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Insecure Property Rights and Conflicts: How to Solve Them? †

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Abstract: According to the leading literature, the valid enforcement of property rights is a key ingredient for economic development. However, their enforcement can be problematic in international relations, which can be a valid approximation of an anarchic or *state of nature* environment. In such a context, we do not have a third party that may sanction any illegal behaviour, since the existing international organizations may lack the necessary power to force countries to behave in a specific way. A large variety of papers have attempted to provide a self-enforcing solution to a conflict among players by defining a bargaining range, which may prevent the emergence of a war. Hence, players renounce the fight and leave peacefully, enforcing de facto property rights. In contrast, we propose a model in which contestants decide to solve their dispute by forming a union. The latter can be interpreted in a broad sense, also encompassing the possibility that they form a new political entity. We highlight the welfare implications of that solution and define the non-empty set of parameters, which support such a decision in the long run. Intuitively, from a dual perspective, the model also discloses the circumstances that may lead players to deviate from the union path and split. Therefore, our paper contributes to the literature about the formation and breakdown of countries, although our primary concern is to present a model with an innovative solution to conflicts. Moreover, our work stresses the importance of the enforcement of property rights to guarantee the peaceful development of relations among countries.

Keywords: property rights; anarchy; conflict; forming a union; Nash equilibrium; Nash bargaining solution

MSC: 91A05; 91A10; 91A80

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1. Introduction

It is established in the literature [1–3] that economic development can be boosted by enforcing property rights. If the latter are insecure, players are forced to allocate some resources to protecting their output, which could be subject to expropriation, meaning a war is likely to explode. This, in turn, reduces their welfare, since they have fewer resources for their productive activities. Eventually, insecure property rights and the risk of expropriation are a strong disincentive to production, generating economic backwardness. Hence, it is urgent to understand under which circumstances a conflict may occur and how it can be solved to enforce property rights.

The study of conflict, its genesis, and its consequences have been the focus of a large interdisciplinary literature, spanning from mathematics to biology, economics, and political science. Such interest stems from the fact that modelling a conflict allows us to characterise the behaviour of agents in a social context and to evaluate the steps that lead them to take some particular decisions. From an economic point of view, a conflict, for instance, can be considered as a type of competition, although much more intensive and destructive, since in this case "...contenders try to hamper, disable, or destroy rivals" [4]. The interest in modelling conflicts and finding solutions stems from the fact that they may arise in a variety of situations [5].

Our contribution borrows from papers in different fields and consists of developing a mathematical model, mainly a game-theoretical one, focusing on the relations between two heterogeneous players, living in an anarchic context. Heterogeneity is determined by the different military and productive skills of each contestant.

In such a scenario, a war becomes appealing since contestants may steal the output that they produce separately. The main difference between our contribution from the existing literature is that we attempt not to determine the existence of a bargaining range, which may guarantee that players reach a peaceful agreement, but, instead, we investigate the possibility that they may decide to form a union. (The latter can take different forms. It can be seen as a customs union, a merger between firms, or, if we focus on international relations, the formation of a new political entity). We stress the advantage of such a decision, highlighting the improvement in welfare that it brings about.

Clearly, any solution to a conflict, including the one that we propose, requires that players talk to each other. Communication is crucial in international relations and may develop in different ways [6].

We obtain the following results. First, we show that forming a union increases players' welfare. This result stems from the fact that the solution concept, which we propose, solves the typical dilemma, faced by players, who need to share their resources between an internal, productive use and an external, military purpose. The decision to form a union makes more resources available for production, generating *direct* and immediate benefits with respect to the war [7]. In addition, such a decision makes players even better off than when they decide to find a peaceful agreement without creating a union. Finally, we study the long-term evolution of such an agreement, stressing that the set of parameters that may drive contestants to split is not empty.

Our contribution fits within the theory of conflict resolution, but it is also consistent with the literature on country formation. Moreover, from a dual perspective, we can also study the conditions that may determine secession among countries.

The topic of country formation has received much consideration in the economic and political science literature from several points of view. For instance, Riker (1964) [8] and Gilpin (2001) [9] argue that unification is an appropriate response to external threats. Instead, secession could be explained as the lack of common values and interests that could support a life together [10–12]. From such a perspective, we can evaluate the process that led to the breakdown of Yugoslavia and the USSR, although the lack of common values is an important but not an exclusive reason for secession. In our model, some parameters can be acknowledged as proxies of cultural heterogeneity, playing an important role in explaining the decisions that countries may take with respect to the formation/secession of a new political entity. In this vein, our model can also explain the process of re-unification in Germany at the beginning of the 1990s.

Finally, as indicated above, our contribution stresses the importance of property rights for economic development. The lack of their enforcement produces sub-optimal equilibria, reducing the economic incentive to invest and, consequently, generating poor economic development. Instead, it has been proved that countries where the rule of law is in place experience high development and sustained growth.

Our model is able to capture other phenomena in the real world, although some adjustments are required. For instance, it could be adopted to study the interactions between firms in the market. Hannah and Eisenhardt (2018) [13] point out that the balance between competition and cooperation in an ecosystem is vital for the development of firms. In an ecosystem, a group of firms produce different and complementary components, that together offer a coherent solution. As the authors highlight, cooperation consists of achieving mutual interests and benefits, while competition postulates that firms try to satisfy their own interests. Therefore, the cooperation aims to jointly create benefits, while competition tends to subtract resources from others. Such a behaviour is not dissimilar from the one that we model in our paper, since players need to decide between a conflict (competition) and following a union (cooperation). As mentioned previously, adopting our model to the behaviour of firms requires some adjustments. Firms operate within a market, which provides the institutional framework that drives their behaviour. This environment is different from the anarchic context that we use to model the interaction of two players. Such differences appear to be crucial to understand the functioning of our model.

Our paper is structured as follows. In Section 2, we review the main literature from different fields, while in Section 3, we present the main characteristics of our model. In Section 4, we study the relations between players in an anarchic context, while in Section 5, we analyse the decision to form a union. In Section 6, we present some case studies which highlight the applicability of our model in the real world, while Section 7 concludes.

2. Literature Review

The majority of contributions in different fields agree on the distinction between internal (or intrastate or social) and interstate conflicts. While we model the latter type of conflict, we also take into consideration the literature that deals with internal conflict. The latter offers us the possibility of evaluating the choices and behaviour of agents that eventually may display similar attitudes toward conflicts, independently of their dimension.

From a mathematical point of view, many authors have tried to characterise the development of a war, offering models that provide useful information about its duration and evolution. Kress (2020) [14] provides an overview of the mathematical tools that have been employed to characterise conflicts among groups. He uses a set of Lanchester equations, which model the dynamics of two contestants, offering a possible interpretation of the outcome of a war. The author also uses their model to explain the insurgence of conflicts and how it may produce a different outcome when many players are involved.

In an interesting contribution, Clauset (2018) [15] analyses the statistics of conflicts over a 200-year span and notices that periods of peace and war alternate with each other. Moreover, a long peace, which is usually followed by sharp violence, seems to be more fragile than expected. Although the alternation of war and peace is a common trend, it does not testify to a change in the conflict-generating process.

The use of statistics is a useful approach to modelling social interactions and dynamics. For instance, Diep et al. (2020) [16] analyse the structure of social conflicts, using tools that are usually employed in statistical physics and applied to social phenomena, such as culture dynamics, crowd behaviour, information dissemination, and social conflict. The authors notice that when one group interacts with another, actions and reactions are not proportional, but, instead, unequal. Moreover, the impact of some specific behaviours vanishes when averaged. Instead, only common characteristics can be detected. The most interesting result, which emerges from the application of different tools such as time-dependent mean-field theory and Monte Carlo simulations, is that there exists a periodic fluctuation between negotiation and conflict. Moreover, the analysis reveals a chaotic behaviour, which generates unpredictability in conflict outcomes.

Other authors adopt different mathematical tools to model conflicts. For instance, Petukhov et al. (2017) and Petukhov et al. (2018) [17,18] apply diffusion equations to model social interactions, trying to isolate and interpret the ethnical–social and religious implications of conflicts. Specifically, they adopt a Langevin diffusion equation, assuming that individuals interact through a communication field. Such an approach allows authors to disentangle the characteristic laws of conflicts and the role that social distance may have in shaping them. Similarly to other contributions from other fields, such as economics and political science, it is possible to identify stable regions, which makes the interaction among individuals stable and peaceful.

On the other hand, Bellomo et al. (2013) [19] adopt kinetic theory for active particles to model the non-linear interaction among individuals, within a game-theoretical framework. Specifically, they study people's competition for wealth, introducing the effect of strong support/opposition to political elites. Such a model attempts to analyse the emergence of some unpredictable outcomes (Black Swans). While the latter contribution shares some features with our work, since it grounds the analysis in a game-theoretical model and aims to explain how individuals interact to distribute wealth in society, it critically departs from ours in that we explicitly model the interaction in an anarchic context, where there is no third party (the government for instance) that may influence individuals' actions.

The idea of adopting cooperative behaviour, in a way that is conceptually similar to ours, is investigated in Salam and Takahashi (2010) [20]. The authors model a multi-opponent conflict, studying the related dynamics. Then, they introduce the possibility that some of the contestants start to cooperate, evaluating how the equilibrium may vary using a computer experiment.

However, as mentioned previously, the analysis of conflicts has generated interdisciplinary interest, which covers different fields, beyond mathematics and physics. For instance, economics and political science adopt the common idea that interstate conflicts are generated by an anarchic or *state of nature* framework in which players live. In such a situation, a third-party authority may not exist or may not have the necessary power to impose specific actions. This marks an important difference between interstate and intrastate conflicts since, in the latter case, a central authority is still in charge and may attempt to mitigate the conflict. For instance, this is what has occurred in Syria in recent years, or what we may observe regarding the tribal/ethnic/religious disputes in Afghanistan.

Although, in some cases, the eruption of violence may determine the collapse of legal institutions, the difference between intrastate and interstate conflicts is crucial and displays its effects in the way in which war is modelled.

The *state of nature* approach used in economics and political science stresses not only the mathematical properties of models, but also the welfare implications of a war and, in general, its economic and institutional features. Since contestants may not be punished for their actions, the *rule of strength* characterises an anarchic society, where property rights are constantly threatened and insecure. Such an approach appears to be consistent with the evolution of the relations among countries, which may take unilateral decisions, although they sit in international bodies, which, in principle, should mitigate extreme actions. An example of this situation emerged in the decision taken by US and its allies to invade Iraq in 2003, without receiving explicit support from the United Nations (UN).

Moreover, different from the analysis of conflict in other fields, economics and, particularly, political science has tried to provide a rational explanation for war. In this respect, the mainstream literature, for instance, which is grounded in rationalist theory, proposes several justifications, which are based on the fact that countries live in anarchy. Such an explanation has found room in the literature since Waltz (1959) [21], who claims that: "…war occurs because there is nothing to prevent it. […] Among states, as among men, there is no automatic adjustment of interests. In the absence of a supreme authority, there is then the constant possibility that conflicts will be settled by force."

Within such a stream of literature, Fearon (1995) [22] recognises the critical role played by anarchy in explaining the emergence of war, although it cannot be considered the only explanation about why contenders do not reach a peaceful agreement through negotiation. Therefore, he proposes several further rationalist explanations. Even in the presence of anarchy, it is still possible to find a bargaining range, where contestants may negotiate an agreement, which makes them better off even in a conflict scenario. Nonetheless, the world still suffers from episodes of sharp violence.

The reasons for the emergence of war are even more complex because they imply that players embark on a situation that, by definition, is costly and leads to a sub-optimal result. Garnett (2007) [23] suggests a variety of reasons that may justify a war, ranging from proximate to ultimate causes, from sufficient to necessary ones, and so on. Lopez and Johnson (2020) [24] offer a comprehensive and updated survey about the determinants of war from the point of view of political scientists. Nonetheless, there exists a large consensus in the literature that conflict is hardly avoidable. Instead, it represents the natural development in countries' relations, when commitment is limited [25].

It is indisputable, however, that many authors support the idea that anarchy is the main driver of the relationships between countries. For instance, Oye (1985) [26] observes that international relations can be evaluated as an appropriate representation of the state of nature elaborated by Hobbes. In such a framework, one country could be under attack from a neighbour. In the same vein, Gilpin (1981) [27] notes that international relations are a "recurring struggle for wealth and power among independent actors in a state of anarchy". Given such a structure, if an agreement is reached, it must be self-enforcing, since, in anarchy, there is no third party that may impose specific behaviour upon players.

However, although there is a large consensus, the central role of anarchy in international relations has been recently under debate. Donnelly (2015) [28], for instance, reject the centrality of anarchy, preferring the concept of hierarchy. Instead, Lechner (2017) [29] challenges the latter conclusions, offering new support for the centrality of anarchy in the field of international relations. Such a view is further reinforced by recent contributions, which note that international institutions may exacerbate conflicts by creating a hierarchical structure among countries because of a certain distrust that they generate [30].

Generally speaking, according to Cronin (1999) [31], we can rank international relations on a seven-point scale, with the extreme points being the international state of nature, where we observe a war of all against all, and the collective security system, characterised by collective action.

Although the economics and political science literature attempt to explain the roots and emergence of a conflict, they also adopt analytical tools, especially from game theory, to model interstate and intrastate conflicts. For instance, Skaperdas (1992); Powell (1993); Grossman and Kim (1995); Grossman and Mendoza (2001); Hirshleifer (2001); Muthoo (2004); McBride and Skaperdas (2014); and Herbst, Konrad and Morath (2017) [7,32–38] ground their analysis of conflict in game theory models to explain their emergence and characterise their main features. All of the above contributions propose a similar solution, consisting of the definition of a set of parameters that drive players to opt for a peaceful settlement of their disputes. Clearly, in all of the aforementioned models, the proposed agreements must be self-enforcing and usually do not require the characterization of a defensive structure to contrast future attacks.

3. Model Framework

We model the interaction of two players, owing an initial endowment, $R_i, \forall i \in \{1,2\}$, which cannot be consumed directly. Instead, it can be used as an input to produce a consumption good, C_i. In a peaceful environment, all of the endowment is employed for the production of C_i.

However, contestants may not always live in peace. Instead, they operate in an anarchic context, where the consumption of goods can be stolen. Therefore, only a portion of R is used to produce C_i. Given that production is insecure, the sum of C_1 and C_2 represents a common pool, over which players may fight to gain control. Hence, they should use a proportion of their initial endowment for the production of weaponry to be used in the fight. The allocation problem consists of choosing how much of R should be employed for the production of (*butter*)—C—and how much for the production of *guns*—G. As Powell (1993) [7] clarifies, this is indeed the typical trade-off, that any contestant needs to solve in their interaction with others. Obviously, allocating resources to the war has only an *indirect* benefit, since that share of resources determines the probability of being successful in a future war. Instead, using R to produce the consumption good generates a *direct* advantage, although, in turn, this reduces the probability of success in a war. It follows that in the anarchic context, the initial endowment will be distributed as follows:

$$R_i = \frac{C_i}{\beta_i} + \frac{G_i}{\gamma_i}. \tag{1}$$

where C_i and $G_i \in (0, R_i)$ and β_i and $\gamma_i \in (0,1)$. The latter can be interpreted as measures of marginal productivity for the two alternative uses—the production of the consumption goods and weaponry—and mirrors the efficiency of the employed technology. More specifically, γ_i captures players' "…productivity in transforming fighting resources into effective units of weaponry" [39]. (Differently from other contributions, for example, Grossman and Kim (1995) and Grossman (2004) [32,40], we do not distinguish between defensive and attacking technologies). Equation (1) indicates how good each contestant is in producing the consumption and military outputs. If, for instance, one unit of C_i (G_i) can be produced using only one unit of input, the less productive one player is, the more it needs to allocate to produce that specific output. Hence, a smaller amount of resources is left for the production of the other good. Therefore, C_i/β_i and G_i/γ_i indicate the *effective* amount of resources allocated by each player in the two alternative uses.

Rearranging Equation (1) in terms of C_i yields

$$C_i = \beta_i \left[R_i - \frac{G_i}{\gamma_i} \right] \tag{2}$$

It follows that the production of C_i depends directly on R_i and on β_i, the efficiency of the technology, employed in its production. Interestingly, C_i is also positively related to γ_i, since, as indicated above, other things being equal, the more efficient contestant i is at producing arms, the more it can allocate to the production of the consumption good, without modifying its probability of success in a future war. It follows that the total amount of consumption goods, which is produced in the world, is

$$F(C_1; C_2) = \sum_{i=1}^{2} \beta_i \left[R_i - \frac{G_i}{\gamma_i} \right] \tag{3}$$

Since the world is insecure, the above amount represents the common pool of consumption goods, which is subject to appropriation. When moving to a war, the probability of success depends on how much a player allocates to the production of arms and on the same choice operated by its opponent. We can model such a probability as a standard contest success function (CSF), which has been axiomatised in Skaperdas (1996) [41] and explicitly takes the following form:

$$p_i = \frac{\left(\frac{G_i}{\gamma_i}\right)^m}{\left(\frac{G_i}{\gamma_i}\right)^m + \left(\frac{G_j}{\gamma_j}\right)^m} \tag{4}$$

where m is an effectiveness parameter, which measures the sensitivity of the probability of success p_i to a change in the effective amount of resources allocated to the war. As Hirshleifer (1995) [4] points out, $m \in (0,1)$ ensures a stable interior equilibrium. From an economic point of view, the probability (4) weights the share of the common pool that player i expects to gain from the fight. It is also an extension of the approach suggested by Tullock (1980) [42] and largely employed in the literature, mapping in a simple way the effort of each contestant into a positive probability [43]. The difference in the CSF originally proposed by Tullock (1980) [42] relies on the fact that the impact of the effort of each contestant on that probability is weighted by their military skill. A similar function has been employed extensively [44,45] and aims to capture the heterogeneity among contestants, followed by a growing and recent trend in the literature, which attempts to make the theoretical models more consistent with reality [46,47]. Jia et al. (2013) [48] discuss different CSFs, their theoretical foundations, and their empirical tractability.

A number of issues should be noted. First, the common pool, i.e., the prize for the contestants, is endogenous, since the quantity of C produced by the two contestants depends on the allocation of the initial endowment. Second, the cost of the war is implicit in the formulation of the model. While we do not introduce any parameter capturing it, allocating a share of the initial endowment to the war implies a reduction in the resources for the production of the consumption good. This fact measures the (indirect) cost of the trade-off in the *butter–guns* model, which is used in our paper [49].

Although fighting is the most common and immediate approach to solving disputes between contestants, we aim to explore a different solution concept. Our idea is to investigate the possibility that players discuss before fighting and decide to avoid the war by forming a union. The latter may generate some advantages since players can jointly produce the consumption good and share their technologies. It follows that after forming a union, they adopt the most efficient technology to produce the consumption good and share the control over the available military technology. Then, the joint production is divided according to a specific rule.

The most important point to be stressed is that any agreement must be self-enforcing since there is no third party that may impose a specific behaviour to avoid war. In such a framework, the strategy for each contestant consists of various elements, since they need to decide:

1. How to allocate their initial endowment between the production and the war;
2. How to divide the commonly produced good if they decide to form a union;
3. Whether to maintain or to break down their union agreement.

As suggested by Grossman (2004) [40], each player picks up an element in its strategy set, which is relevant to the specific situation it faces. In other words, if contestants fight over the control of the common pool, they decide the optimal allocation of their initial endowment between *butter* and *guns*. Instead, if they reach an agreement and form a union, they set up an optimal sharing rule, which fairly distributes the joint production.

We set some assumptions to make the tractability of the model as simple as possible. As we will clarify in their discussion, although they may look strong, they are still consistent with many real situations.

Assumption 1. *$R_1 > R_2$; $\beta_1 > \beta_2$; $\gamma_1 < \gamma_2$, $C_i, G_i < R_i$ and $\beta_2 R_2 \in \left[\frac{1}{3}\beta_1 R_1; \frac{2}{3}\beta_1 R_1\right)$.*

Assumption 1 defines the characteristics of contestants. It indicates that player 1 is richer than its opponent (first inequality) and owns the most efficient technology to produce the consumption good (second inequality). In turn, its opponent displays more effective military skills (third inequality). While such an assumption does not imply specialization in one activity, it underlines that player 1 finds it marginally more convenient to produce the commodity good (*direct benefit*) rather than embarking on a fight with its opponent. The fourth inequality, $C_i, G_i < R_i$, imposes the constraint that both players always allocate a fraction of their endowment to both production and war. This avoids the possibility of a corner solution, which occurs when both contestants (or even only one of them) allocate all of their resources to either the production of the consumption good or the war. Under some specific circumstances, both players allocate all of their resources to the production of the consumption good, which is a case that has been extensively analysed in the literature, for instance, by Skaperdas (1992) [41] and Muthoo (2004) [35], and has emerged as a repeated equilibrium. Moreover, the possibility that one contestant allocates all of their resources to the war, while its opponent chooses to allocate its endowment to production, is meaningless, given our scenario. Since players live in an anarchic environment and the rule of strength is in power, they do not have any incentive to produce only the consumption good since it would be subject to expropriation. Finally, the last inequality implies that although $\beta_2 R_2 < \beta_1 R_1$, the amount of consumption goods produced by contestant 2 in a peaceful scenario is not smaller than $\frac{1}{3}$ and not greater than $\frac{2}{3}$. This assumption is quite important to understand the development of our model and, in general, the characterization of a war equilibrium. The misalignment between wealth and military capabilities is necessary to justify the emergence of a war. If a player is richer and more powerful than the other, it might be unlikely (although still possible) that it will fight against its opponent [7]. Instead, being more powerful but less rich than the other contestant implies that a player would be interested in re-balancing the distribution of wealth, especially in an anarchic context. In other words, Assumption 1 implies that the probability of a war is not zero, letting us exclude other situations, in which a fight cannot emerge.

Assumption 2. *Each player acts as a unitary agent.*

The second assumption indicates that we do not model the process, which leads to a specific decision for each player. While some scholars focus on the political process, which determines the formation of preferences [50–54], Assumption 2 is not uncommon in international relations [24,40]. Nonetheless, although we do not investigate the political process, which leads to a specific decision, our model does not lose generality. In particular, we assume that when a player decides to fight a war or to live in a peaceful environment, it has already solved the internal issues that may have determined such a decision. In addition, even though internal discussions have not been solved yet, we observe that externally a player behaves in a specific manner. A typical example is offered by the recent Russian actions. The decision to start a special operation in Ukraine generated a lot of internal discussions about its opportunity. In fact, those discussions are still going on, although the international position of Russia is clear, as well as it is unambiguous in its final decision. The choice of not modelling the internal debate about a specific action should be understood as a way to keep our model as simple as possible. Eventually, we are interested in the consequence of a specific action. Whether a certain decision is internally under debate does not affect our analysis. Modelling the political process can offer us insights about under which circumstances a specific decision is supported/opposed by the majority. Hence, if we removed Assumption 2, we could investigate the variety of factors that may lead to specific deliberation. However, this is beyond the scope of the present work, where we want to analyse the consequence of a decision, that, in a certain way, we consider exogenously given.

Assumption 3. *Interaction between players is characterised by perfect information.*

Some scholars [55,56] develop models, which are characterised by asymmetric (or imperfect) information among players. In our model, we take on board the approach that is not uncommon, especially in international relations, that players share the same amount of information [57] about the structure of the game they are going to play and the characteristics of their opponents. Someone could argue that such an assumption is too strong and possibly unrealistic. However, this may not be the case. On one hand, perfect information implies that players are fully aware of the possible development of their relations. We assume that they know about the necessity of choosing the optimal allocation of their resources if they move to the war. Moreover, if they talk to each other to decide whether to form a union, they are fully aware of the timing of the game: they know that they need to decide on a sharing rule, to produce the consumption good jointly and to share it. They also know that they could deviate from such a cooperative equilibrium and are aware that if this happens, their opponent will deviate from the union equilibrium forever (trigger strategy). While perfect information about the structure of the game is not likely to generate concerns, the assumption that players can be perfectly informed about the characteristics of their opponents could appear unrealistic. Moreover, in this case, however, this is not necessarily true. The key point in our model is that one player is richer and owns a better production technology than its opponent, while the latter displays better military skills. In international relations, which is possibly the best example of an anarchic environment, it is not uncommon that this is exactly the case. It is well-known, for instance, that some countries are richer than others. Moreover, it is not ambiguous that some have better productive skills than others. Instead, it might be less clear whether they have similar/different military skills. This could be an issue that may compromise the validity of our assumption. However, while players may tend to hide their military skills, it is still likely that such information will spill out, albeit not officially confirmed. For instance, it is well-known that Israel owns nuclear weapons. There is not any doubt among Israeli allies that the country can be considered a nuclear-armed state [58]. Nonetheless, nobody has ever officially confirmed this. However, intelligence was also able to provide an estimate of the number of nuclear bombs the country owns. If a conflict occurs between Israel and some other countries, would it be a surprise if Israeli military command decides to strike a nuclear attack? The answer is obvious. It is clearly true that in some circumstances, a player may underestimate the true military skills of their opponent. For instance, during the US–Vietnam conflict, US military command were likely to have estimated the capability of its opponent incorrectly. A similar situation can be viewed in the recent developments of the Russia–Ukraine conflict. However, it must be acknowledged that such erroneous evaluations could eventually be the consequence of a miscalculation, not the result of a lack of information.

The argument developed above offers support in favour of our assumptions. It is worth underlining once again that we introduced them to simplify the structure of our game and, therefore, to make our model more tractable. Nonetheless, as argued above, they are not necessarily too far-fetched, but they appear consistent with a large variety of real situations.

4. State of Nature Equilibrium

We start by characterizing the equilibrium in the anarchic (or state of nature) environment, which will be used as a benchmark. In equilibrium, each player allocates the initial endowment between *butter* and *guns* to maximise the output that is expected to be obtained from the war, subject to the resources constraint (1) and taking into consideration the probability of success in the war. Using Equation (2), the maximization problem can be written as follows:

$$\max_{G_i} V_i = \frac{\left(\frac{G_i}{\gamma_i}\right)^m}{\left(\frac{G_i}{\gamma_i}\right)^m + \left(\frac{G_j}{\gamma_j}\right)^m} \sum_{i=1}^{2} \beta_i \left[R_i - \frac{G_i}{\gamma_i}\right] \; \forall i \neq j \in \{1,2\}. \tag{5}$$

Maximization yields the following set of first-order conditions (FOCs):

$$\frac{\frac{m}{\gamma_1}\left(\frac{G_1}{\gamma_1}\right)^{m-1}\left(\frac{G_2}{\gamma_2}\right)^m}{\left[\left(\frac{G_1}{\gamma_1}\right)^m + \left(\frac{G_2}{\gamma_2}\right)^m\right]^2} \sum_{i=1}^{2} \beta_i \left[R_i - \frac{G_i}{\gamma_i}\right] \quad = \quad \frac{\left(\frac{G_1}{\gamma_1}\right)^m}{\left(\frac{G_1}{\gamma_1}\right)^m + \left(\frac{G_2}{\gamma_2}\right)^m} \frac{\beta_1}{\gamma_1}, \tag{6}$$

$$\frac{\frac{m}{\gamma_2}\left(\frac{G_1}{\gamma_1}\right)^{m}\left(\frac{G_2}{\gamma_2}\right)^{m-1}}{\left[\left(\frac{G_1}{\gamma_1}\right)^m + \left(\frac{G_2}{\gamma_2}\right)^m\right]^2} \sum_{i=i}^{2} \beta_i \left[R_i - \frac{G_i}{\gamma_i}\right] \quad = \quad \frac{\left(\frac{G_2}{\gamma_2}\right)^m}{\left(\frac{G_1}{\gamma_i}\right)^m + \left(\frac{G_2}{\gamma_2}\right)^m} \frac{\beta_2}{\gamma_2}, \tag{7}$$

In the above equations, the left-hand side (lhs) indicates the marginal return from the fight, while the right-hand side (rhs) displays the marginal cost. Moreover, they define the best response of each player in terms of the opponent's optimal choice. Differentiating G_i with respect to G_j yields, after rearranging:

$$\frac{\partial G_i^{BR}(G_j)}{\partial G_j} = \frac{m^2}{\gamma_j} \frac{\left(\frac{G_i}{\gamma_i}\right)^{m-1}\left(\frac{G_j}{\gamma_j}\right)^{m-1}}{\left(\left(\frac{G_i}{\gamma_i}\right)^m + \left(\frac{G_j}{\gamma_j}\right)^m\right)^2} \sum_{i=1}^{2} \beta_i \left[R_i - \frac{G_i}{\gamma_i}\right] - \frac{\left(\frac{G_i}{\gamma_i}\right)^{-1}\left(\frac{G_j}{\gamma_j}\right)^m}{\left(\left(\frac{G_i}{\gamma_i}\right)^m + \left(\frac{G_j}{\gamma_j}\right)^m\right)} \frac{\beta_j}{\gamma_j}$$

where G_i^{BR} denotes the best response of player i in terms of j's optimal choice. We note that the above derivative is positive:

$$\frac{m^2}{\gamma_j}\frac{\left(\frac{G_i}{\gamma_i}\right)^{m-1}\left(\frac{G_j}{\gamma_j}\right)^{m-1}}{\left(\left(\frac{G_i}{\gamma_i}\right)^m+\left(\frac{G_j}{\gamma_j}\right)^m\right)^2}\sum_{i=1}^{2}\beta_i\left[R_i-\frac{G_i}{\gamma_i}\right] > \frac{\left(\frac{G_i}{\gamma_i}\right)^{-1}\left(\frac{G_j}{\gamma_j}\right)^m}{\left(\left(\frac{G_i}{\gamma_i}\right)^m+\left(\frac{G_j}{\gamma_j}\right)^m\right)}\frac{\beta_j}{\gamma_j}$$

$$m^2\frac{\left(\frac{G_i}{\gamma_i}\right)^{m}\left(\frac{G_j}{\gamma_j}\right)^{-1}}{\left(\left(\frac{G_i}{\gamma_i}\right)^m+\left(\frac{G_j}{\gamma_j}\right)^m\right)}\sum_{i=1}^{2}\beta_i\left[R_i-\frac{G_i}{\gamma_i}\right] > \beta_j$$

$$\frac{m^2}{\beta_j}\frac{p_i}{G_j}\sum_{i=1}^{2}\beta_i\left[R_i-\frac{G_i}{\gamma_i}\right] > 0$$

The latter result clearly indicates that a peaceful relationship between players can be compromised by expanding the amount of resources allocated to the war. If contestants make such a decision, they are likely to start an arms race. In particular, this may occur when a strong contestant faces a new rising power, suggesting that the incumbent is not willing to accept the new status of its opponent. Under these considerations, war becomes the most appealing outcome. A number of examples from the real world fit into such a situation. For instance, a striking example is offered by the choices made by the US and USSR during the Cold War or by India and Pakistan between 1998 and 2002, when each of them tried to confirm their supremacy in the region. While the India–Pakistan arms race did not develop into a formal war, the risk has been estimated to be high, since they had long-lasting harsh relations.

Another useful information provided by Equations (6) and (7) is that, in equilibrium, the ratio of the resources that each player allocates to the war, after simplification, is:

$$\frac{G_i^*}{G_j^*}=\left(\frac{\beta_j}{\beta_i}\right)^{\frac{1}{1+m}}\frac{\gamma_i}{\gamma_j} \tag{8}$$

Equation (8) indicates that, in equilibrium, the amount of resources allocated to the war depends on the productivity of each contestant in the two alternative activities. Moreover, given Assumption 1, i allocates fewer resources to the war than its opponent j (i.e., $G_i^* < G_j^*$). In other words, the player that displays better skills in the production of the consumption good prefers to allocate more resources to that activity. This obviously translates into a lower probability of success in the war. Using Equation (8) and noting that the CSF can be written in terms of the ratio of the effective allocations decided by the players, we obtain:

$$p_1^*=\frac{1}{1+\left[\frac{\beta_1}{\beta_2}\right]^{\frac{m}{m+1}}}<\frac{1}{2}<\frac{1}{1+\left[\frac{\beta_2}{\beta_i}\right]^{\frac{m}{m+1}}}=p_2^* \tag{9}$$

The probability of success for i decreases with the ability to produce the consumption good, while it increases with the opponent's skills. This is because, the more efficiently a player is able to produce C, the less convenient it is to allocate resources to the war. Interestingly, the probability of success does not depend directly on military technology. Given the argument above, if one contestant displays a more efficient technology in one of the two alternative productions, it will reduce the number of resources allocated to the less productive activity. Solving Equations (6) and (7) for G_1 and G_2 yields the following result:

Proposition 1. *In the context of the state of nature, the Nash equilibrium in the pure strategy is characterised by the following pair of equations:*

$$G_1^* = \frac{\frac{m}{m+1}\gamma_1 R^*}{\beta_1\left[1+\left(\frac{\beta_2}{\beta_1}\right)^{\frac{m}{m+1}}\right]} \tag{10}$$

$$G_2^* = \frac{\frac{m}{m+1}\gamma_2 R^*}{\beta_2\left[1+\left(\frac{\beta_1}{\beta_2}\right)^{\frac{m}{m+1}}\right]} \tag{11}$$

where $R^* = \beta_1 R_1 + \beta_2 R_2$*. The above is the unique and interior Nash equilibrium of the game.*

Proof. In the Appendix A. □

Through the paper, we use the symbol "∗" to mark the equilibrium values, which are derived in the war game. The efficiency of the two production technologies parametrises the opportunity cost of allocating more resources to the war than to the production of the consumption good and, *viceversa*, imposes a balance in the allocation of resources. As Powell (1993) [7] points out, allocating more to production has a *direct* effect, since this choice directly impacts the wealth of a player. However, choosing to allocate resources to the war has an *indirect* effect, since a contestant may gain more if successful in the war. Clearly, each contestant needs to strike a balance between these two alternatives, the choice being determined by their skills. Hence, the players' heterogeneity has the consequence that they choose a different distribution of initial resources between the alternative uses.

The equilibrium solution allows us to study how much the optimal allocation changes when players experience a variation in their productive skills. Equations (10) and (11) clarify that the optimal allocation of the initial endowment depends on each player's own military skills, on the productive skills of both, and on the effective parameter m. Therefore, given the argument developed above regarding the value of the probabilities in equilibrium, the allocation for one player is independent of how good its opponent is at producing weaponry. Therefore, the following results can be established In the Appendix B, we propose a sensitivity analysis, which appears to confirm the results that we prove mathematically in our propositions. Readers that are interested in this approach can look at it):

Proposition 2. *If one of the contestants develops a better military technology, it will increase the share of resources allocated to the war:*

$$\frac{\partial G_i^*}{\partial \gamma_i} > 0 \tag{12}$$

Instead, if a contestant develops a more efficient technology to produce C, it will reduce the resources allocated to the war, while its opponent will increase its effort in terms of military activity:

$$\frac{\partial G_i^*}{\partial \beta_i} < 0 \quad \textit{and} \quad \frac{\partial G_j^*}{\partial \beta_i} > 0 \tag{13}$$

Finally, if m increases, both players employ more resources in the war, provided that the difference in their productive skills is not too small:

$$\frac{\partial G_i^*}{\partial m} > 0 \tag{14}$$

Proof. In Appendix A. □

The result in Proposition 2 stems directly from the situation that contestants face and is generated by the trade-off between *butter* and *guns*. When a more efficient military technology is developed, i increases the share of R, which is dedicated to the war. In this case, military activity becomes comparatively more appealing than the production of the consumption good. This, in turn, generates further costs for society, reducing the total amount of C. Such behaviour is further confirmed by the fact that player i allocates more resources to the war if it develops a better production technology. On the contrary, in this case, its opponent finds it more convenient to increase its effort in the war, since any improvement in its opponent's technology to produce the consumption good expands the common pool, making the war more attractive.

Finally, the previous result indicates that an increase in m makes the war more appealing since the resources employed in the war become more effective in generating a larger probability of success. It should be noted that this result always holds for player 1, while it is true for its opponent only if its productive skills are not too small in comparison. It should be kept in mind that the allocation of the initial endowment generates a direct (production of C) and an indirect effect (success in the war). Hence, contestants need to balance these two alternative uses. When m increases, lower production skills could be compensated for by an increase in the allocation of resources to the production of C without reducing (or even possibly increasing) the probability of success in the war. Instead, if β_2 is sufficiently large—$\beta_2 \in \left[\frac{\beta_1}{2}; \beta_1\right)$—then player 2 would find it more beneficial to increase the allocation of resources to the war to secure a larger probability of success.

In both cases discussed above, the war becomes more likely to occur, since at least one player displays a greater interest in that outcome.

Welfare Implication of the War

The model allows us to measure the extent of welfare loss generated by the war. Replacing Equations (10) and (11) in the objective function (5) and rearranging them yields:

$$V_1^* = p_1^* \left[\frac{R^*}{1+m}\right] \tag{15}$$

$$V_2^* = p_2^* \left[\frac{R^*}{1+m}\right] \tag{16}$$

The latter implies that in equilibrium, the total amount of consumption good is

$$V^* = \frac{R^*}{1+m} \tag{17}$$

The welfare loss is parametrised by m. The larger it is, i.e., the larger the sensitivity of the CSF to m, the lower the total amount of the consumption good produced by players. If $m \to 0$, the probability of success for a player is unresponsive to the amount of resources allocated to the war. Eventually, players, showing an identical probability of success ($p_i = p_j = \frac{1}{2}$), would allocate the total amount of resources to the production of the consumption good, which, in total, would be equal to R^*, corresponding to the amount that would be produced in a peaceful setting. Instead, if $m \to 1$, the war becomes an appealing outcome, since the probability of success is fully responsive to the amount of resources allocated by players. In this case, p depends on the difference in the efficiency of the technologies employed by players, who will find it convenient to allocate as many resources as they can to the war. Eventually, the total amount of consumption goods produced in the world would be equal to half of that produced in a peaceful setting.

From Equations (15) and (16), the following results can be established:

Proposition 3. *If player i develops a more efficient technology to produce the consumption good, both contestants increase their expected outcome from the war:*

$$\frac{\partial V_i^*}{\partial \beta_i} > 0 \quad and \quad \frac{\partial V_j^*}{\partial \beta_i} > 0 \tag{18}$$

Moreover, any increase in m reduces the expected payoff:

$$\frac{\partial V_i}{\partial m} < 0 \tag{19}$$

Proof. In Appendix A. □

The first result in Proposition 3 stems from the fact that any improvement in the technology used to produce the consumption good enlarges the dimension of the common pool over which players fight. Therefore, other things being equal, the (expected) outcome of the war will increase.

The second result, instead, is an immediate consequence of the fact that players will find it more convenient to allocate as many resources as they can to the war if the effective parameter, m, increases. Hence, this reduces the availability of resources for the production of the consumption good. In this case, the dimension of the common pool shrinks, reducing the welfare of both contestants.

5. Union Set Up

As anticipated at the beginning of the paper, we depart from the standard view that players may avoid the war by enforcing a peaceful agreement, which is the result of an infinitely repeated game, where they "learn" that the fight is not an efficient outcome. Instead, we want to take a step further and investigate the possibility that they meet and discuss before the fight, reaching an agreement which consists of forming a union.

Such an agreement has several consequences, the most important being that contestants share their technologies. However, if they do not reach an agreement, fighting is the only available choice for both of them. Such a scenario is modelled as a two-stage game, with the following timing:

1: In the first stage, players bargain over a sharing rule to divide the jointly produced commodity good;
2: In the second stage:
  - 2a: If they are unable to reach an agreement, the war is the only option to solve the dispute and contestants continue to live in anarchy;
  - 2b: If they reach an agreement in the first stage, they jointly produce the consumption good and share it according to the agreed rule.

The second stage is characterised by two alternative situations, depending on whether players agreed/did not agree on a sharing rule in the first stage. If they did not reach an agreement and moved to war, the emerging equilibrium is that derived in Section 4.

In this part of the paper, we first carry out a one-shot analysis, assuming that the agreement lasts for one day only. We may interpret this situation as an opportunity to explore the advantages of forming a union. Moreover, the analysis gives us the possibility to evaluate the impact of some specific parameters of the model. In this context, we do not observe any share of technology.

Secondly, we perform a long-run analysis to derive the conditions to guarantee that the agreement is self-enforcing. In this case, a transfer of technologies takes place.

5.1. One-Shot Analysis

It is worthwhile to stress that players accept forming a union if and only if this choice makes them better off. The model is solved backwards.

Second stage

If contestants reach an agreement in the first stage, in the second, they jointly produce the consumption good using only the most efficient production technology. In this case, all of their initial endowment is allocated to the production of C. Therefore, the produced amount is (The symbol "$**$" is used to denote the equilibrium values when forming a union):

$$V^{**} = \beta_1 [R_1 + R_2] \tag{20}$$

Immediately, we can notice that players have a comparative advantage when forming a union; otherwise, they would not be willing to do so. Hence:

Lemma 1. *The joint production of the consumption good is always larger than that in the anarchic context. Moreover, it is even larger than the one that players would obtain if they lived in peace without creating a union.*

The previous lemma is straightforward. If contestants do not fight and do not form a union, each of them would enjoy an amount of consumption good equal to $\beta_i R_i \ \forall i \in \{1,2\}$. Moreover, using the fact that $\beta_1 > \beta_2$, we may note that:

$$\left[\frac{\beta_1 R_1 + \beta_2 R_2}{1+m}\right] \leq [\beta_1 R_1 + \beta_2 R_2] < \beta_1 [R_1 + R_2] \tag{21}$$

The lhs of inequality (21) measures the production of the consumption good if a war is fought, while the term in the middle indicates the amount of consumption good when players neither fight nor form a union. Finally, the rhs is the amount of production when they form a union. The latter is unambiguously larger than the previous two values.

Inequality (21) guarantees the existence of a comparative advantage from a union agreement, but it also stresses that this solution generates (jointly) the largest wealth for contestants.

First stage

Solving the game backwards, in the first stage, we can derive the optimal sharing rule, x, through the following maximization:

$$\max_x [xV^{**} - V_1^*]^{\phi} [(1-x)V^{**} - V_2^*]^{1-\phi} \tag{22}$$

The equation above corresponds to the Nash bargaining solution (NBS), which is a cornerstone concept in game theory. It is used to describe a situation in which two players attempt to maximise a joint surplus cooperatively. Specifically, they try to find a rule to share joint production (V^{**} in our case), conditional on some disagreement outcomes (in our case, what they would obtain if they fought, i.e., V_1^* and V_2^*) if they fail to reach an agreement. The above maximization indicates that players will agree on a sharing rule, provided that it will guarantee to each of them an amount of output not smaller than what they would receive after the war. Hence, the payoff from the fight is meant to indicate the minimum amount of the joint product that they will be ready to accept to reach an agreement. Therefore, the advantage of the NBS is that it explicitly takes into consideration the consequence of not reaching an agreement. Assuming that players will fight forever if they do not reach an agreement is reasonable since if they fail to strike an agreement when seated at the same table, it is quite unlikely that they can reach an agreement otherwise. Moreover, in Equation (22) ϕ and $(1-\phi)$ measure the bargaining power of each player, introducing a further asymmetry between contestants. Maximization yields:

$$x^{**} = \phi + \frac{(1-\phi)V_1^* - \gamma V_2^*}{V^{**}} \tag{23}$$

Making use of the fact that $V_1^* + V_2^* = V^*$, the following result can be established:

Proposition 4. *If contestants form a union, they consume the following amounts:*

$$\begin{aligned} V_1^{**} &= V_1^* + \phi(V^{**} - V^*) \\ V_2^{**} &= V^{**} - V_1^* - \phi(V^{**} - V^*) \end{aligned} \tag{24}$$

In equilibrium, player 1 obtains the amount of commodity goods that it would obtain after the war, augmented by a fraction of the difference between joint production and what is globally produced if it came to war. That portion is determined by its bargaining power. Consequently, player 2 receives the remaining part.

As expected, the distribution of the bargaining power between players is a key determinant of the outcome. If $\phi \to 0$, the previous set of equations reduces to:

$$V_1^{**} = V_1^*$$
$$V_2^{**} = V^{**} - V_1^*$$

In this case, player 1 obtains precisely the same (expected) output that it would obtain after the war, while its opponent receives the remaining portion. The situation is reverted when $\phi \to 1$. Hence:

Corollary 1. *Any decrease in ϕ reduces the gain from a union agreement for player 1. The opposite occurs if ϕ increases.*

Interestingly, the distribution of gains after forming a union critically affects the decision to follow that path. As noted before, players reach a union agreement if and only if this choice has a positive impact on their welfare. Since contestant 1's gain reduces when ϕ decreases, the smaller its bargaining power, the smaller its gain in this scenario and, consequently, the less inclined, it is toward this solution. Eventually, if its bargaining power is 0, it will decide not to form a union but to remain in the state of nature. The same applies to player 2 when ϕ increases.

While ϕ is simply considered as a measure of bargaining power, it can also be interpreted in a deeper way. Binmore, Rubinstein, and Wolinsky (1986) [59] suggest that the asymmetry introduced by a different bargaining power can be due to a variety of reasons, including some possible contestants' specific beliefs about the details of the environment in which they operate. They also show that the asymmetric Nash bargaining solution is the unique perfect equilibrium of a negotiation process between two parties with asymmetric beliefs about the likelihood of a breakdown when the length of each bargaining period becomes infinitely small. In other words, the higher player 1's estimate of the probability of breakdown, the lower ϕ is. Therefore, the bargaining power can be interpreted as a measure of the probability that each contestant gives to the possibility of successful bargaining. This explanation is consistent with the content of corollary 1 since a small value of ϕ implies a small gain for player 1 from forming a union. In the limit, when $\phi = 0$, contestant 1 gains nothing from the union and prefers to live in anarchy since they display little or even no confidence in the success of bargaining.

Another interesting interpretation of ϕ follows an institutional approach suggested by Svejnar (1986) [60]. The author argues that the bargaining power can be "…influenced by institutional, economic, and other variables…" Hence, the author includes in the theory of bargaining the possibility that ϕ parametrises other characteristics of parties, which are not explicitly included in their utility functions and in the bargaining process. Such an interpretation is quite interesting in our context, since, among other aspects, it introduces a further element of heterogeneity. Although we are not specifically interested in determining which variables affect ϕ, we may still offer some suggestions to interpret it.

For instance, Assumption 2 of the model states that the formation of preferences is characterised by unanimity. Nonetheless, one of the determinants of the bargaining power of a player could be precisely the extent of homogeneity in the individual preferences. A large ethnic/religious fractionalization may reduce the bargaining power and, eventually, diminish the possibility of negotiating an agreement.

Another interesting possibility would be to connect ϕ to the degree of cultural homogeneity. Contestants with similar cultures may have a greater chance of reaching and maintaining a union agreement in the long run. This idea relies on the extensive literature in economics and political science, which explores the impact of cultural traits on different issues, such as economic development [61], saving behaviour [62], technological progress [63], the functioning of the labour market [64], and international relations [11,65,66].

Interpreting the bargaining power in terms of cultural homogeneity can help us to explain several cases that have occurred in recent decades. For instance, the lack of similar cultural values determined, among other things, the breakdown of former Yugoslavia. At the end of the First World War, the Balkans lived in a period of strong distress. The proposed solution was to create a new political entity by merging together the countries of Serbia, Croatia, Slovenia, Macedonia, Bosnia, and Montenegro. It is interesting to note that these countries were deeply different from a cultural and religious point of view. As a matter of fact, during the Great War, Slovenia and Croatia supported the Austrian Empire, while the other countries allied with Great Britain, France, and Russia. The deep differences among them emerged dramatically at the beginning of the 1990s, determining the breakdown of Yugoslavia. A similar process occurred in the USSR, and, again, the lack of a similar cultural identity can explain, to a certain extent, the breakdown of the country.

On the other hand, the presence of common cultural values favoured the reunification of West and East Germany, which at the end of the Second World War were divided because of political reasons. In a similar fashion, albeit with obvious differences, we can interpret the process of integration that took place in the 1950s among some European countries. While in this case, the process was determined by economic reasons, it rapidly developed in the following decades, although the European Union cannot yet be considered an established new political entity. Nonetheless, to be successfully admitted within the EU, among other things, according to the Treaty of the European Union, candidates need to commit to the respect of freedom, democracy, and the state of law. Hence, to be part of that community, countries are required to share common values and beliefs, enforcing cultural homogeneity among its members.

5.2. Long-Term Analysis

If players form a union, they should be able to support the agreement in the long run. In this section, we investigate this possibility, deriving the incentive compatibility requirements that make this possible.

It is important to remember that keeping the union, in the long run, implies some consequences in terms of the new allocation of technologies. While player 2 partially gives over control to the most efficient military, its opponent shares the best technology to produce the consumption good. In addition, we assume that player 2 cannot instantaneously obtain the latter, but it embarks on a learning process, which in the long run should allow it to use this technology fully.

The above issues have some relevant consequences, especially when we consider the possibility that contestants deviate from the union path. If they break down their agreement, they will enjoy a different endowment of technology with respect to the one that they had before they formed a union.

Given a parameter $0 \leq \rho \leq 1$, each player will obtain the following amount of military technology, respectively (The symbol "ˆ" is used to denote the parameters and the equilibrium values which emerge in the anarchic environment, where players adopt new technologies):

$$\hat{\gamma}_1 = \rho[\gamma_1 + \gamma_2] \tag{25}$$
$$\hat{\gamma}_2 = (1-\rho)[\gamma_1 + \gamma_2] \tag{26}$$

Equations (25) and (26) imply that the two military technologies are not overlapping. This may simply occur when each contestant specialises in a specific technology, which becomes available to both of them. Instead, regarding the technology used to produce the consumption good, if players split, they control, respectively:

$$\hat{\beta}_1 = \beta_1 \tag{27}$$
$$\hat{\beta}_2 = \beta_2 + \lambda\beta_1 \tag{28}$$

While player 1 maintains its own technology, player 2 increases its production skills. In particular, the parameter $\lambda \in \{0,1\}$ parametrises the ability of player 2 to learn the new technology. Moreover, if contestants deviate from the union path, we assume that contestant 1 still owns the best production technology. In other words:

$$\beta_2 + \lambda\beta_1 < \beta_1 \tag{29}$$

Equation (29) implies:

$$\lambda < \frac{\beta_1 - \beta_2}{\beta_1}$$

To satisfy the previous inequality, we assume that $\lambda = {}^{(\beta_1 - \beta_2)}/_{\kappa\beta_1}$, where $\kappa > 1$ captures the learning error of player 2. Substituting this value into Equation (28) yields:

$$\hat{\beta}_2 = \frac{\beta_1 + (\kappa - 1)\beta_2}{\kappa} \tag{30}$$

Clearly, the technology used by player 2 to produce the consumption good decreases in κ, since any increase in the learning error results in a less efficient technology after leaving the union.

Based on the elements above, we can focus on the conditions that support the decision to maintain the union when players repeatedly interact with each other. Life in such a new scenario develops as follows. After forming a union, players jointly produce the consumption good, which is divided according to an agreed sharing rule. However, before they share the joint production, one contestant may steal the entire output. If this occurs, the other contestant will return to anarchy. Hence, they use a trigger strategy: they remain together unless one of them deviates from that path, the latter option determining that a new anarchic equilibrium, characterised by a new distribution of technologies, takes place.

We set δ as the common rate at which players discount their expected amount of output V_i^{**}. Player i does not deviate if and only if:

$$V^{**} + \sum_{k=1}^{\infty} \delta_i^k \hat{V}_i \leq \sum_{k=0}^{\infty} \delta_i^k V_i^{**} \tag{31}$$

The rhs of Equation (31) indicates that if i deviates from the union path and steals all the daily jointly produced output (V^{**}), from the day after, it will enjoy the (expected) discounted amount of the consumption good obtained from the war ($\sum_{k=1}^{\infty} \delta^k \hat{V}_i$). It is important to remark again that in this case, the amount of consumption goods obtained by players in the anarchy is not V_i^*, but $\hat{V}_i$ since we need to take into consideration the new distribution of technologies.

Solving Equation (31) for δ yields the critical discount factors, which guarantee that contestants do not have any incentive to deviate:

Proposition 5. *Players sustain the union path in the long run if and only if:*

$$\delta^{**} \geq \max\left\{\underline{\delta}_i, \underline{\delta}_j\right\} \; \forall i \neq j = 1,2$$

where

$$\underline{\delta}_i = \frac{V_j^{**}}{V^{**} - \hat{V}_i} \tag{32}$$

Proof. The discount factors in Equation (32) are calculated by solving the incentive-compatibility constraint, after rewriting it as follows:

$$V^{**} + \frac{\delta}{1-\delta}\hat{V}_i \leq \frac{1}{1-\delta} V_i^{**}$$

Solving the above inequality for δ yields $\underline{\delta}_1$ and $\underline{\delta}_2$. □

We can interpret the content of the above proposition as follows. Player i (j) does not deviate if the discount factor is not smaller than $\underline{\delta}_i$ ($\underline{\delta}_j$). Hence, δ^{**} must be at least as large as the highest critical discount factor to avoid any incentive to deviate for both players.

It is interesting to note that the possibility of maintaining the union is affected by the distribution of the bargaining power. More specifically, if ϕ is very low (and possibly equal to 0), $\underline{\delta}_1$ the discount factors approach 1. Instead, if it is very high (and possibly equal to 1), $\underline{\delta}_2 \to 1$. This implies that an unequal distribution of the bargaining power makes at least one of the contestants less inclined toward the union path. This is an immediate consequence of the fact that when ϕ takes extreme values, the gain from forming a union is reduced for at least one of the players, making this solution less appealing.

This point also emerges from a comparison of the two discount factors, with ϕ determining which of them is larger:

$$
\begin{aligned}
\underline{\delta}_1 &\lesseqgtr \underline{\delta}_2 \\
1 - \frac{\phi\left(V^{**} - \widehat{V}\right)}{\left(V^{**} - \widehat{V}_1\right)} &\lesseqgtr \frac{\left[\widehat{V}_1 + \phi\left(V^{**} - \widehat{V}\right)\right]}{\left(V^{**} - \widehat{V}_2\right)} \\
1 &\lesseqgtr \frac{\phi\left(V^{**} - \widehat{V}\right)}{\left(V^{**} - \widehat{V}_1\right)} + \frac{\left[\widehat{V}_1 + \phi\left(V^{**} - \widehat{V}\right)\right]}{\left(V^{**} - \widehat{V}_2\right)} \\
1 &\lesseqgtr \frac{\widehat{V}_1}{V^{**} - \widehat{V_2}} + \frac{\phi\left(V^{**} - \widehat{V}\right)}{V^{**} - \widehat{V_1}} + \frac{\phi\left(V^{**} - \widehat{V}\right)}{V^{**} - \widehat{V_2}}
\end{aligned}
$$

$$
\begin{aligned}
1 &\lesseqgtr \frac{\widehat{V}_1}{V^{**} - \widehat{V_2}} + \phi\left(V^{**} - \widehat{V}\right)\left[\frac{2V^{**} - \widehat{V}}{\left(V^{**} - \widehat{V_1}\right)\left(V^{**} - \widehat{V_2}\right)}\right] \\
\left(V^{**} - \widehat{V_2}\right) &\lesseqgtr \widehat{V}_1 + \phi\left(V^{**} - \widehat{V}\right)\left[\frac{2V^{**} - \widehat{V}}{\left(V^{**} - \widehat{V_1}\right)}\right] \\
\left(V^{**} - \widehat{V}\right) &\lesseqgtr \phi\left(V^{**} - \widehat{V}\right)\left[\frac{2V^{**} - \widehat{V}}{\left(V^{**} - \widehat{V_1}\right)}\right] \\
1 &\lesseqgtr \phi\left[\frac{2V^{**} - \widehat{V}}{\left(V^{**} - \widehat{V_1}\right)}\right]
\end{aligned}
$$

When $\phi \to 0$, the lhs is larger than the rhs, indicating that contestant 1 is less willing to follow the union path than its opponent. The opposite happens when $\phi \to 1$. From the above discussion, the following corollary can be established:

Corollary 2. *The sustainability of the union path, in the long run, depends on the distribution of the bargaining power between players, other things being equal.*

5.3. Relationship between κ and the Critical Discount Factors

The sustainability of the union, in the long run, implies that a change in the main parameters of the model can produce a variation in the discount rate.

Specifically, we want to verify how $\underline{\delta}_1$ and $\underline{\delta}_2$ vary when κ, the learning error, increases. Preliminarily, to simplify the calculation, we can rewrite the two discount factors as follows:

$$\begin{aligned}
\underline{\delta}_1 &= \frac{V_2^{**}}{V^{**} - \widehat{V}_1} \\
&= \frac{V^{**} - \widehat{V}_1 - \phi\left(V^{**} - \widehat{V}\right)}{V^{**} - \widehat{V}_1} \\
&= 1 - \frac{\phi\left(V^{**} - \widehat{V}\right)}{V^{**} - \widehat{V}_1} \\
\underline{\delta}_2 &= \frac{V_1^{**}}{V^{**} - \widehat{V}_2} \\
&= \frac{V_1^{*} + \phi(V^{**} - V^{*})}{V^{**} + \widehat{V}_2} \\
&= \phi + \frac{(1-\phi)\widehat{V}_1}{V^{**} - (1-p_1)\widehat{V}}
\end{aligned}$$

Differentiating $\underline{\delta}_1$ and $\underline{\delta}_2$ with respect to κ yields the following result:

Proposition 6. *When player* 2 *displays a larger error in learning the most efficient technology to produce the consumption good, the union can be supported in the long run if m is sufficiently low, i.e.,*

$$\frac{\partial \underline{\delta}_1}{\partial \kappa} \lesseqgtr 0 \quad \textit{if} \quad m \lesseqgtr \widehat{m} \tag{33}$$

$$\frac{\partial \underline{\delta}_2}{\partial \kappa} < 0 \tag{34}$$

Proof. In Appendix A. □

The last proposition indicates that player 2 always becomes more patient when κ rises. This result is expected since any increase in κ reduces the availability of the most effective production technology if the union breaks down. On the other hand, the same result holds for contestant 1, if m is sufficiently low. This finding appears less intuitive since it indicates that player 1 is willing to be more cooperative if its bargaining power is sufficiently low. However, this result can be explained by considering that the union solution always implies an improvement in the contestants' welfare. Therefore, player 1 is always better off enforcing such a solution instead of opting for war. Since the player's probability of success is smaller than that of its opponent and the war can have a negative outcome, it is always better to obtain something that is certain and larger than the (expected) profit from the war. On the contrary, if $m > \widehat{m}$, player 1 may become more impatient since it can erroneously believe that the union solution benefits its opponent more than itself since such a solution implies that player 2 can exploit the capability of its opponent to produce the consumption good. Therefore, player 1 becomes impatient, compromising the possibility of maintaining the union equilibrium in the long run.

Remarkably, deriving the conditions that guarantee the sustainability of the union in the long run shapes our understanding of the reasons that may lead players to break down. Hence, this offers an explanation about why some unions may break down at a certain point. This leads us (albeit indirectly) to contribute to the analysis of the unification/secession of countries.

6. Discussion

The novelty of our model is that players may choose to form a union to avoid an infinite fight since they live in an anarchic world. Our solution is different from the one usually proposed in the literature, which consists of finding an equilibrium where players renounce war and produce the consumption good in a peaceful environment. While in both cases, contestants "learn" the respect of property rights, our solution produces an improvement in their welfare.

Our model can be seen as a formal analysis of situations that may arise in the real world. This is particularly true within international relations since they can be a valid approximation of an anarchic environment, where players may not be forced to adopt a specific behaviour because of a lack of a third party. With some adjustments, the model can explain other cases, such as the merger of firms or banks. These situations are slightly different since firms and/or banks do not operate in a *state of nature* context, but within an institutionalised framework, which influences (and possibly sanctions) any illegal behaviour.

In this section, we provide some case studies to stress the applicability of our model to the real world, using a couple of examples from international relations. Specifically, we analyse the decision taken by East and West Germany to create a unified state at the beginning of the 1990s. Moreover, we discuss the disadvantage stemming from the choice of the UK to leave the EU in recent years. While our model analyses the consequences of forming a union (or, in the case of international relations, a new political entity), it also highlights from a dual perspective the negative consequences of its breakdown.

It is important to stress that our model offers a simplified picture of the world. Therefore, it does not pretend to address all of the issues that are connected to the decision to form a union or to leave it. Nonetheless, its main message displays a general validity that we try to address in the discussion of the case studies in the next subsections.

6.1. Germany

On 3 October 1990, West (FRG) and East Germany (GDR) announced their re-unification after more than 40 years. This choice was acknowledged as an incredible political success. From an economic point of view, the consequences of German re-unification are still under debate, since some open issues remain on the table, even after four decades.

When the re-unification was announced, a massive economic plan was launched to guarantee a quick and successful transition, which was intended to improve the social and economic conditions of the East. Those living in East Germany experienced a long-lasting period when socialist principles and rules were applied, which eventually depressed the GDR, compared with the FRG.

At the end of the Second World War, the country was separated into four areas of influence. West Germany was under the control of France, the UK, and the US, while East Germany was supervised by the USSR. This created difficult relations, which culminated in the building of the Berlin Wall in 1961. This action testified to the worsening of the relations between the two countries and the fact that they were not peaceful [67]. This obviously generated a large flow of resources in both countries designed to protect their borders from reciprocal interferences. Hence, from this point of view, the situation characterizing the relations between GDR and FRG is similar to that which we highlighted in the war scenario in our model.

After the re-unification, a large flow of resources was devoted to ameliorating the economic conditions of the East. For instance, at the end of the 1980s, the per capita GDP in the East was almost 40% less than in the West. Furthermore, labour productivity was approximately 30% lower in the GDR than in the FRG. In 1991, one year after the re-unification, the per capita GDP in the East was almost 32% of that in the West, but it increased up to 70% by the end of 2019. Even the unemployment rate constantly declined, with a larger reduction being noted in the former GDR. Moreover, a large training programme took place, with the aim of improving the productivity of former East Germany. This could be seen as a transfer of technology, which eventually produced an improvement in productivity. It was not possible to close the gap between the two areas, due to the different characteristics of the two economies. However, living standards have improved, and some new economic growth centres have emerged in former East Germany (for instance, Leipzig, Dresden, or Jena). While costly, especially in terms of public funding [68], the decision to unify the two countries produced a boost in both economies.

It can be stressed that re-unification was possible because of the large cultural homogeneity between the two countries. This is echoed in our model in the parameter ϕ, representing the bargaining power of the two countries.

6.2. United Kingdom

In June 2016, a referendum was held in the UK on leaving the European Union. Surprisingly, a slight majority of people voted to leave, possibly because they were convinced by Brexit supporters of the large welfare improvements that this choice could bring about.

After the referendum, a 5-year negotiation started between the UK and EU to regulate the future relationship between them. Although the risk of no deal was on the table, eventually, the two parties reached an agreement, and at the beginning of 2021, the UK and EU officially split.

Brexit supporters were confident that a larger amount of resources would be available for the internal market and to support people, leading to strong economic and social development.

This challenged the widespread pessimistic view that Brexit could generate negative consequences. For instance, using a multi-country, multi-sector, computable general equilibrium, Van Reenen (2016) [69] suggested that leaving the EU could generate a welfare loss between 1.3% and 2.6%, although a dynamic model including productivity effects offered a more pessimistic forecast, quantifying the loss between 6.3% and 9.5%.

While there could be some discrepancies in the correct amount, after Brexit took place, there is no longer any doubt that this choice had severe consequences on welfare in the UK. Springford (2022) [70] demonstrates that the British GDP, depurated by the effect of the pandemic, falls behind those of the most advanced economies by about 4.9%, suggesting that Brexit depressed the UK economy. In general, the per capita GDP in the UK has increased by 3.8% since the referendum, which is less than half of the growth in the EU zone (about 8.5%).

The reason for such a decline is mainly due to the Trade and Cooperation Agreement, signed by the UK and EU, introduced to regulate future commercial relations, given that Britain lost access to the European market. The aforementioned agreement imposed tariffs and trade barriers between the two parties, generating a consistent reduction in the trade between UK and EU, as stressed by Felbermayr, Groschl, and Steininger (2022) [71] and Fusacchia, Salvaticci, and Winters (2022) [72], who focus on textiles, motor vehicles, and the service industry.

The British market appears today to be closer and less competitive than it was before Brexit, generating a steady increase in consumer price, quantifiable at approximately 2.9% [73].

The situation does not change if we have a closer look at living standards. The British Office of National Statistics underlines that social spending remains below the level before the pandemic, testifying to the inability of the British economy to recover. In addition, the majority of students have faced great problems due to higher living costs.

All of the above facts attest to the difficulties faced by the UK following the decision to leave the EU. Although the pandemic hit the country and worsened its economic conditions, this cannot be judged as the only explanation for the aforementioned difficulties in the UK. The pandemic could be considered as a stress test, verifying the recovery capabilities of the country–the UK apparently failed to pass this test. In addition, leaving the EU precluded the possibility of accessing the Next Generation EU programme, which offers strong financial support to other European countries, helping them to recover from the pandemic.

Consistent with our model, the British choice of withdrawing from the European Union displays a (possibly) myopic advantage (in our model, this consists of enjoying a large amount of wealth in one period), but in the long run, it has produced a strong loss of welfare.

7. Concluding Remarks

Our paper contributes to the large literature about conflict, stressing that the lack of any enforcement of property rights generates a welfare loss. Specifically, we model a world characterised by anarchy (or *state of nature*), populated by two players, who produce a consumption good using an initial endowment. However, given the characteristics of the context in which they live, property rights are not enforced and, therefore, production is insecure and subject to expropriation. Therefore, contestants need to allocate a share of their resources to protecting their production, since war appears to be the most likely outcome.

The main features of the model appear to be consistent with many real situations, particularly those that arise in the context of international relations, which are considered to be a valid approximation of anarchy.

The existing literature on conflict resolution stresses the conditions that may prevent a fight. The majority of contributions in different fields argue that the set of values of parameters may drive contestants to discard the possibility that the war is non-empty. Hence, under some conditions, there exists the possibility that players decide not to fight, and instead produce their consumption goods peacefully, without forming a union.

In this paper, we attempt to investigate a different solution, deriving and analysing the possibility that players form a union. As clarified in this work, the latter can take different forms, such as a customs union or even a new political entity, if we focus on international relations. To the best of our knowledge, our paper represents the first attempt to explore such a different solution concept, representing its novelty.

Our model allowed us to derive a set of interesting results. The first and more important refers to the fact that players may greatly increase their welfare if they decide to form a union. This happens because only the most efficient technology is used in the production of the consumption good, and contestants may employ all of their initial endowment for that activity. In addition, we derive the incentive–compatible conditions which support this equilibrium in the long run. If players reach such an agreement, it must be self-enforcing, since, in anarchy, there is no third party that may impose any specific behaviour. Deriving the conditions that guarantee the maintenance of a long-term equilibrium gives us the possibility to investigate the conditions that may lead to the breakdown of the union since the set of parameters that may produce that result is not empty. Specifically, the latter cannot be achieved at all, or, if reached, it could collapse rapidly, bringing players back to anarchy.

In addition to the literature on conflict, our paper can also be understood as a contribution to the literature on the reasons why players may decide to form a union or to split, using an approach that has been poorly investigated in the literature. This is particularly relevant if we focus on international relations, which are the most typical example of the environment, as depicted in our model. Moreover, this focuses on the importance of the enforcement of property rights to avoid an undesirable result, such as war.

We acknowledge that our model introduces a simplified picture of the world. For instance, if we apply it to international relations, it does not take into consideration issues that are related to internal politics. For instance, we do not model the process that leads players to decide to take a certain action (for instance, to go to war or to form a union). While this can be seen as a weakness of our approach, it is nonetheless consistent with the economics, and political science literature [24,40]. Our model is intended to provide a message, stressing the advantage that forming a union may have on players' welfare compared with any other solution available in the literature. The discussion of some case studies in Section 6 offers an application of our model to the real world, stressing the beneficial effect of forming a union or, conversely, the negative consequences of breaking it down.

Our paper can be seen as a starting point for future research. For instance, additional issues can be addressed, and useful information can be obtained if we try to model the internal process that leads players to make a specific decision. In terms of countries, does the degree of heterogeneity (cultural, religious, or ethnic) impact the results of the model? Does the political orientation of the government play any role? Does the rule to aggregate preferences (majority or proportional) influence our results? A future extension of the model should take these points into account.

Furthermore, what are the consequences if we introduce a third player and allow for the formation of coalitions? Such a coalition could determine the specialization of each player in one activity (butter or guns). In this case, it would be interesting to analyse the conditions that make such a coalition stable in the long run, and how they would affect the general result of our model.

Another possible extension consists of assuming that the initial endowment is endogenous rather than exogenous. In a multi-period setup, for instance, we may investigate the possibility that players save a fraction of their resources for the next period. In other words, we may introduce an investment function, which explains how the endowment evolves over time.

Finally, a further possible direction for our research could consist of assuming that the transfer in technologies does not occur instantaneously but over time. Introducing dynamics to the transfer of technologies may contribute to making our model more consistent with the real world. What are the consequences of a smooth transfer? May this impact the sustainability of a union in the long run?

All of the above questions can be seen as further directions for our future research, which cannot be addressed within the current work. Indeed, they represent our research agenda for further work on the topic.

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Appendix A. Proof of Propositions

Proof of Proposition 1. The equilibrium in Proposition 1 can be derived by using Equation (9) and the fact that the FOCs can be rewritten as follows:

$$\frac{mp_j}{G_i}F(C_i;C_j) = \frac{\beta_i}{\gamma_i}$$

In this proof, we analyse the conditions that guarantee that the solution is an interior and unique one. By Assumption 1, $R_1 > R_2$ and $\beta_1 > \beta_2$; thus, a necessary and sufficient condition for an interior solution requires that the following inequality is satisfied:

$$R_2 - \frac{\frac{m}{m+1}(\beta_1 R_1 + \beta_2 R_2)}{\beta_2\left[1 + \left(\frac{\beta_1}{\beta_2}\right)^{\frac{m}{m+1}}\right]} > 0 \tag{A1}$$

Inequality (A2) implies:

$$\beta_2 R_2\left[1 + (m+1)\left(\frac{\beta_1}{\beta_2}\right)^{\frac{m}{m+1}}\right] - m\beta_1 R_1 > 0 \tag{A2}$$

It can be noted that:

$$lim_{m\to 0}\ \beta_2 R_2\left[1+(m+1)\left(\frac{\beta_1}{\beta_2}\right)^{\frac{m}{m+1}}\right]-m\beta_1 R_1 = 2\beta_2 R_2 > 0$$

$$lim_{m\to 1}\ \beta_2 R_2\left[1+(m+1)\left(\frac{\beta_1}{\beta_2}\right)^{\frac{m}{m+1}}\right]-m\beta_1 R_1 = \beta_2 R_2\left[2\sqrt{\frac{\beta_1}{\beta_2}}+1\right]-\beta_1 R_1 > 0$$

Inequality (A3) is satisfied when $m \to 0$. However, it also holds when $m \to 1$, since, by Assumption 1, the first term after the equality sign is larger than $3\beta_2 R_2$, because $\beta_1 > \beta_2$. Therefore, given continuity, inequality (A3) holds for any value of m.

Regarding uniqueness, Skaperdas and Syropoulos (1997) [74] shows that the pure strategy equilibrium is unique if the production function of the consumption good, $F(\cdot)$, is homogeneous of degree 1, $\frac{\partial^2 p_i}{\partial G_i^2} p_i \leq \left(\frac{\partial p_i}{\partial G_i}\right)^2$ and the solutions are interior.

The linearity of the production function satisfies the first requirement, while the third assumption has been proved above. Regarding the second requirement, it is satisfied, provided that $\frac{\partial^2 p_i}{\partial G_i^2}$ is negative since both p_i and $\left(\frac{\partial p_i}{\partial G_i}\right)^2$ are positive. Therefore:

$$\begin{aligned}\frac{\partial^2 p_i}{\partial G_i^2} &= \frac{\frac{m(m-1)}{\gamma_i^2}\left(\frac{G_i}{\gamma_i}\right)^{m-2}\left(\frac{G_j}{\gamma_j}\right)^{m}\left[\left(\frac{G_i}{\gamma_i}\right)^{m}+\left(\frac{G_j}{\gamma_j}\right)^{m}\right]}{\left[\left(\frac{G_i}{\gamma_i}\right)^{m}+\left(\frac{G_j}{\gamma_j}\right)^{m}\right]^4} \\ &- \frac{\frac{2m^2}{\gamma_i^2}\left(\frac{G_i}{\gamma_i}\right)^{2(m-1)}\left(\frac{G_j}{\gamma_j}\right)^{m}\left[\left(\frac{G_i}{\gamma}\right)^{m}+\left(\frac{G_j}{\gamma_j}\right)^{m}\right]}{\left[\left(\frac{G_i}{\gamma_i}\right)^{m}+\left(\frac{G_j}{\gamma_j}\right)^{m}\right]^4}\end{aligned}$$

By rearranging and simplifying the above, we obtain:

$$\begin{aligned}\frac{\partial^2 p_i}{\partial G_i^2} &= \frac{\frac{m}{\gamma_i^2}\left(\frac{G_i}{\gamma_i}\right)^{(m-2)}\left(\frac{G_j}{\gamma_j}\right)^{m}\left[(m-1)-2m\left(\frac{G_i}{\gamma_i}\right)^{m}\right]}{\left[\left(\frac{G_i}{\gamma_i}\right)^{m}+\left(\frac{G_j}{\gamma_j}\right)^{m}\right]^3} \\ &= -\frac{m p_i p_j\left[(1-m)+2m\left(\frac{G_i}{\gamma_i}\right)^{m}\right]}{G_i^2\left[\left(\frac{G_i}{\gamma_i}\right)^{m}+\left(\frac{G_j}{\gamma_j}\right)^{m}\right]} < 0\end{aligned}$$

□

Proof of Proposition 2. In this proof, we will focus on the set of derivatives (13) and (14), since the result in Equation (12) is immediate. Differentiating G_i^* with respect to β_i and β_j yields, respectively:

$$\frac{\partial G_i^*}{\beta_i} = -\frac{m\gamma_1}{(1+m)^2}\frac{\left[(1+m)\beta_j R_j + \left(\frac{\beta_j}{\beta_i}\right)^{\frac{m}{m+1}}(\beta_j R_j - m\beta_i R_i)\right]}{\left\{\beta_i\left[1+\left(\frac{\beta_j}{\beta_i}\right)^{\frac{m}{m+1}}\right]\right\}^2} < 0$$

$$\frac{\partial G_i^*}{\beta_j} = \frac{m\gamma_i}{(1+m)^2\beta_i}\frac{\left[(1+m)\beta_j R_j + \left(\frac{\beta_j}{\beta_i}\right)^{\frac{m}{m+1}}(\beta_j R_j - m\beta_i R_i)\right]}{\beta_j\left\{\left[1+\left(\frac{\beta_j}{\beta_i}\right)^{\frac{m}{m+1}}\right]\right\}^2} > 0$$

Both derivatives are satisfied if the numerator is positive. This is the case regardless of the value of m. If $m \to 0$, both derivatives are 0. Instead, when $m \to 1$, the numerator becomes:

$$2\beta_j R_j + \left(\frac{\beta_j}{\beta_i}\right)^{\frac{1}{2}}(\beta_j R_j - \beta_i R_i) > 0$$

The latter is always satisfied. If $j = 1$ and $i = 2$, given Assumption 1, the above inequality holds. This is also true if $i = 1$ and $j = 2$, because in this case $\frac{\beta_2}{\beta_1}$ is smaller than 1. Therefore, if players display a large difference in their production skills, $\frac{\beta_2}{\beta_1}$ approaches 0. Instead, if their skills are almost equal, the latter inequality becomes:

$$2\beta_j R_j + \left(\frac{\beta_j}{\beta_i}\right)^{\frac{1}{2}}(\beta_j R_j - \beta_i R_i) \approx 3\beta_j R_j - \beta_i R_i$$

The inequality above is, again, positive by Assumption 1. Finally, differentiating G_i with respect to m yields for players 1 and 2, respectively:

$$\frac{\partial G_1^*}{\partial m} = \frac{\gamma_1 p_1^2 R^*}{\beta_1(m+1)^3}\left[\frac{1+m}{\widehat{p}_1} - m\left(\ln\frac{\beta_2}{\beta_1}\right)\left(\frac{\beta_2}{\beta_1}\right)^{\frac{m}{m+1}}\right] > 0 \quad \text{(A3)}$$

$$\frac{\partial G_2^*}{\partial m} = \frac{\gamma_2 p_2^2 R^*}{\beta_2(m+1)^3}\left[\frac{1+m}{\widehat{p}_2} - m\left(\ln\frac{\beta_1}{\beta_2}\right)\left(\frac{\beta_1}{\beta_2}\right)^{\frac{m}{m+1}}\right] > 0 \quad \text{(A4)}$$

The first derivative is always positive, since $\beta_2 < \beta_1$. The second is satisfied, provided that the difference in the production skill is not particularly small. Specifically, if $\beta_2 \in \left[\frac{\beta_1}{2};\ \beta_1\right)$, then the second term in the square brackets of Equation (A4) is smaller than 1, while the first term is always larger than unity. □

Proof of Proposition 3. Differentiating V_i^* with respect to β_i yields:

$$\frac{\partial V_i^*}{\partial \beta_i} = \frac{\widehat{p}_i^2}{(1+m)^2\beta_i}\left[\beta_i R_i(1+m) + \left(\frac{\beta_i}{\beta_j}\right)^{\frac{m}{1+m}}(\beta_i R_i - m\beta_j R_j)\right] > 0$$

The above derivative is always satisfied for the same argument developed to prove Proposition 2. Instead, taking the derivative of V_i^* with respect to β_j yields:

$$\frac{\partial V_i^*}{\partial \beta_j} = \frac{\widehat{p}_i^2}{(1+m)^2\beta_j}\left\{\beta_j R_j(1+m) + \left(\frac{\beta_i}{\beta_j}\right)^{\frac{m}{1+m}}[\beta_j R_j(1+2m) + m\beta_i R_i]\right\} > 0 \quad \text{(A5)}$$

Finally, differentiating V_i^* with respect to m yields:

$$\frac{\partial V_i^*}{\partial m} = -\widehat{p}_i^2 \frac{\tilde{R}}{(1+m)^3}\left[\frac{(1+m)}{\widehat{p}_i} + ln\left(\frac{\beta_i}{\beta_j}\right)\left(\frac{\beta_i}{\beta_j}\right)^{\frac{m}{m+1}}\right] \tag{A6}$$

The latter inequality is always satisfied. If $i = 1$ and $j = 2$, given Assumption 1, the above inequality holds, since $ln\left(\frac{\beta_1}{\beta_2}\right) > 0$. Instead, if $i = 2$ and $j = 1$, the value in the square brackets is positive if the difference in the production skills is not too small, by the same argument developed in Proposition 2 regarding the derivative (A4). □

Proof of Proposition 6. Taking the derivative of Equation (33) with respect to κ yields:

$$\begin{aligned}
\frac{\partial \underline{\delta}_1}{\partial \kappa} &= -\phi \frac{-\frac{\partial \widehat{\beta}_2}{\partial \kappa}\frac{R_2}{1+m}\left(V^{**} - \widehat{V}_1\right) + \left(V^{**} - \widehat{V}\right)\left(\frac{\partial \widehat{p}_1}{\partial \kappa}\widehat{V} + \widehat{p}_1 \frac{\partial \widehat{\beta}_2}{\partial \kappa}\frac{R_2}{1+m}\right)}{\left(V^{**} - \widehat{V}_1\right)^2} \\
&= -\phi \frac{-\frac{\partial \widehat{\beta}_2}{\partial \kappa}\frac{R_2}{1+m}\widehat{p}_2 V^{**} + \frac{\partial \widehat{p}_1}{\partial \kappa}\hat{V}\left(V^{**} - \widehat{V}\right)}{\left(V^{**} - \widehat{V}_1\right)^2}
\end{aligned} \tag{A7}$$

It can be noted that:

$$\begin{aligned}
\frac{\partial \widehat{\beta}_2}{\partial \kappa} &< 0 \\
\frac{\partial \widehat{p}_1}{\partial \kappa} &< 0
\end{aligned}$$

Therefore, the sign of the above derivative depends on the numerator. The latter can be rewritten as follows:

$$\begin{aligned}
-\frac{\partial \widehat{\beta}_2}{\partial \kappa}\frac{R_2}{1+m}\widehat{p}_2 V^{**} &\lesseqgtr -\frac{\partial \widehat{p}_1}{\partial \kappa}\hat{V}\left(V^{**} - \widehat{V}\right) \\
\frac{V^{**}}{\hat{V}\left(V^{**} - \widehat{V}\right)} &\lesseqgtr \frac{\frac{\widehat{p}_1^2 m \widehat{\beta}_1\left(\widehat{\beta}_1 - \widehat{\beta}_2\right)}{(m+1)\left[\widehat{\beta}_1 + (\kappa-1)\widehat{\beta}_2\right]^2\left[\kappa\frac{\widehat{\beta}_1}{\widehat{\beta}_1 + (\kappa-1)\widehat{\beta}_2}\right]^{\frac{1}{m+1}}}}{\frac{R_2}{1+m}\widehat{p}_2\frac{1}{\kappa^2}\left(\widehat{\beta}_1 - \widehat{\beta}_2\right)} \\
\frac{V^{**}}{\hat{V}\left(V^{**} - \widehat{V}\right)} &\lesseqgtr \frac{\widehat{p}_1^2 m \widehat{\beta}_1\left(\widehat{\beta}_1 - \widehat{\beta}_2\right)}{(m+1)\left[\widehat{\beta}_1 + (\kappa-1)\widehat{\beta}_2\right]^2\left[\kappa\frac{\widehat{\beta}_1}{\widehat{\beta}_1 + (\kappa-1)\widehat{\beta}_2}\right]^{\frac{1}{m+1}}}\frac{(1+m)\kappa^2}{R_2\widehat{p}_2\left(\widehat{\beta}_1 - \widehat{\beta}_2\right)} \\
\frac{R_2 V^{**}}{\hat{V}\left(V^{**} - \widehat{V}\right)} &\lesseqgtr \frac{\widehat{p}_1^2 m}{\widehat{p}_2}\frac{\widehat{\beta}_1}{\widehat{\beta}_2^2}\left(\frac{\widehat{\beta}_1}{\widehat{\beta}_2}\right)^{-\frac{1}{m+1}} \\
\frac{\widehat{\beta}_2 R_2 V^{**}}{\hat{V}\left(V^{**} - \widehat{V}\right)} &\lesseqgtr \frac{\widehat{p}_1^2 m}{\widehat{p}_2}\left(\frac{\widehat{\beta}_1}{\widehat{\beta}_2}\right)^{\frac{m}{1+m}}
\end{aligned}$$

The rhs of the last inequality is equal to 0, when $m \to 0$:

$$\lim_{m \to 0}\frac{\widehat{p}_1^2 m}{\widehat{p}_2}\left(\frac{\widehat{\beta}_1}{\widehat{\beta}_2}\right)^{\frac{m}{1+m}} = 0$$

On the other hand, when $m \to 1$, we have:

$$\lim_{m\to 1}\frac{\widehat{p}_1^2 m}{\widehat{p}_2}\left(\frac{\beta_1}{\widehat{\beta}_2}\right)^{\frac{1}{1+m}} = \frac{1}{\sqrt{\frac{\widehat{\beta}_1}{\widehat{\beta}_2}}+1}\left(\sqrt{\frac{\widehat{\beta}_2}{\widehat{\beta}_1}}+1\right)\sqrt{\frac{\widehat{\beta}_1}{\widehat{\beta}_2}} = 1$$

Hence, the numerator of Equation (A7) is positive when $m \to 0$ and negative when $m \to 1$. Therefore, given continuity in m, there must exist a threshold value, $\widehat{m}$, such that the derivative is negative if $m < \widehat{m}$ and positive if $m > \widehat{m}$.

Instead, regarding the second derivative in the proposition, we have:

$$\begin{aligned}
\frac{\partial \delta_2}{\partial \kappa} &= (1-\phi)\frac{\left(\frac{d\widehat{p}_1}{\partial\kappa}\widehat{V} + \widehat{p}_1\frac{d\widehat{\beta}_2}{\partial\kappa}\frac{R_2}{1+m}\right)\left(V^{**}-\widehat{V}_2\right) - \widehat{V}_1\left(\frac{\partial\widehat{p}_1}{\partial\kappa}\widehat{V} - (1-p_1)\frac{\partial\widehat{\beta}_2}{\partial\kappa}\frac{R_2}{1+m}\right)}{\left(V^{**}-\widehat{V}_2\right)^2} \\
&= (1-\phi)\frac{\frac{\partial\widehat{p}_1}{\partial\kappa}\widehat{V}\left(V^{**}-\hat{V}_2\right) + \widehat{p}_1\frac{\partial\widehat{\beta}_2}{\partial\kappa}\frac{R_2}{1+m}V^{**} - \hat{V}_1\frac{\partial\widehat{p}_1}{\partial\kappa}\widehat{V}}{\left(V^{**}-\hat{V}_2\right)^2} \\
&= (1-\phi)\frac{\frac{\partial\widehat{p}_1}{\partial\kappa}\widehat{V}\left(V^{**}-\widehat{V}+\widehat{V}_1\right) + \widehat{p}_1\frac{\partial\widehat{\beta}_2}{\partial\kappa}\frac{R_2}{1+m}V^{**} - \widehat{V}_1\frac{\partial\widehat{p}_1}{\partial\kappa}\widehat{V}}{\left(V^{**}-\widehat{V}_2\right)^2} \\
&= (1-\phi)\frac{\frac{\partial\widehat{p}_1}{\partial\kappa}\widehat{V}\left(V^{**}-\widehat{V}\right) + \widehat{p}_1\frac{\partial\widehat{\beta}_2}{\partial\kappa}\frac{R_2}{1+m}V^{**}}{\left(V^{**}-\widehat{V}_2\right)^2} < 0
\end{aligned}$$

□

Appendix B. Sensitivity Analysis

In this Appendix, we provide a sensitivity analysis for some of the results reported in the main text. Specifically, we discuss the change in the equilibrium values in Propositions 2, 3, and 6, when the main parameters vary. In the reported sensitivity analysis, we select the initial values for the parameters as indicated in Table A1:

Table A1. Values for the sensitivity analysis.

R_1	10
R_2	8
β_1	0.2
β_2	0.1
γ_1	0.1
γ_2	1
m	0.3
κ	2
ϕ	0.5

Clearly, not all of them will be used in each simulation. For instance, we will make use of the last two values, when we discuss the variation of the critical discount factors.

We start by considering how the allocation of the initial endowments changes when the parameters β_1 and β_2 vary. This is, basically, the content of Proposition 2. The result of our analysis is reported in Table A2. It should be noted that when evaluating the aforementioned changes, we need to take into consideration some additional issues. For instance, by Assumption 1 in the text, in a peaceful environment without forming a union, player 2 is going to produce a total amount of consumption good, $\beta_2 R_2$, which is not smaller than 1/3 and not larger than 2/3 of R_1. In addition, again by Assumption 1, $\beta_1 > \beta_2$. Therefore, in the following Table, we mark in bold the values that are consistent with the aforementioned assumptions. Moreover, we consider three alternative scenarios, characterised by a different value of γ_1, the military skills of player 1. In the first panel, we assume a low level of military productivity ($\gamma_1 = 0.1$), while we increase it in the second ($\gamma_1 = 0.4$) and in the third one ($\gamma_1 = 0.7$). Consistently with Assumption 1, player 2 always owns a better military technology ($\gamma_2 = 1$ in our sensitivity analysis).

Table A2. Changes in the optimal allocation of resources.

				$\gamma_1 = 0.1$					
β_2 \ β_1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.1	**0.1744**	**0.1646**	**0.1604**	**0.1584**	**0.1574**	**0.1570**	**0.1569**	**0.1568**	**0.1567**
0.2	0.2077	**0.1852**	**0.1744**	**0.1684**	**0.1646**	**0.1621**	**0.1604**	**0.1592**	**0.1584**
0.3	0.2420	0.2077	**0.1907**	**0.1808**	**0.1744**	**0.1700**	**0.1669**	**0.1646**	**0.1628**
0.4	0.2761	0.2305	0.2077	**0.1941**	**0.1852**	**0.1790**	**0.1744**	**0.1710**	**0.1684**
0.5	0.3097	0.2534	0.2248	0.2077	**0.1964**	**0.1884**	**0.1825**	**0.1780**	**0.1744**
0.6	0.3428	0.2761	0.2420	0.2214	0.2077	**0.1980**	**0.1907**	**0.1852**	**0.1808**
0.7	0.3755	0.2985	0.2591	0.2351	0.2191	0.2077	**0.1992**	**0.1926**	**0.1874**
0.8	0.4078	0.3208	0.2761	0.2488	0.2305	0.2175	0.2077	**0.2001**	**0.1941**

				$\gamma_1 = 0.4$					
β_2 \ β_1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.1	**0.6977**	**0.6583**	**0.6417**	**0.6337**	**0.6297**	**0.6279**	**0.6272**	**0.6273**	**0.6270**
0.2	0.8308	**0.7408**	**0.6977**	**0.6734**	**0.6583**	**0.6484**	**0.6417**	**0.637**	**0.6337**
0.3	0.9679	0.8308	**0.7630**	**0.7233**	**0.6977**	**0.6802**	**0.6676**	**0.6583**	**0.6513**
0.4	1.1042	0.9222	0.8308	**0.7764**	**0.7408**	**0.7159**	**0.6977**	**0.6840**	**0.6734**
0.5	1.2388	1.0135	0.8993	0.8308	**0.7854**	**0.7534**	**0.7298**	**0.7118**	**0.6977**
0.6	1.3714	1.1042	0.9679	0.8856	0.8308	**0.7919**	**0.7630**	**0.7408**	**0.7233**
0.7	1.5021	1.1941	1.0363	0.9405	0.8764	0.8308	**0.7967**	**0.7704**	**0.7496**
0.8	1.6310	1.2832	1.1042	0.9953	0.9222	0.8699	0.8308	**0.8005**	**0.7764**

				$\gamma_1 = 0.7$					
β_2 \ β_1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.1	**1.2210**	**1.1521**	**1.1230**	**1.1089**	**1.1020**	**1.0987**	**1.0976**	**1.0978**	**1.0988**
0.2	1.4538	**1.2964**	**1.2210**	**1.1785**	**1.1521**	**1.1348**	**1.1230**	**1.1147**	**1.1089**
0.3	1.6939	1.4538	**1.3352**	**1.2658**	**1.2210**	**1.1903**	**1.1683**	**1.1521**	**1.1398**
0.4	1.9324	1.6138	1.4538	**1.3587**	**1.2964**	**1.2528**	**1.2210**	**1.1970**	**1.1785**
0.5	2.1678	1.7737	1.5738	1.4538	**1.3745**	**1.3185**	**1.2772**	**1.2457**	**1.2210**
0.6	2.3999	1.9324	1.6939	1.5498	1.4538	**1.3858**	**1.3352**	**1.2964**	**1.2658**
0.7	2.6286	2.0897	1.8135	1.6459	1.5338	1.4538	**1.3942**	**1.3482**	**1.3118**
0.8	2.8543	2.2456	1.9324	1.7418	1.6138	1.5223	1.4538	**1.4008**	**1.3587**

As a general remark, we may notice that the optimal amount of resources allocated to the war increases for higher values of γ_1, since, in this case, player 1 will find marginally more convenient to fight a war than producing the consumption good. However, given the above consideration, developing a better productive technology (i.e., any increase in β_1) reduces the portion of resources employed in the war. This indicates that producing the consumption good is (comparatively) more convenient for the contestant rather than moving to the war. Not surprisingly, when its opponent develops a better productive technology (increases in β_2, the extension of the common pool over which players fight is larger. Therefore, other things being equal, player 1's optimal strategy consists of allocating more resources to the fight. This can be noted by observing how much G_1^* increases for each value of β_1, when β_2 goes up. Eventually, when $\beta_1 = 1$, player 1 will allocate the lowest amount of resources to the war in all of the three reported scenarios.

While we present the sensitivity analysis for G_1^*, the results are unaffected if we focus on changes of G_2^*, when we vary β_1 and β_2.

In Table A3, we run a sensitivity analysis, which confirms the findings in Proposition 3. Specifically, we check how the (expected) outcome in the war scenario is affected by a change in the parameters capturing the productive skills of contestants. Moreover, we consider again three alternative situations ($m = 0.3$; $m = 0.5$ and $m = 0.7$), characterised by an increased sensitivity of the CSF to the allocation of the resources to the war.

Table A3. Changes in the expected outcome of the war.

					$m = 0.3$				
β_2 \ β_1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.1	**0.9910**	**1.2773**	**1.5533**	**1.8212**	**2.0823**	**2.3375**	**2.5878**	**2.8336**	**3.0755**
0.2	1.3846	**1.6865**	**1.9820**	**2.2711**	**2.5545**	**2.8329**	**3.1067**	**3.3764**	**3.6424**
0.3	1.7714	2.0769	**2.3799**	**2.6786**	**2.9729**	**3.2630**	**3.5492**	**3.8318**	**4.1109**
0.4	2.1596	2.4637	2.7692	**3.0727**	**3.3730**	**3.6701**	**3.9639**	**4.2545**	**4.5422**
0.5	2.5508	2.8508	3.1561	3.4615	**3.7653**	**4.0666**	**4.3653**	**4.6614**	**4.9549**
0.6	2.9452	3.2394	3.5428	3.8485	4.1538	**4.4577**	**4.7597**	**5.0596**	**5.3572**
0.7	3.3427	3.6300	3.9304	4.2351	4.5409	4.8462	**5.1502**	**5.4526**	**5.7532**
0.8	3.7432	4.0228	4.3192	4.6222	4.9275	5.2332	5.5385	**5.8426**	**6.1453**
					$m = 0.5$				
β_2 \ β_1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.1	**0.9342**	**1.1573**	**1.3653**	**1.5616**	**1.7483**	**1.9270**	**2.0989**	**2.2650**	**2.4260**
0.2	1.3846	**1.6316**	**1.8684**	**2.0957**	**2.3145**	**2.5259**	**2.7306**	**2.9295**	**3.1231**
0.3	1.8240	2.0769	**2.3255**	**2.5675**	**2.8026**	**3.0314**	**3.2543**	**3.4718**	**3.6843**
0.4	2.2650	2.5164	2.7692	**3.0186**	**3.2631**	**3.5025**	**3.7369**	**3.9664**	**4.1915**
0.5	2.7102	2.9559	3.2089	3.4615	**3.7114**	**3.9574**	**4.1994**	**4.4372**	**4.6711**
0.6	3.1599	3.3975	3.6480	3.9013	4.1538	**4.4040**	**4.6510**	**4.8947**	**5.1349**
0.7	3.6140	3.8419	4.0881	4.3404	4.5938	4.8462	**5.0965**	**5.3442**	**5.5891**
0.8	4.0722	4.2893	4.5301	4.7798	5.0328	5.2862	5.5385	**5.7889**	**6.0372**
					$m = 0.7$				
β_2 \ β_1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.1	**0.8783**	**1.0414**	**1.1874**	**1.3204**	**1.4433**	**1.5579**	**1.6657**	**1.7677**	**1.8648**
0.2	1.3846	**1.5769**	**1.7565**	**1.9246**	**2.0828**	**2.2325**	**2.3748**	**2.5107**	**2.6409**
0.3	1.8763	2.0769	**2.2713**	**2.4572**	**2.6348**	**2.8047**	**2.9676**	**3.1243**	**3.2753**
0.4	2.3690	2.5689	2.7692	**2.9646**	**3.1537**	**3.3364**	**3.5130**	**3.6838**	**3.8493**
0.5	2.8657	3.0603	3.2616	3.4615	**3.6575**	**3.8486**	**4.0345**	**4.2153**	**4.3913**
0.6	3.3672	3.5534	3.7526	3.9541	4.1538	**4.3502**	**4.5425**	**4.7306**	**4.9143**
0.7	3.8732	4.0494	4.2444	4.4452	4.6466	4.8462	**5.0428**	**5.2360**	**5.4256**
0.8	4.3836	4.5485	4.7379	4.9365	5.1379	5.3391	5.5385	**5.7353**	**5.9292**

Once again, we mark in bold those values that are consistent with the assumptions you made. First, we notice that other things being equal, an increase in m reduces the expected outcome of the war. This stems directly from the fact that players are more inclined to increase the allocation of resources to the war, reducing, in turn, the extension of the common pool over which they fight. In fact, globally, a lower amount of consumption goods is produced. However, such a reduction in the expected outcome is offset by the development of better productive technology. In fact, when β_1 and/or β_2 increase, although players may reduce the number of resources employed in the war, the production of the consumption good may still increase, since each unit of R generates a larger amount of C. Such a pattern is clear in Table A3, when we observe, for instance, how much player 1 may expect from the war when $\beta_1 = 1$ and β_2 constantly increases. Different to the mathematical result we obtained in Proposition 3, we can now calculate exactly the payoff from the war. Moreover, given that for each value of the parameters, the common pool has a fixed extension, a look at Table A3 displays also how much its opponent is expected to receive. Dividing all the entries in the Table by the extension of the common pool for each pair of β_1 and β_2 produces the probability of being successful in the war for player 1 and, consequently, for player 2. For instance, when $\beta_1 = 1$, $\beta_2 = 0.8$ and $m = 0.3$, $V^* = 16.4$. Since $V_1^* = 6.1453$, we can immediately calculate that $p_1^* = 0.375$ and $p_2^* = 0.625$.

Finally, in Table A4, we analyse how the critical discount factors, $\underline{\delta}_1$ and $\underline{\delta}_2$ are simultaneously affected by a change in m, the sensitivity of the CSF to the amount of resources employed in the war, and κ, player 2's error in learning the most efficient production technology. In the first panel, we observe the variation of $\underline{\delta}_1$. The latter decreases, when κ increases. However, as we showed in the proof of Proposition 6, this is true, provided that m is sufficiently low. Specifically, we may observe that for a value of m between 0.5 and 0.6, any increase in κ indicates that player 1 becomes more impatient. In other words, the possibility that the union can be sustained in the long run reduces for the reasons that we explained in the main text. It can be noted that this never happens for player 2— the second panel in Table A4: for that player, any increase in κ and m reduces the critical discount factor. On one hand, a larger κ indicates that it has more difficulties in learning the most efficient production technology. On the other, any increase in m enlarges the probability of being successful in the war, this putting it in a better bargaining condition with respect to its opponent. Hence, finding an agreement appears to be a better pattern to follow.

Table A4. Changes in the critical discount factors.

					$\underline{\delta}_1$					
m \ κ	**1.5**	**2**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
0.0	0.9259	0.8889	0.8667	0.8519	0.8413	0.8333	0.8272	0.8222	0.8182	0.8148
0.1	0.8427	0.8098	0.7901	0.7771	0.7678	0.7609	0.7554	0.7511	0.7476	0.7447
0.2	0.7740	0.7449	0.7276	0.7162	0.7080	0.7019	0.6972	0.6934	0.6904	0.6878
0.3	0.7165	0.6908	0.6756	0.6655	0.6584	0.6531	0.6490	0.6457	0.6430	0.6408
0.4	0.6675	0.6449	0.6316	0.6228	0.6165	0.6119	0.6083	0.6055	0.6031	0.6012
0.5	0.6254	0.6055	0.5938	0.5862	0.5808	0.5768	0.5736	0.5712	0.5692	0.5675
0.6	0.6267	0.6377	0.6467	0.6477	0.6687	0.6827	0.7057	0.7147	0.7257	0.7387
0.7	0.6347	0.6457	0.6567	0.6687	0.6757	0.6887	0.7197	0.7307	0.7317	0.7507
0.8	0.6417	0.6517	0.6677	0.6777	0.6887	0.7037	0.7327	0.7507	0.7457	0.7617
0.9	0.6527	0.6627	0.6817	0.6957	0.7097	0.7267	0.7547	0.7607	0.7607	0.7717
1.0	0.6540	0.6827	0.6837	0.6987	0.7119	0.7281	0.7563	0.7627	0.7623	0.7734

					$\underline{\delta}_2$					
m \ κ	**1.5**	**2**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
0.0	0.9310	0.9000	0.8824	0.8710	0.8630	0.8571	0.8526	0.8491	0.8462	0.8438
0.1	0.8627	0.8378	0.8234	0.8141	0.8076	0.8027	0.7990	0.7960	0.7936	0.7916
0.2	0.8124	0.7914	0.7792	0.7712	0.7656	0.7614	0.7581	0.7556	0.7535	0.7517
0.3	0.7739	0.7555	0.7448	0.7377	0.7327	0.7289	0.7260	0.7237	0.7218	0.7203
0.4	0.7435	0.7269	0.7172	0.7108	0.7062	0.7028	0.7001	0.6980	0.6963	0.6949
0.5	0.7188	0.7036	0.6946	0.6887	0.6844	0.6813	0.6788	0.6768	0.6752	0.6739
0.6	0.6984	0.6843	0.6758	0.6702	0.6662	0.6632	0.6609	0.6591	0.6575	0.6563
0.7	0.6813	0.6679	0.6599	0.6546	0.6508	0.6480	0.6457	0.6440	0.6425	0.6413
0.8	0.6667	0.6539	0.6463	0.6412	0.6375	0.6348	0.6327	0.6310	0.6296	0.6284
0.9	0.6541	0.6418	0.6345	0.6295	0.6260	0.6234	0.6213	0.6197	0.6183	0.6172
1.0	0.6431	0.6312	0.6241	0.6194	0.6159	0.6134	0.6114	0.6098	0.6085	0.6074

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