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New Classes of δ-Continuous Mappings in Neutrosophic Topological Spaces

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Abstract. We introduce and study two classes of neutrosophic continuous mappings: the neutrosophic irresolute δ-continuous mappings ($NIr\ \delta{-}CM$) and the neutrosophic contra δ-continuous mappings ($NC\ \delta{-}CM$). We establish their fundamental properties and provide characterizations in terms of preimages of δ-open and δ –closed sets. The interplay between the two notions is analyzed through implication chains, (non-)equivalences under mild hypotheses, and stability results under composition, subspaces, and products. We then extend the framework to the setting by defininneutrosophic irresolute δ-continuous mappings ($NIr\ \delta{-}CM$) and neutrosophic contra δ-continuous mappings ($NC\ \delta{-}CM$), showing how core properties lift to the case and where genuinely new phenomenaarise. Throughout, examples and counterexamples are provided to separate the classes and to illustrate the sharpness of the obtained results.

Keywords: Contra δ –mappings, Irresolute δ –mappings, Generalized continuousneutrosophic mappings.

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1. Introduction

The concept of *intuitionistic fuzzy sets*, introduced by Atanassov [13] as a generalization of the classical fuzzy sets of Zadeh [36], opened the way to several extensions and refinements in the study of uncertainty. Building on these foundations, Smarandache [31] proposed in 1999 the theory of *neutrosophic sets*, which explicitly model truth, falsity and indeterminacy, and rapidly attracted the attention of many researchers worldwide.

Neutrosophic topology emerged soon after: Salama and Alblowi [29] introduced neutrosophic topological spaces and explored their basic properties, while Salama et al. [30] investigated continuous mappings in this new framework. Further applications and developments can be found for instance in [6, 11, 12]. The algebraic side of the theory was first outlined by Kandasamy and Smarandache [24], and later enriched by Abed et al. [1, 2] through the interplay between neutrosophic structures and module theory. Complex-valued neutrosophic sets were then studied and applied to real-world problems in [10, 27].

Topological investigations have also played a central role. Hatir and Noiri [17] examined the notion of $\delta-\beta$-open sets, a novel type of generalized open sets providing a fertile ground for new separation axioms and continuity concepts. Al-Jumaili et al. [7, 23] analyzed new families of mappings with strongly closed networks in classical topological spaces, while Ishwarya and Bageerathi [21] focused on neutrosophic semi-open sets. Other important contributions on weakly neutrosophic crisp continuity and generalized separation axioms were given by Imran et al. [18–20], Abdulkadhim et al. [3, 4], and Al-Obaidi et al. [8], together with further recent studies such as [5, 16, 32–35].

In a parallel line of research, Chen et al. [14] introduced the notion of *fuzzy sets* (FS), capable of handling multi-criteria and multi-source information. To address simultaneously the multi-attribute setting and the intrinsic indeterminacy of neutrosophic data, the concept of *m-polar neutrosophic sets* (NS) has been proposed: it extends the bipolar neutrosophic set and allows each of the m attributes to carry independent degrees of truth, falsity and indeterminacy. On NSs one can define suitable operations and scoring functions for comparing neutrosophic values.

Recent works have further expanded the neutrosophic landscape. Nordo and collaborators, for instance, contributed to the development of soft and neutrosophic sets [25], to the design of computational tools for neutrosophic topologies in Python [26], to implication-based neutrosophic finite state machines [22], and to new classes of neutrosophic crisp generalized closed functions [9], showing the breadth of current applications and computational perspectives.

Within this rich framework, the present paper introduces two new classes of generalized continuous neutrosophic mappings: the *neutrosophic irresolute $\delta-\beta^*$-continuous mappings* (NIr $\delta-\beta^*$ CM) and the *neutrosophic contra $\delta-\beta^*$-continuous mappings* (NC $\delta-\beta^*$ CM). We investigate their fundamental properties, analyze the relationships between these two notions, and finally extend the study to the setting by defining m-polar neutrosophic irresolute and contra δ–β^*-continuous mappings, highlighting the specific features that arise in the multi-attribute context.

2. Preliminaries

In this section we recall the basic notions on neutrosophic sets and neutrosophic topologies that will be used throughout the paper, together with their counterparts.

Definition 2.1 ([29]). Let $\mathfrak{V}$ be a nonempty set. A *neutrosophic set* (NS) $\mathcal{C}$ on $\mathfrak{V}$ is a set of ordered quadruples

$$\mathcal{C} = \left\{ \left\langle \mathcal{J},\, \mu_{\mathcal{C}}(\mathcal{J}),\, \sigma_{\mathcal{C}}(\mathcal{J}),\, v_{\mathcal{C}}(\mathcal{J}) \right\rangle : \mathcal{J} \in \mathfrak{V} \right\},$$

where, for every $\mathcal{J} \in \mathfrak{V}$, the functions $\mu_{\mathcal{C}}$, $\sigma_{\mathcal{C}}$, and $v_{\mathcal{C}}$ assign, respectively, the degrees of membership, indeterminacy, and non-membership in $[0,1]$.

Definition 2.2 ([29]). Let $\mathcal{H} = \{\langle \mathcal{J}, \mu_{\mathcal{H}}(\mathcal{J}), \sigma_{\mathcal{H}}(\mathcal{J}), v_{\mathcal{H}}(\mathcal{J})\rangle : \mathcal{J} \in \mathfrak{V}\}$ and $\mathcal{K} = \{\langle \mathcal{J}, \mu_{\mathcal{K}}(\mathcal{J}), \sigma_{\mathcal{K}}(\mathcal{J}), v_{\mathcal{K}}(\mathcal{J})\rangle : \mathcal{J} \in \mathfrak{V}\}$. Then:

(1) $0_{\mathcal{N}} = \{\langle \mathcal{J}, 0, 0, 1\rangle : \mathcal{J} \in \mathfrak{V}\}$ and $1_{\mathcal{N}} = \{\langle \mathcal{J}, 1, 0, 0\rangle : \mathcal{J} \in \mathfrak{V}\}$;

(2) $\mathcal{H} \subseteq \mathcal{K}$ iff, for all $\mathcal{J} \in \mathfrak{V}$, $\mu_{\mathcal{H}}(\mathcal{J}) \le \mu_{\mathcal{K}}(\mathcal{J})$, $\sigma_{\mathcal{H}}(\mathcal{J}) \le \sigma_{\mathcal{K}}(\mathcal{J})$, and $v_{\mathcal{H}}(\mathcal{J}) \ge v_{\mathcal{K}}(\mathcal{J})$;

(3) $\mathcal{H} = \mathcal{K}$ iff $\mathcal{H} \subseteq \mathcal{K}$ and $\mathcal{K} \subseteq \mathcal{H}$;

(4) $\mathcal{H}^c = \{\langle \mathcal{J},\, v_{\mathcal{H}}(\mathcal{J}),\, 1 - \sigma_{\mathcal{H}}(\mathcal{J}),\, \mu_{\mathcal{H}}(\mathcal{J})\rangle : \mathcal{J} \in \mathfrak{V}\}$;

(5) $\mathcal{H} \cup \mathcal{K} = \{\langle \mathcal{J},\, \mu_{\mathcal{H}}(\mathcal{J}) \vee \mu_{\mathcal{K}}(\mathcal{J}),\, \sigma_{\mathcal{H}}(\mathcal{J}) \vee \sigma_{\mathcal{K}}(\mathcal{J}),\, v_{\mathcal{H}}(\mathcal{J}) \wedge v_{\mathcal{K}}(\mathcal{J})\rangle : \mathcal{J} \in \mathfrak{V}\}$.

Definition 2.3 ([29]). A *neutrosophic topology* (NT) on a nonempty set $\mathcal{C}$ is a collection τ_N of neutrosophic sets on $\mathcal{C}$ such that:

(i) $0_N, 1_N \in \tau_N$;

(ii) if $\mathcal{K}_1, \mathcal{K}_2 \in \tau_N$, then $\mathcal{K}_1 \cap \mathcal{K}_2 \in \tau_N$;

(iii) if $\{\mathcal{K}_j\}_{j\in I} \subseteq \tau_N$, then $\bigcup_{j\in I} \mathcal{K}_j \in \tau_N$.

Members of τ_N are called *neutrosophic open sets* (NOS). A neutrosophic set $\mathcal{K}$ is *neutrosophic closed* (NCS) iff its complement $\mathcal{K}^c$ is NOS. The pair $(\mathcal{C}, \tau_N)$ is a *neutrosophic topological space* (NTS).

Definition 2.4 ([29]). Let $\mathcal{H}$ be an NS on the NTS $(\mathcal{C}, \tau_N)$. The *neutrosophic closure* and *neutrosophic interior* of $\mathcal{H}$ are

$$NCl(\mathcal{H}) = \bigcap\{\mathcal{D} :\ \mathcal{D} \text{ is NCS in } \mathcal{C} \text{ and } \mathcal{H} \subseteq \mathcal{D}\},$$

$$NInt(\mathcal{H}) = \bigcup\{\mathcal{P} :\ \mathcal{P} \text{ is NOS in } \mathcal{C} \text{ and } \mathcal{P} \subseteq \mathcal{H}\}.$$

They satisfy $NCl(\mathcal{H}^c) = (NInt(\mathcal{H}))^c$ and $NInt(\mathcal{H}^c) = (NCl(\mathcal{H}))^c$.

Proposition 2.5 ([15]). *Let $(\mathcal{C}, \tau_N)$ be an NTS. Then:*

(1) *every NOS is an $N\beta$-open set;*

(2) *every $N\alpha$-open set is $N\beta$-open.*

Definition 2.6 ([34]). Let $(\mathcal{C}, \tau_N)$ be an NTS. A neutrosophic set $\mathcal{H}$ is *neutrosophic δ-open* (briefly, $N\delta$OS) if

$$\mathcal{H} = N\delta Int(\mathcal{H}),$$

where $N\delta Int$ denotes the δ-interior operator associated with the $N\delta$-open sets.

Definition 2.7 ([28]). Let $\mathfrak{G}$ be a nonempty reference set and $m \in \mathbb{N}$. An *m-polar neutrosophic set* (MPNS) on $\mathfrak{G}$ is

$$\mathbb{M}_{\mathcal{G}} = \Big\{ \big(\mathcal{J},\ \mu_\alpha(\mathcal{J}),\ \sigma_\alpha(\mathcal{J}),\ v_\alpha(\mathcal{J})\big) : \mathcal{J} \in \mathfrak{G},\ \alpha = 1, 2, \ldots, m \Big\},$$

with $\mu_\alpha, \sigma_\alpha, v_\alpha : \mathfrak{G} \to [0,1]$ and $0 < \mu_\alpha(\mathcal{J}) + \sigma_\alpha(\mathcal{J}) + v_\alpha(\mathcal{J}) \leq 3$ for each α and $\mathcal{J}$. The triple $\mathfrak{h}_\alpha(\mathcal{J}) = (\mu_\alpha(\mathcal{J}), \sigma_\alpha(\mathcal{J}), v_\alpha(\mathcal{J}))$ is an *m-polar neutrosophic numeral* (NN).

Definition 2.8 ([28]). Let $\text{mpn}({}^1\mathbb{M}_{\mathcal{G}})$ denote the family of all NSs over $\mathfrak{G}$. A subfamily $T_{\mathbb{M}_{\mathcal{G}}} \subseteq \text{mpn}({}^1\mathbb{M}_{\mathcal{G}})$ is an *neutrosophic topology* on $\mathfrak{G}$ if:

(i) ${}^0\mathbb{M}_{\mathcal{G}}, {}^1\mathbb{M}_{\mathcal{G}} \in T_{\mathbb{M}_{\mathcal{G}}}$ (the empty and whole NS);

(ii) if $\{(MN)_\gamma\}_{\gamma \in \Delta} \subseteq T_{\mathbb{M}_{\mathcal{G}}}$, then $\bigcup_{\gamma \in \Delta} (MN)_\gamma \in T_{\mathbb{M}_{\mathcal{G}}}$;

(iii) if $\mathbb{M}_{\mathcal{G}_1}, \mathbb{M}_{\mathcal{G}_2} \in T_{\mathbb{M}_{\mathcal{G}}}$, then $\mathbb{M}_{\mathcal{G}_1} \cap \mathbb{M}_{\mathcal{G}_2} \in T_{\mathbb{M}_{\mathcal{G}}}$.

Then $(\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}})$ is an *m-neutrosophic topological space* (MNTS). Members of $T_{\mathbb{M}_{\mathcal{G}}}$ are open MNSs; their complements are closed MNSs.

Example 2.9. Let $\mathfrak{G} = \{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, \mathcal{J}_4\}$ (e.g., four books) and consider two 4-NSs:

$$\begin{aligned}
\mathbb{M}_{\mathcal{G}_1} = \Big\{ & \big(\mathcal{J}_1, \langle 0.853, 0.423, 0.401\rangle, \langle 0.328, 0.416, 0.311\rangle, \langle 0.191, 0.222, 0.111\rangle\big), \\
& \big(\mathcal{J}_2, \langle 0.551, 0.147, 0.411\rangle, \langle 0.523, 0.149, 0.107\rangle, \langle 0.624, 0.503, 0.331\rangle\big), \\
& \big(\mathcal{J}_3, \langle 0.631, 0.352, 0.242\rangle, \langle 0.826, 0.123, 0.532\rangle, \langle 0.522, 0.442, 0.209\rangle\big), \\
& \big(\mathcal{J}_4, \langle 0.522, 0.388, 0.805\rangle, \langle 0.827, 0.333, 0.407\rangle, \langle 0.909, 0.319, 0.509\rangle\big) \Big\},
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{M}_{\mathcal{G}_2} = \Big\{ & \big(\mathcal{J}_1, \langle 0.623, 0.499, 0.623\rangle, \langle 0.301, 0.545, 0.428\rangle, \langle 0.147, 0.402, 0.304\rangle\big), \\
& \big(\mathcal{J}_2, \langle 0.246, 0.209, 0.521\rangle, \langle 0.447, 0.226, 0.308\rangle, \langle 0.542, 0.524, 0.447\rangle\big), \\
& \big(\mathcal{J}_3, \langle 0.435, 0.483, 0.334\rangle, \langle 0.651, 0.229, 0.363\rangle, \langle 0.304, 0.593, 0.327\rangle\big), \\
& \big(\mathcal{J}_4, \langle 0.228, 0.501, 0.793\rangle, \langle 0.734, 0.513, 0.529\rangle, \langle 0.742, 0.442, 0.626\rangle\big) \Big\}.
\end{aligned}$$

Then $T_{\mathbb{M}_{\mathcal{G}}} = \{{}^0\mathbb{M}_{\mathcal{G}}, {}^1\mathbb{M}_{\mathcal{G}}, \mathbb{M}_{\mathcal{G}_1}, \mathbb{M}_{\mathcal{G}_2}\}$ is a 4-neutrosophic topology on $\mathfrak{G}$.

Theorem 2.10 ([28])**.** *Let $(\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}})$ be an MNTS. Then:*

(i) ${}^{0}\mathbb{M}_{\mathcal{G}}$ *and* ${}^{1}\mathbb{M}_{\mathcal{G}}$ *are closed MNSs;*

(ii) *arbitrary unions of open MNSs are open;*

(iii) *finite intersections of closed MNSs are closed.*

Definition 2.11 ([28])**.** Let $\mathbb{M}_{\mathcal{G}} \in \text{mpn}({}^{1}\mathbb{M}_{\mathcal{G}})$ and $(\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}})$ be an MNTS. Then:

(i) the *interior* of $\mathbb{M}_{\mathcal{G}}$, denoted $\mathbb{M}_{\mathcal{G}}{}^{\circ}$, is the union of all open MNSs contained in $\mathbb{M}_{\mathcal{G}}$; it is the largest open MNS contained in $\mathbb{M}_{\mathcal{G}}$;

(ii) the *closure* of $\mathbb{M}_{\mathcal{G}}$, denoted $\overline{\mathbb{M}_{\mathcal{G}}}$, is the intersection of all closed MNSs containing $\mathbb{M}_{\mathcal{G}}$; it is the least closed MNS containing $\mathbb{M}_{\mathcal{G}}$.

3. Main results and extensions

We introduce two notions of neutrosophic δ-continuity: the *neutrosophic irresolute δ—-continuous mappings* ($NIr\,\delta$–CM) and the *neutrosophic contra δ-continuousmappings* ($NC\,\delta-\beta^*$ CM). We establish basic properties, relationships, and stability under composition, and we indicate the m-counterparts.

Definition 3.1. Let $(\mathcal{C}, \tau_N)$ and $(\mathcal{O}, \varrho_N)$ be NTSs and $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ a mapping. We say that h is *neutrosophic irresolute δ-continuous* ($NIr\,\delta$–β^* CM) if for every neutrosophicδ -open set $\mathcal{H}$ in $(\mathcal{O}, \varrho_N)$, the preimage $h^{-1}(\mathcal{H})$ is δ-open in $(\mathcal{C}, \tau_N)$.

Example 3.2. Let $\mathcal{C} = \{\mathfrak{p}, \mathfrak{q}\}$ and $\mathcal{O} = \{\mathfrak{s}, \mathfrak{r}\}$. Let $\tau_N = \{0_N, 1_N, \mathcal{Q}, \mathcal{H}\}$ and $\varrho_N = \{0_N, 1_N, \mathcal{P}, \mathcal{K}\}$ be neutrosophic topologies on $\mathcal{C}$ and $\mathcal{O}$, respectively, where

$$\mathcal{Q} = \langle \mathfrak{p}; (0.3, 0.4, 0.3), (0.2, 0.2, 0.2), (0.7, 0.8, 0.7)\rangle,$$

$$\mathcal{H} = \langle \mathfrak{q}; (0.3, 0.5, 0.3), (0.3, 0.5, 0.3), (0.7, 0.7, 0.7)\rangle,$$

$$\mathcal{P} = \langle \mathfrak{s}; (0.6, 0.6, 0.6), (0.5, 0.5, 0.5), (0.4, 0.4, 0.4)\rangle,$$

$$\mathcal{K} = \langle \mathfrak{r}; (0.2, 0.4, 0.3), (0.4, 0.6, 0.5), (0.7, 0.7, 0.6)\rangle.$$

Define $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ by $h(\mathfrak{p}) = \mathfrak{s}$ and $h(\mathfrak{q}) = \mathfrak{r}$. If $\mathcal{W}$ is δ-open in $\mathcal{O}$, then $h^{-1}(\mathcal{W})$is δ-open in $\mathcal{C}$ (for instance via $NCl(NInt(NCl(h^{-1}(\mathcal{P})))) = 1_N$ when $h^{-1}(\mathcal{P}) \subseteq \mathcal{H}$). Hence h is $NIr\,\delta-\beta^*$ CM.

Theorem 3.3. *Every NIr-CM is $NIr\,\delta$–CM.*

Proof. Let $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ be $NIr\,\beta^*$ CM. If $\mathcal{P}$ is δ-open in $\mathcal{O}$, then it is in particularβ_*-open. By irresoluteness, $h^{-1}(\mathcal{P})$ is-open in $\mathcal{C}$; since δ-openness is weaker, $h^{-1}(\mathcal{P})$ is δ-open as well. Hence h is $NIr\,\delta$–CM. □

Remark 3.4. The converse of Theorem 3.3 does not hold in general (see Example 3.5).

Example 3.5. In the setting of Example 3.2, one may arrange ϱ_N so that some-open set in $\mathcal{O}$ fails to be δ-open; then h is $NIr\,\delta$CM but not NIrCM, because the preimage of that-open set need not beopen in $\mathcal{C}$.

Theorem 3.6. *If $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ and $\mathcal{S} : (\mathcal{O}, \varrho_N) \to (\mathcal{G}, \rho_N)$ are both NIrCM, then $\mathcal{S} \circ h$ is $NIr\,\delta$–CM.*

Proof. Let $\mathcal{H}$ be δ-open in $(\mathcal{G}, \rho_N)$. Since $\mathcal{S}$ is NIrCM, $\mathcal{S}^{-1}(\mathcal{H})$ is-open in $(\mathcal{O}, \varrho_N)$;applying h, we get $h^{-1}(\mathcal{S}^{-1}(\mathcal{H}))$-open in $(\mathcal{C}, \tau_N)$. Therefore it is δ-open in $(\mathcal{C}, \tau_N)$,showing that $\mathcal{S} \circ h$ is $NIr\,\delta$–CM. □

Theorem 3.7. *Let $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ be $NIr\,\delta - \beta^*$ CM and $\mathcal{S} : (\mathcal{O}, \varrho_N) \to (\mathcal{G}, \rho_N)$ be $NIr\,\delta$–CM. Then $\mathcal{S} \circ h$ is $NIr\,\delta$–CM.*

Proof. If $\mathcal{H}$ is δ-open in $(\mathcal{G}, \rho_N)$, then $\mathcal{S}^{-1}(\mathcal{H})$ is δ-open in $(\mathcal{O}, \varrho_N)$ and $h^{-1}(\mathcal{S}^{-1}(\mathcal{H}))$is δ-open in $(\mathcal{C}, \tau_N)$. Hence $\mathcal{S} \circ h$ is $NIr\,\delta$–CM. □

Definition 3.8. Let $(\mathcal{C}, \tau_N)$ and $(\mathcal{O}, \varrho_N)$ be NTSs. A mapping $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ is *neutrosophic contra δ-continuous* ($NC\,\delta$–CM) if for every δ-open set $\mathcal{H}$ in $(\mathcal{O}, \varrho_N)$ the preimage $h^{-1}(\mathcal{H})$ is δ-closed in $(\mathcal{C}, \tau_N)$.

Remark 3.9. Every NCCM is $NC\,\delta$–CM. The converse is, in general, false (see Example 3.10).

Example 3.10. Let $\mathcal{C} = \{\mathfrak{p}, \mathfrak{q}, \mathfrak{g}\}$ and $\mathcal{O} = \{\mathfrak{s}, \mathfrak{r}\}$. Consider neutrosophic topologies $\tau_N = \{0_N, 1_N, \mathcal{Q}, \mathcal{H}, \mathcal{F}\}$ on $\mathcal{C}$ and $\varrho_N = \{0_N, 1_N, \mathcal{P}, \mathcal{K}\}$ on $\mathcal{O}$, where (triples are $\langle \mu, \sigma, v \rangle$)

$$\mathcal{Q} = \langle \mathfrak{s}; (0.3, 0.5, 0.3), (0.5, 0.5, 0.5), (0.3, 0.4, 0.3) \rangle,$$

$$\mathcal{H} = \langle \mathfrak{r}; (0.6, 0.4, 0.4), (0.4, 0.5, 0.4), (0.6, 0.5, 0.4) \rangle,$$

$$\mathcal{F} = \langle \mathfrak{g}; (0.8, 0.3, 0.5), (0.5, 0.4, 0.5), (0.7, 0.2, 0.3) \rangle,$$

$$\mathcal{P} = \langle \mathfrak{g}; (0.3, 0.4, 0.3), (0.5, 0.4, 0.4), (0.3, 0.7, 0.5) \rangle,$$

$$\mathcal{K} = \langle \mathfrak{g}; (0.6, 0.4, 0.3), (0.3, 0.4, 0.5), (0.5, 0.5, 0.4) \rangle.$$

Define $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ by $h(\mathfrak{p}) = \mathfrak{s} = h(\mathfrak{g})$ and $h(\mathfrak{q}) = \mathfrak{r}$. If $\mathcal{W}$ is δ-open in $\mathcal{O}$, one checks that $h^{-1}(\mathcal{W})$ is δ-closed in $\mathcal{C}$ (for instance $NCl(NInt(NCl(h^{-1}(\mathcal{P})))) = 1_N \subseteq \mathcal{Q}$), hence h is $NC\,\delta$–CM.

Theorem 3.11. *If $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ and $\mathcal{S} : (\mathcal{O}, \varrho_N) \to (\mathcal{G}, \rho_N)$ are both NCCM, then $\mathcal{S} \circ h$ is $NC\,\delta$–CM.*

Proof. Let $\mathcal{H}$ be δ-open in $(\mathcal{G}, \rho_N)$. Then $\mathcal{S}^{-1}(\mathcal{H})$ is-open in $(\mathcal{O}, \varrho_N)$ and, since h isNC CM, $h^{-1}(\mathcal{S}^{-1}(\mathcal{H}))$ is-closed in $(\mathcal{C}, \tau_N)$, hence δ-closed. Thus $\mathcal{S} \circ h$ is $NC\,\delta$–CM.

□

Theorem 3.12. *Let $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ be $NC\,\delta$–CM and $\mathcal{S} : (\mathcal{O}, \varrho_N) \to (\mathcal{G}, \rho_N)$ be $NC\,\delta$–CM. Then $\mathcal{S} \circ h$ is $NC\,\delta$–CM.*

Proof. As in Theorem 3.11, using δ-open preimages and contra continuity. □

Theorem 3.13. *Let $h : (\mathcal{C}, \tau_N) \to (\mathcal{O}, \varrho_N)$ be $NIr\,\delta$–CM and $\mathcal{S} : (\mathcal{O}, \varrho_N) \to (\mathcal{G}, \rho_N)$ be $NC\,\delta$–CM. Then $\mathcal{S} \circ h$ is $NC\,\delta$–CM.*

Proof. Let H be δ-open in (G, ρ_N). Since S is $NC\,\delta$–CM, $S^{-1}(H)$ is δ-closed in (O, ϱ_N). Because h is $NIr\,\delta$–CM, h^{-1} preserves δ-open sets and hence also δ-closedsets (preimage commutes with complements). Thus $h^{-1}(S^{-1}(H))$ is δ-closed in (C, τ_N),i.e.,$S{\circ}h$ is $NC\,\delta$–CM. □

Remark 3.14. The implications among the classes of mappings follow the arrows $NIr \Rightarrow NIr\,\delta$and $NC \Rightarrow NC\,\delta$; converses generally fail.

Definition 3.15. Let $(\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}})$ and $(\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}})$ be two MPNTSs. For any MPN δ-open set $\mathcal{H}$ of $(\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}})$, if $h^{-1}(\mathcal{H})$ is δ-open in $(\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}})$ with respect to its neutrosophic topology, then

$$h : (\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}}) \to (\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}})$$

is an $MPNIr\,\delta$–CM.

Example 3.16. Let $\mathfrak{G} = \{\mathfrak{t}_1, \mathfrak{t}_2, \mathfrak{t}_3, \mathfrak{t}_4\}$ and $\mathfrak{L} = \{\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3\}$. Assume $T_{\mathbb{M}_{\mathcal{G}}}$ and $T_{\mathbb{N}_{\mathcal{G}}}$ are MNTs on these sets. Let mpn$(\mathfrak{G}^1)$ and mpn$(\mathfrak{L}^1)$ denote the collections of all MNSs over $\mathfrak{G}$ and $\mathfrak{L}$.

Define the following two 4-polar neutrosophic sets on $\mathfrak{G}$:

$$\begin{aligned}\mathfrak{G}_1 = \Big\{ &(\mathfrak{t}_1, \langle 0.313, 0.368, 0.401\rangle, \langle 0.303, 0.288, 0.333\rangle, \langle 0.361, 0.426, 0.406\rangle),\\ &(\mathfrak{t}_2, \langle 0.482, 0.392, 0.299\rangle, \langle 0.592, 0.418, 0.571\rangle, \langle 0.308, 0.516, 0.428\rangle),\\ &(\mathfrak{t}_3, \langle 0.319, 0.455, 0.518\rangle, \langle 0.529, 0.309, 0.419\rangle, \langle 0.391, 0.409, 0.414\rangle),\\ &(\mathfrak{t}_4, \langle 0.472, 0.402, 0.317\rangle, \langle 0.428, 0.319, 0.466\rangle, \langle 0.327, 0.316, 0.345\rangle)\Big\},\end{aligned}$$

$$\begin{aligned}\mathfrak{G}_2 = \Big\{ &(\mathfrak{t}_1, \langle 0.313, 0.345, 0.442\rangle, \langle 0.319, 0.328, 0.311\rangle, \langle 0.401, 0.424, 0.511\rangle),\\ &(\mathfrak{t}_2, \langle 0.523, 0.417, 0.431\rangle, \langle 0.427, 0.518, 0.526\rangle, \langle 0.518, 0.477, 0.317\rangle),\\ &(\mathfrak{t}_3, \langle 0.409, 0.426, 0.481\rangle, \langle 0.318, 0.369, 0.383\rangle, \langle 0.407, 0.456, 0.515\rangle),\\ &(\mathfrak{t}_4, \langle 0.527, 0.408, 0.472\rangle, \langle 0.517, 0.427, 0.513\rangle, \langle 0.491, 0.333, 0.382\rangle)\Big\}.\end{aligned}$$

Define on $\mathfrak{L}$:

$$\begin{aligned}\mathfrak{L}_1 = \Big\{ &(\mathfrak{r}_1, \langle 0.313, 0.368, 0.401\rangle, \langle 0.327, 0.311, 0.308\rangle, \langle 0.271, 0.272, 0.295\rangle),\\ &(\mathfrak{r}_2, \langle 0.307, 0.382, 0.437\rangle, \langle 0.371, 0.387, 0.392\rangle, \langle 0.427, 0.391, 0.403\rangle),\\ &(\mathfrak{r}_3, \langle 0.527, 0.418, 0.428\rangle, \langle 0.417, 0.369, 0.383\rangle, \langle 0.491, 0.333, 0.391\rangle)\Big\},\end{aligned}$$

$$\begin{aligned}\mathfrak{L}_2 = \Big\{ &(\mathfrak{r}_1, \langle 0.402, 0.317, 0.308\rangle, \langle 0.427, 0.361, 0.419\rangle, \langle 0.518, 0.477, 0.317\rangle),\\ &(\mathfrak{r}_2, \langle 0.319, 0.455, 0.444\rangle, \langle 0.333, 0.392, 0.299\rangle, \langle 0.319, 0.444, 0.520\rangle),\\ &(\mathfrak{r}_3, \langle 0.417, 0.451, 0.326\rangle, \langle 0.429, 0.318, 0.381\rangle, \langle 0.482, 0.326, 0.377\rangle)\Big\}.\end{aligned}$$

Define $h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}})$ by

$$h(\mathfrak{t}_1) = \mathfrak{r}_1 = h(\mathfrak{t}_2), \qquad h(\mathfrak{t}_3) = \mathfrak{r}_3, \qquad h(\mathfrak{t}_4) = \mathfrak{r}_2.$$

Let $T_{\mathbb{M}_\mathcal{G}} = \{\mathfrak{G}^0, \mathfrak{G}^1, \mathfrak{G}_1, \mathfrak{G}_2\}$ and $T_{\mathbb{N}_\mathcal{G}} = \{\mathfrak{L}^0, \mathfrak{L}^1, \mathfrak{L}_1, \mathfrak{L}_2\}$. Then

$$h^{-1}(\mathfrak{L}_1) = \mathfrak{G}_1, \quad h^{-1}(\mathfrak{L}_2) = \mathfrak{G}_2, \quad h^{-1}(\mathfrak{L}^0) = \mathfrak{G}^0, \quad h^{-1}(\mathfrak{L}^1) = \mathfrak{G}^1,$$

and each preimage shown is δ-open in $T_{\mathbb{M}_\mathcal{G}}$. Hence h is $MNIr\,\delta$–CM.

Theorem 3.17. *Let $h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}})$ and $\mathcal{S} : (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}}) \to (\mathfrak{J}, T_{\mathfrak{D}_\mathcal{G}})$ be $MNIr\,\delta$–CM. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{J}, T_{\mathfrak{D}_\mathcal{G}})$ is $MNIr\,\delta$–CM.*

Proof. Immediate from Theorem 3.6. □

Theorem 3.18. *Let $h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}})$ be $MN\,\delta-\beta^*$ CM and $\mathcal{S} : (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}}) \to (\mathfrak{J}, T_{\mathfrak{D}_\mathcal{G}})$ be $MNIr\,\delta$–CM. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{J}, T_{\mathfrak{D}_\mathcal{G}})$ is $MNIr\,\delta$–CM.*

Proof. By Theorem 3.7. □

Definition 3.19. Let $(\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}})$ and $(\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}})$ be two MPNTSs. For any MN δ-open set$\mathcal{H}$of ($\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}})$, if $h^{-1}(\mathcal{H})$ is δ-closed in $(\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}})$ with respect to its neutrosophic topology, then

$$h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}})$$

is an $MPNC\,\delta$–CM.

Theorem 3.20. *Let $h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}})$ and $\mathcal{S} : (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}}) \to (\mathfrak{J}, T_{\mathfrak{D}_\mathcal{G}})$ be $MNC\,\delta$–CM. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{J}, T_{\mathfrak{D}_\mathcal{G}})$ is $MPNC\,\delta$–CM.*

Proof. By Theorem 3.11. □

Example 3.21. Let $\mathfrak{G} = \{\mathfrak{t}_1, \mathfrak{t}_2, \mathfrak{t}_3\}$ and $\mathfrak{L} = \{\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3\}$. Let $T_{\mathbb{M}_\mathcal{G}}$ and $T_{\mathbb{N}_\mathcal{G}}$ be MNTs, and mpn($\mathfrak{G}^1$), mpn($\mathfrak{L}^1$) the corresponding families of MNSs. Define:

$$\begin{aligned}\mathfrak{G}_1 = \Big\{ &(\mathfrak{t}_1, \langle 0.613, 0.627, 0.711\rangle, \langle 0.501, 0.551, 0.524\rangle, \langle 0.609, 0.617, 0.583\rangle),\\ &(\mathfrak{t}_2, \langle 0.613, 0.627, 0.711\rangle, \langle 0.627, 0.519, 0.618\rangle, \langle 0.541, 0.617, 0.681\rangle),\\ &(\mathfrak{t}_3, \langle 0.451, 0.525, 0.595\rangle, \langle 0.725, 0.618, 0.723\rangle, \langle 0.511, 0.619, 0.634\rangle)\Big\},\end{aligned}$$

$$\begin{aligned}\mathfrak{G}_2 = \Big\{ &(\mathfrak{t}_1, \langle 0.613, 0.627, 0.711\rangle, \langle 0.319, 0.328, 0.311\rangle, \langle 0.401, 0.424, 0.511\rangle),\\ &(\mathfrak{t}_2, \langle 0.523, 0.417, 0.431\rangle, \langle 0.427, 0.518, 0.526\rangle, \langle 0.518, 0.477, 0.317\rangle),\\ &(\mathfrak{t}_3, \langle 0.409, 0.426, 0.481\rangle, \langle 0.318, 0.369, 0.383\rangle, \langle 0.407, 0.456, 0.515\rangle)\Big\},\end{aligned}$$

and

$$\begin{aligned}\mathfrak{L}_1 = \Big\{ &(\mathfrak{r}_1, \langle 0.613, 0.627, 0.711\rangle, \langle 0.547, 0.516, 0.563\rangle, \langle 0.671, 0.634, 0.674\rangle),\\ &(\mathfrak{r}_2, \langle 0.595, 0.561, 0.523\rangle, \langle 0.482, 0.456, 0.490\rangle, \langle 0.518, 0.477, 0.618\rangle),\\ &(\mathfrak{r}_3, \langle 0.725, 0.618, 0.627\rangle, \langle 0.723, 0.692, 0.614\rangle, \langle 0.619, 0.612, 0.745\rangle)\Big\},\end{aligned}$$

$$\begin{aligned}\mathfrak{L}_2 = \Big\{ &(\mathfrak{r}_1, \langle 0.501, 0.551, 0.534\rangle, \langle 0.551, 0.524, 0.644\rangle, \langle 0.608, 0.651, 0.811\rangle),\\ &(\mathfrak{r}_2, \langle 0.627, 0.672, 0.781\rangle, \langle 0.517, 0.519, 0.728\rangle, \langle 0.681, 0.627, 0.617\rangle),\\ &(\mathfrak{r}_3, \langle 0.614, 0.813, 0.725\rangle, \langle 0.851, 0.735, 0.817\rangle, \langle 0.689, 0.728, 0.744\rangle)\Big\}.\end{aligned}$$

Define

$$h : (\mathfrak{G}, T_{\mathbb{M}_\mathcal{G}}) \to (\mathfrak{L}, T_{\mathbb{N}_\mathcal{G}}), \qquad h(\mathfrak{t}_1) = \mathfrak{r}_3, \quad h(\mathfrak{t}_2) = \mathfrak{r}_1, \quad h(\mathfrak{t}_3) = \mathfrak{r}_2.$$

Let $T_{\mathbb{M}_\mathcal{G}} = \{\mathfrak{G}^0, \mathfrak{G}^1, \mathfrak{G}_1, \mathfrak{G}_2\}$ and $T_{\mathbb{N}_\mathcal{G}} = \{\mathfrak{L}^0, \mathfrak{L}^1, \mathfrak{L}_1, \mathfrak{L}_2\}$. Then

$$h^{-1}(\mathfrak{L}_1) = \mathfrak{G}_1, \qquad h^{-1}(\mathfrak{L}_2) = \mathfrak{G}_2, \qquad h^{-1}(\mathfrak{L}^0) = \mathfrak{G}^0, \qquad h^{-1}(\mathfrak{L}^1) = \mathfrak{G}^1,$$

and the preimages asserted are δ-closed in $T_{\mathbb{M}_\mathcal{G}}$; hence h is $MNC\,\delta$–CM.

Theorem 3.22. *Let* $h : (\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}}) \to (\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}})$ *be* $MN\,\delta$*–CM and* $\mathcal{S} : (\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}}) \to (\mathfrak{J}, T_{\mathfrak{O}_{\mathcal{G}}})$ *be* $MNC\,\delta$*–CM. Then* $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}}) \to (\mathfrak{J}, T_{\mathfrak{O}_{\mathcal{G}}})$ *is* $MNC\,\delta$*–CM.*

Proof. By Theorem 3.11. □

Theorem 3.23. *Let* $\mathcal{S} : (\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}}) \to (\mathfrak{J}, T_{\mathfrak{O}_{\mathcal{G}}})$ *be* $MNC\,\delta$*–CM and* $h : (\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}}) \to (\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}})$ *be* $MNIr\,\delta$*–CM. Then* $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}}) \to (\mathfrak{J}, T_{\mathfrak{O}_{\mathcal{G}}})$ *is* $MNC\,\delta$*–CM.*

Proof. The claim follows from Theorem 3.13. □

Remark 3.24. Figure 1 illustrates the relationships among the various classes for the map $h : (\mathfrak{G}, T_{\mathbb{M}_{\mathcal{G}}}) \to (\mathfrak{L}, T_{\mathbb{N}_{\mathcal{G}}})$.

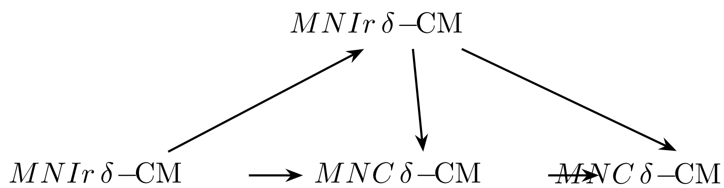

Figure 1. Relationships between different types of MNTS continuous mappings.

4. Conclusions

We studied new classes of neutrosophic δ-continuous mappings, namely the irresolute and contra versions, and analyzed their fundamental properties and interrelations. We also discussed stability under composition and provided counterexamples showing that certain converses fail. Finally, we extended the framework to the setting, introducing $MNIr\,\delta$–CM and $MNC\,\delta$–CM and showing how the main results lift to MNTSs.

The comparison with the δ(non-star) counterparts clarifies the hierarchy among the notions ($NIr \Rightarrow NIr\,\delta$and $NC \Rightarrow NC\,\delta$, but not conversely) and highlights how irresolute and contra behaviors diverge. These findings contribute to a more refined picture of continuity in neutrosophic topology and suggest further applications in multi-criteria settings modeled via neutrosophic structures.

Conflict of Interest

No conflicts of interest are disclosed by the authors.

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