



Robust participation incentives in dynamic kidney exchange[☆]

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ABSTRACT

Robustness to plausible distributions of outcomes helps rationalize straightforward participation incentives for Compatible patient–donor Pairs in dynamic kidney exchange programs. The CP faces ambiguity on the distribution of outcomes in every matching round and evaluates participation conditions – minimum guaranteed quality and committed number of rounds before fallback – according to every plausible distribution. When the decay of the donor's organ quality between rounds is negligible, the ex-ante and ex-post individually rational socially optimal contract involves a minimum guarantee exactly equal to the fallback option and an unlimited, self-enforced stay at the pool. The model also easily explains that no contract can attract a CP with sufficiently high reservation value to the exchange pool.

1. Introduction

Increasing theoretical work has examined how to incentivize *compatible* patient–donor pairs (CPs) to participate in Kidney Exchange Programs (KEPs), e.g., [Sönmez et al. \(2020\)](#) and [Akbarpour et al. \(2025\)](#). The *quality* of the transplanted organ is a key determinant of patient survival. The Living Kidney Donor Profile Index (LKDPI, see [Massie et al. 2016](#)) and related measures such as Expected Graft Survival (EGS, [Li et al. 2019](#)) are widely used to assess organ quality.² Welfare gains from including compatible pairs are substantial.³

We adopt this perspective and study the optimal “minimum quality gain” offered to a compatible pair.⁴ [Nicolò et al. \(2025\)](#) simulate scenarios in which CP participate in KEP when they expect quality gains, and they operate exchanges only if they entail a minimum 20-point LKDPI reduction, roughly two-year survival time gain according to [Li et al. \(2019\)](#). Such ex-ante and ex-post minimum quality gains constitute our focus.

We also analyze the optimal commitment duration within the KEP. While [Li et al. \(2019\)](#) assume that CPs remain in the pool for a single round, we allow longer commitments.

[Agarwal et al. \(2021\)](#) and [Su and Zenios \(2004\)](#) analyze waitlist procedures where patients may reject available kidneys, modeled as

optimal stopping problems. By contrast, our model assumes that a contract guarantees a search for a kidney meeting a minimum committed quality; any offered kidney satisfying this guarantee must be accepted.

CPs may hesitate to participate due to uncertainty. Concerns include preserving the emotional bond of directed donation, fear of chain failures or donor renegeing, delays in transplantation, and travel requirements ([Fortin et al., 2021](#)).

Participation in a dynamic exchange pool entails uncertainty regarding future match quality. In practice, a CP has limited information about the distribution of attainable qualities in future rounds. This situation involves not merely risk with known probabilities, but genuine *ambiguity*. We therefore adopt the conservative max–min approach ([Gilboa and Schmeidler, 1989](#)), rather than a smooth second-order aggregator ([Klibanoff et al., 2005](#)). The max–min framework implies a robust notion of individual rationality: participation must be optimal for every plausible distribution of outcomes.

The model is tractable and yields several insights. First, interestingly, *long-term contracts benefit compatible pairs*.⁵ Second, if donor organ quality does not significantly deteriorate across rounds, the optimal contract provides no guarantee beyond the reservation value. Third, the model identifies high-reservation-value types that never participate. A back-of-the-envelope calculation, assuming a uniform distribution of exchange qualities and a discount factor $\beta = 0.95$ —

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² An earlier theoretical contribution on optimal dynamic allocation explicitly incorporating kidney quality is [Zenios \(2002\)](#).

³ [Ferrari et al. \(2017\)](#), [Basu et al. \(2020\)](#), [Tafulo et al. \(2020\)](#) show that CP participation may improve outcomes even for the compatible recipient herself.

⁴ For example, matching to a younger donor ([Bingaman et al., 2012](#)).

⁵ This is a feature known in recent work on optimal queue design ([Che and Tercieux, 2023](#)).

consistent with five-year expected survival without transplantation if matching occurs quarterly — yields a 27.6% probability that a CP does not join the KEP.

Before presenting the model, we note that risk aversion alone does not replicate the implications of ambiguity aversion. With discounting and extreme risk aversion, participation would typically remain optimal for a narrower range of reservation values than under ambiguity aversion.

2. Model

A patient–donor pair has reservation value $r \in [0, 1)$, representing the health gain from direct transplantation. A value of zero corresponds to incompatibility.

Time is indexed by matching rounds. Over time, both patient health and donor kidney quality may decline, captured by discount factors $\beta \in (0, 1)$ and $\delta \in (0, 1]$. The parameter β may be interpreted as the probability that the patient remains eligible in the next round, while δ measures the remaining donor organ quality if transplantation is delayed.

In each round $\tau = 1, 2, \dots$, the pair considers a set \mathcal{F} of plausible distributions of health gains from exchange. This set is invariant across rounds. Each $F \in \mathcal{F}$ has support $[0, 1]$, possibly with mass at zero, and density f on $(0, 1)$.⁶

For distributions F and G , F Hazard-rate Dominates (HRD) G if $\frac{1-F(v)}{1-G(v)}$ is increasing in $v \in (0, 1)$ and $F(0) \leq G(0)$. We assume that \mathcal{F} admits a minimum element \underline{F} under HRD.

A social planner manages the exchange pool. A **contract** is a pair $(b, t) \in [0, 1) \times \mathbb{N} \cup \infty$. The parameter b denotes the minimum guaranteed exchange quality, and t the number of rounds during which the pair commits to remain before exercising the reservation option. If accepted, the pair remains for t rounds unless matched earlier.

We impose minimal structure on the planner’s objective. Conditional on participation, the planner prefers lower b and higher t , as both expand feasible matches. If the set of individually rational contracts⁷ I contains (b^*, t^*) with $b^* = \inf\{b : (b, t) \in I\}$ and $t^* = \sup\{t : (b, t) \in I\}$, then (b^*, t^*) is optimal.

3. Participation constraints

Ex-post individual rationality requires that the contract never yield an outcome inferior to direct transplantation. Formally, $b \geq \delta^\tau r$ for all $\tau = 1, \dots, t$, which simplifies to $b \geq \delta r$.

For a distribution F , define $V_F(b) \equiv \mathbb{E}_F[v \mid v \geq b] \geq b$. Note that, by HRD, we have $V_{\underline{F}}(b) \leq V_F(b)$ for all $F \in \mathcal{F}$. Moreover, and as a consequence of HRD, $F(v) \geq \underline{F}(v)$ for all $v \in [0, 1)$.

The subjective utility under an ex-post individually rational contract (b, t) is

Subjective (worst-case) utility:

$$\begin{aligned}
 U^{\text{SUBJ}}(b, t) &= \min_{(F_1, \dots, F_t) \in \mathcal{F}^t} \left\{ \sum_{\tau=1}^t \left[\prod_{h=1}^{\tau-1} F_h(b) \right] [1 - F_\tau(b)] \beta^\tau V_{\underline{F}}(b) \right. \\
 &\quad \left. + \left[\prod_{\tau=1}^t F_\tau(b) \right] \beta^t \delta^t r \right\} \\
 &\geq \min_{(F_1, \dots, F_t) \in \mathcal{F}^t} \left\{ \sum_{\tau=1}^t \left[\prod_{h=1}^{\tau-1} F_h(b) \right] [1 - F_\tau(b)] \beta^\tau V_{\underline{F}}(b) \right. \\
 &\quad \left. + \left[\prod_{\tau=1}^t F_\tau(b) \right] \beta^t \delta^t r \right\}
 \end{aligned}$$

⁶ Results apply to restricted structures $\mathcal{P} \subset \mathcal{F}^t$ containing an infimum in the HRD sense defined below.

⁷ Characterized below.

$$\begin{aligned}
 &= \min_{(F_1, \dots, F_t) \in \mathcal{F}^t} \left\{ \sum_{\tau=1}^{t-1} \left[\prod_{h=1}^{\tau-1} F_h(b) \right] [1 - F_\tau(b)] \beta^\tau V_{\underline{F}}(b) \right. \\
 &\quad \left. + \left[\prod_{\tau=1}^{t-1} F_\tau(b) \right] \left([1 - F_t(b)] \beta^t V_{\underline{F}}(b) + F_t(b) \beta^t \delta^t r \right) \right\} \\
 &\geq \min_{(F_1, \dots, F_t) \in \mathcal{F}^t} \left\{ \sum_{\tau=1}^{t-1} \left[\prod_{h=1}^{\tau-1} F_h(b) \right] [1 - F_\tau(b)] \beta^\tau V_{\underline{F}}(b) \right. \\
 &\quad \left. + \left[\prod_{\tau=1}^{t-1} F_\tau(b) \right] \left([1 - \underline{F}(b)] \beta^t V_{\underline{F}}(b) + \underline{F}(b) \beta^t \delta^t r \right) \right\} \\
 &\geq \dots \geq \sum_{\tau=1}^t \underline{F}(b)^{\tau-1} (1 - \underline{F}(b)) \beta^\tau V_{\underline{F}}(b) + \underline{F}(b)^t \beta^t \delta^t r \\
 &= [1 - [\beta \underline{F}(b)]^t] \frac{\beta - \beta \underline{F}(b)}{1 - \beta \underline{F}(b)} V_{\underline{F}}(b) + [\beta \underline{F}(b)]^t \delta^t r
 \end{aligned}$$

The first inequality comes from $V_{\underline{F}}(b) \leq V_F(b)$ due to HRD. Using backwards recursive substitution of F_τ with \underline{F} , the remaining inequalities follow. We have provided a first step (noting $V_{\underline{F}}(b) \geq b \geq \delta^t r$), leaving intermediate steps to the reader. By utilizing $\underline{F}(b)$ for every $\tau = 1, \dots, t$,

$$U^{\text{SUBJ}}(b, t) = [1 - [\beta \underline{F}(b)]^t] \frac{\beta - \beta \underline{F}(b)}{1 - \beta \underline{F}(b)} V_{\underline{F}}(b) + [\beta \underline{F}(b)]^t \delta^t r$$

Let $W^{\text{SUBJ}}(b) \equiv \frac{\beta - \beta \underline{F}(b)}{1 - \beta \underline{F}(b)} V_{\underline{F}}(b)$. **Ex ante individual rationality** is formulated as

$$W^{\text{SUBJ}}(b) - r \geq \frac{[\beta \underline{F}(b)]^t}{1 - [\beta \underline{F}(b)]^t} [1 - \delta^t] r$$

We note that ex-ante individual rationality implies the necessary condition $W^{\text{SUBJ}}(b) - r \geq 0$.

4. Results

We characterize the optimal contract under full information. Limited commitment is not optimal. Rather, it is optimal for both the planner and the CP to set $t = \infty$. The planner then chooses the smallest b satisfying ex-post and ex-ante individual rationality. As $\delta \rightarrow 1$, the rule simplifies to $b = r$. There exists a threshold r^* such that pairs with $r > r^*$ do not participate no matter δ (see Fig. 1 for an illustration of the optimal contract.)

Proposition 1. *The optimal contract entails $t = \infty$ and $\min\{b/\delta, W^{\text{SUBJ}}(b)\} = r$. There is r^* that is well defined as the unique solution to $W^{\text{SUBJ}}(r^*) = r^*$, such that, for any $\delta \in (0, 1]$ no patient–donor pair with $r > r^*$ participates in the kidney exchange pool. For $\delta = 1$, the optimal contract simplifies to $b = r$.*

Proof. For $t = \infty$. Let $\bar{U}^{\text{SUBJ}}(b, t) \equiv [1 - [\beta \underline{F}(b)]^t] W^{\text{SUBJ}}(b) + [\beta \underline{F}(b)]^t r$ be an upper bound for $U^{\text{SUBJ}}(b, t)$. It is clear that, as long as $W^{\text{SUBJ}}(b) - r \geq 0$, necessary condition for ex-ante individual rationality, $\bar{U}^{\text{SUBJ}}(b, t)$ is increasing in t . Since $U^{\text{SUBJ}}(b, \infty) = \bar{U}^{\text{SUBJ}}(b, \infty) = W^{\text{SUBJ}}(b)$, $U^{\text{SUBJ}}(b, t)$ reaches its maximum at the limit $t = \infty$. Therefore, under the assumptions of the model, it is in the interest of both the social planner and the patient–donor pair to set $t = \infty$.

For $\min\{b/\delta, W^{\text{SUBJ}}(b)\} = r$. Given $t = \infty$, the ex-ante individual rationality constraint is reduced to $W^{\text{SUBJ}}(b) \geq r$, which combined with ex-post individual rationality, and given that the minimal b accomplishing with both conditions is optimal, yields $\min\{b/\delta, W^{\text{SUBJ}}(b)\} = r$.

Well-defined r^* such that $r > r^*$ implies no participation. $W^{\text{SUBJ}}(b)$ is increasing iff $W^{\text{SUBJ}}(b) > b$. We also have $W^{\text{SUBJ}}(0) = \beta \mathbb{E}_{\underline{F}}(v) > 0$ and $\lim_{b \rightarrow 1} W^{\text{SUBJ}}(b) = 0$. By continuity of W^{SUBJ} , there is r^* that is well defined as the unique solution to $W^{\text{SUBJ}}(r^*) = r^*$. W^{SUBJ} reaches a maximum at this point, thus $r > r^*$ implies that there is no b such that $W^{\text{SUBJ}}(b) \geq r$, thus necessarily violating ex-ante individual rationality.

For $b = r$ as $\delta = 1$. We focus on $r \leq r^*$. For $r \leq r^*$, we have seen in the previous paragraph that, if $b < r^*$, $W^{\text{SUBJ}}(b) > b$. In other words, the ex ante individual rationality constraint is redundant. ■

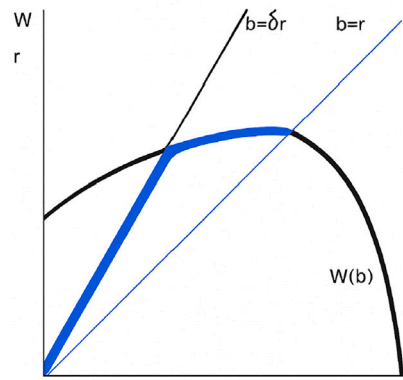


Fig. 1. Simplified illustration – thick blue line – of the optimal choice of b (horizontal axis coordinate.).

5. Limitations and final comments

The easy tractability of our model comes with mandatory acknowledgment of several limitations. Our model abstracts from reneging, imposes constant discounting, and assumes atomistic participation. The last point is relevant: the distribution of potential outcomes depends on the mass and types of CP's accepting to participate through time, and not only on an exogenous flow of incompatible pairs that have to participate. Implicitly, the model assumes a steady-state set of plausible profiles of outcome distributions.⁸

Our study does not design an optimal matching rule, but rather characterizes robust participation incentives through minimum quality guarantees. When donor organ decay is negligible, the optimal mechanism need not promise quality above the reservation value. Compatible pairs are interested in long-term participation under this minimum quality guarantee.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to improve academic English. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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Data availability

No data was used for the research described in the article.

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