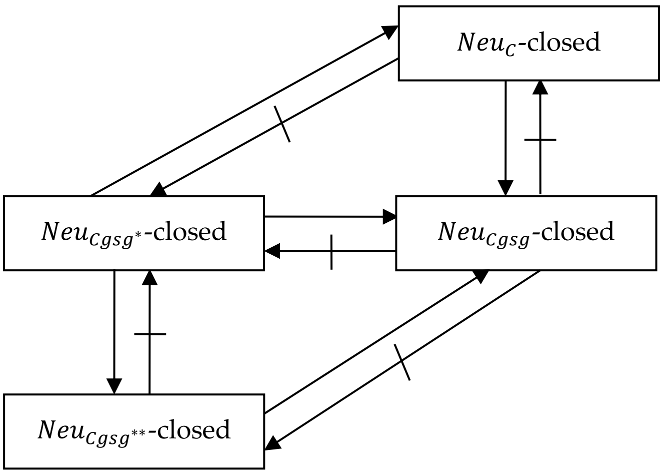

Fig. 3.1

4. Conclusion

This paper included three newfound concepts of neutrosophic crisp gsg-closed functions via neutrosophic crisp gsg-closed sets such as Neu_{Cgsg}-closed, Neu_{Cgsg^*}-closed, and $Neu_{Cgsg^{**}}$-closed functions. Furthermore, it proved certain further thoughts that are required for this research. Consequently, it examined specific of these functions descriptions and illuminated how they attach to other sorts of functions for case Neu_C-continuous surjection, $Neu_{Cgsg^{**}}$-continuous injective, Neu_{Cgsg^*}-continuous surjection, Neu_{Cgsg^*}-continuous injective, $Neu_{Cgsg^{**}}$-continuous surjection. It demonstrated the well depiction of Neu_{Cgsg}-closed functions by employing Neu_C-continuous surjection. In the upcoming, we lie ahead that several further investigations will be capable of being performed in the employing these ideas from Neu_{Cgsg}-closed functions.

The significance lies in the potential applications of generalized closed function concepts in various domains requiring the handling of uncertainty and indeterminacy, such as decision-making processes, artificial intelligence, and fuzzy systems modeling. Through the precise classifications offered by this study, it is possible to improve the understanding and description of phenomena involving high levels of ambiguity or inconsistency.

For future study, promising directions include studying the interactions of g_α-, g_β-, and g_γ-closed functions with other types of generalized open and closed sets, or extending these ideas to cover compactness and connectedness in neutrosophic crisp topologies [12,13]. Additionally, developing algorithmic approaches to detect and classify these functions in computational neutrosophic frameworks could open new avenues for applications in software and applied mathematics [14,15].

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References

[1] A. A. Salama, F. Smarandache and V. Kroumov, Neutrosophic crisp sets and neutrosophic crisp topological spaces. Neutrosophic Sets and Systems, 2(2014), 25-30.

[2] A. A. Salama, Basic structure of some classes of neutrosophic crisp nearly open sets & possible application to GIS topology. Neutrosophic Sets and Systems, 7(2015), 18-22.

[3] R. K. Al-Hamido, Q. H. Imran, K. A. Alghurabi and T. Gharibah, On neutrosophic crisp semi-α-closed sets. Neutrosophic Sets and Systems, 21(2018), 28-35.

[4] A. H. M. Al-Obaidi and Q. H. Imran, On new types of weakly neutrosophic crisp open mappings. Iraqi Journal of Science, 62(8) (2021), 2660-2666.

[5] A. H. M. Al-Obaidi, Q. H. Imran and M. M. Abdulkadhim, On new types of weakly neutrosophic crisp closed functions. Neutrosophic Sets and Systems, 50(2022), 239-247.

[6] Q. H. Imran, R. K. Al-Hamido and A. H. M. Al-Obaidi, On new types of weakly neutrosophic crisp continuity. Neutrosophic Sets and Systems, 38(2020), 179-187.

[7] Q. H. Imran, K. S. Tanak and A. H. M. Al-Obaidi, On new concepts of neutrosophic crisp open sets. Journal of Interdisciplinary Mathematics, 25(2)(2022), 563-572.

[8] Q. H. Imran, A. H. M. Al-Obaidi, F. Smarandache and Md. Hanif PAGE, On some new concepts of weakly neutrosophic crisp separation axioms. Neutrosophic Sets and Systems, 51(2022), 330-343.

[9] Q. H. Imran, M. M. Abdulkadhim, A. H. M. Al-Obaidi and Said Broumi, Neutrosophic crisp generalized sg-closed sets and their continuity. International Journal of Neutrosophic Science, 20(4) (2023), 106-118.

[10] M. M. Abdulkadhim, Q. H. Imran, A. K. Abed and Said Broumi, On neutrosophic generalized alpha generalized separation axioms. International Journal of Neutrosophic Science, 19(1) (2022), 99-106.

[11] M. M. Abdulkadhim, Q. H. Imran, A. H. M. Al-Obaidi and Said Broumi, On neutrosophic crisp generalized alpha generalized closed sets. International Journal of Neutrosophic Science, 19(1) (2022), 107-115.

[12] T. A. Al-Tamimi, L. A. A. Al-Swidi, and A. H. M. Al-Obaidi, Partner Sets for Generalizations of MultiNeutrosophic Sets. International Journal of Neutrosophic Science, 24(1) (2024),08-13.

[13] D. Nihad, and L. A. A. Al-Swidi, Necessary and Sufficient Conditions for a Stability of the Concepts of Stable Interior and Stable Exterior via Neutrosophic Crisp Sets. International Journal of Neutrosophic Science, 24(1) (2024), 87-93.

[14] D. A. Abdulsada, and L. A. A. Al-Swidi, Some Properties of $\mathcal{C}$-Topological Space, 2019 First International Conference of Computer and Applied Sciences (CAS). IEEE, (2019), 52-56.

[15] L. A. A. Al-Swidi, and F. S. Awad, Analysis on the soft Bench points, 2018 International Conference on Advanced Science and Engineering (ICOASE). IEEE, (2018), 330-335.

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