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A Complete Breakdown of Politics Coverage Using the Concept of Domination and Double Domination in Picture Fuzzy Graph

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Abstract: The notion of fuzzy graph (FG) is widely used in many problems arising from partial or incomplete descriptions of the real world and in particular from fields such as engineering, economics, computer science, social disciplines, or medical diagnostics, and has been used in many fields of pure mathematics as well as in several areas of applied sciences such as decision making, statistics and networking. In this paper we will deal with the graph of the picture fuzzy(symmetric) set using the notion of domination in picture fuzzy graph (PFG) as a generalization of both the concept of fuzzy graph domination and intuitionistic fuzzy graph (IFG) domination. The concepts of domination theory (DT) and double domination theory (DDT) of a PFG are introduced, studied and concretely applied to the real case of an election competition to determine the minimum number of citizens a politician should meet in person in order to win the election. The choice of fuzzification (symmetric) and defuzzification (anti-symmetric) methods depends on the specific application and the type of fuzzy sets being used, whether they are symmetric or anti-symmetric. There are various methods for each process, such as centroid, max-min, and weighted average methods for defuzzification. Finally, in the last section, drawing from the application example, the features and benefits of PFGs with respect to fuzzy graphs and intuitionistic fuzzy graphs are compared and discussed.

Keywords: intuitionistic fuzzy set; intuitionistic fuzzy graph; picture fuzzy graph; domination; double domination

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1. Introduction

In 1965, Zadeh [1] introduced the notion of the fuzzy set (FS) as a characteristic function on the unit interval [0,1] whose values express the degree of membership of each element. In 1986, Atanassov [2] generalized this idea by presenting the notion of intuitionistic fuzzy sets (IFs) in which each element corresponds to two real values of the interval [0,1], the membership degree and the non-membership degree, the sum of which must be less than or equal to 1. Since the latter condition is binding for assigning the membership to non-membership degrees of an element in real life problems, further generalizations were provided by Yager [3] with the Pythagorean fuzzy sets (PyFSs) and the of q-rung orthopair (q-ROPFSs). In the particular case of predicting or analyzing voting in electoral contests, it is evident that each voter can choose to abstain, to vote for someone or to vote against someone by expressing his or her preference toward one of the remaining candidates. This type of concrete problem with competing motivations has been addressed and solved by Cuong [4] by means of the notion of picture fuzzy set (PFS). In fact, while in FGs there is only the degree of membership and in IFGs only the degrees of membership and non-membership, PFGs provide four degrees: membership, non-membership, neutrality and rejection.

In 2016, Cuong et al. [5] investigated the idea of the classification of representable t-norm operators for PFSs. In 2020 Garg et al. [6] proposed the notion of generalized geometric aggregation operators based on t-norm operations for complex intuitionistic fuzzy sets and their application to decision-making. Although PFS generalizes both FS and IFS, in real problems, the choice of the three values included in the interval [0,1] and corresponding to the degrees of membership, non-membership and neutrality is not entirely free, being constrained by the fact that their sum must always be less than or equal to 1.

The notion of the fuzzy graph (FG) was introduced and studied by Kaufman [7] and Rosenfield [8] as a generalization of the classical sharp graphs and was later expanded and applied to real-world problem solving by several researchers including Sameena and Sunitha [9], Sunitha and Mathew [10] and Yeh and Bang [11]. The notion of intuitionistic fuzzy graph (IFG) introduced by Parvathi et al. [12,13] generalizes that of FG. Parvathi and Karunambigai introduced the concept of intuitionistic fuzzy graphs, which are a generalization of fuzzy graphs. Fuzzy graphs are a type of graph where the edges and vertices are assigned fuzzy values, allowing for more flexible and nuanced relationships between them. Intuitionistic fuzzy graphs take this concept further by allowing for uncertainty not only in the degree of membership of elements in a graph, but also in the degree of non-membership. The applications in many areas such as decision making and networking [14–19].

The notion of domain is a central topic in graph theory, and in 1998 it was generalized to FGs as well [20,21] and has been the starting point for further declinations on IGFs [22,23]. The concept of dominance in FGs was introduced by Borzooei and Rashmanlou [24] and subsequently investigated by Manjusha and Sunitha [25], Zhang [26], who studied dynamic dominations into fuzzy networks, and Shubatah [27] who dealt with domination in the FG's product. The concept of domination in graph theory has considerable applications in many application branches, having been used, for example, in the medical field by Gupta, Aardal et al. [28,29] to analyze the working principle of medical radars, by Xu et al., in the field of software engineering, by Xu et al. [30] to reduce software errors during collation, Borzooei et al. [31] worked on the semi global domination sets in vague graphs with application, and in networking by Koczy et al. [32] to analyze social networks and the coverage of Wi-Fi networks. An extensive review of the trends and major application areas of FG theory was provided by Pal et al. in [33].

A fuzzy set is frequently generated as the system's output following the fuzzy judgment process. The process of transforming the fuzzy set into a clear output value is known as defuzzification. Defuzzification techniques include the centroid, max-min, and weighted average algorithms, among others. The fuzzy set's center of gravity is determined by the centroid method and the output value that best reflects the fuzzy set's degree of membership is chosen by the max-min approach. The weighted average method calculates the output value by averaging the input values. Terms such as cardinality order, integrity of dominance, PFG neighbors, strength and double domination set are used to define the fundamental operations. Similar to this, PFG-related terms are shown together with their associated attributes. The great application of theory demonstrates the extent to which political innovators may still reach a sizable voter base. The possibility for PF dominance can be quite beneficial in this specific situation. Equivalent analyses highlight the novelty of the proposed structure and the assistance provided by current designs in addressing circumstances in which different devices are disregarded.

This paper consists of six more sections. In Section 2, some preliminary notions regarding FS, IFS, PFS and the corresponding graphs FG, IFG and PFG are presented. In

Sections 3 and 4, we introduce and discuss the notions of domination and double domination in PFGs, respectively. In Section 5, we describe the application of the previously discussed notions to the concrete case of an electoral competition. In Section 6, we discuss and compare the features and benefits of PFGs with respect to FGs and IFGs and finally in Section 7, we provide our concluding reflections.

2. Preliminaries

In this section we recall the main definitions on FS, IFS, PFS and the corresponding FG, IFG, complete IFG and PFG graphs and provide some simple examples.

Definition 1. *Let* $\tilde{V}$ *be a non-empty set, then a fuzzy set (FS) is characterized by* $\hat{A} = \{\H{u}, \check{T}_{\hat{A}}(\H{u}) | \H{u} \in \tilde{V}\}$*, in any where* $\H{u}$ *is a particular element of* $\tilde{V}$ *and* $\check{T}_{\hat{A}}: \tilde{V} \to [0,1]$ *is said to be membership function and* $\check{T}_{\hat{A}}(\H{u})$ *is known as the grade of membership of* $\tilde{V}$ *[1].*

Definition 2. *An IFS* $\hat{A}$ *in* $\tilde{V}$ *is characterized by* $\hat{A} = \{(\H{u}, \check{T}_{\hat{A}}(\H{u}), \hat{R}_{\hat{A}}(\H{u})) | \H{u} \in \tilde{V}\}$ *in anywhere* $\check{T}_{\hat{A}} : \tilde{V} \to [0,1]$ *and* $\hat{R}_{\hat{A}} : \tilde{V} \to \{0,1\}$ *represents the grade of membership and grade of non-membership function, respectively, with the condition.*

$$0 \leq \check{T}_{\hat{A}}(\H{u}) + \hat{R}_{\hat{A}}(\H{u}) \leq 1, for\ all\ \H{u} \in \tilde{V}$$

Furthermore, $1 - (\check{T}_{\hat{A}}(\H{u}) + \hat{R}_{\hat{A}}(\H{u})) \leq 1$ *represents the grade of rejection* $\H{u} \in \tilde{V}$*. The* $(\check{T}, \hat{R})$ *is known as a fuzzy number (FN) [2].*

Definition 3. *A PFS* $\hat{A}$ *in* $\tilde{V}$ *is characterized by* $\hat{A} = \{\tilde{V}, \check{T}_{\hat{A}}(\H{u}), \check{N}_{\hat{A}}(\H{u}), \hat{R}_{\hat{A}}(\H{u}) | \H{u} \in \tilde{V}\}$*,in anywhere* $\check{T}_{\hat{A}}: \tilde{V} \to [0,1], \check{N}_{\hat{A}}: \tilde{V} \to [0,1]$ *and* $\hat{R}_{\hat{A}}: \tilde{V} \to [0,1]$ *denoted by the grade of membership, grade of neutral and non-membership grade functions, respectively, which have the condition.*

$$0 \leq \check{T}_{\hat{A}}(\H{u}) + \check{N}_{\hat{A}}(\H{u}) + \hat{R}_{\hat{A}}(\H{u}) \leq 1, for\ all\ \H{u} \in \tilde{V}$$

Furthermore, $1 - (\check{T}_{\hat{A}}(\H{u}) + \check{N}_{\hat{A}}(\H{u}) + \hat{R}_{\hat{A}}(\H{u})) \leq 1$ *denotes the rejection grade of* $\H{u} \in \tilde{V}$*. The triplet* $(\check{T}, \hat{R}, \check{N})$ *is PFN [4].*

Example 1. *A fuzzy graph in Figure 1.*

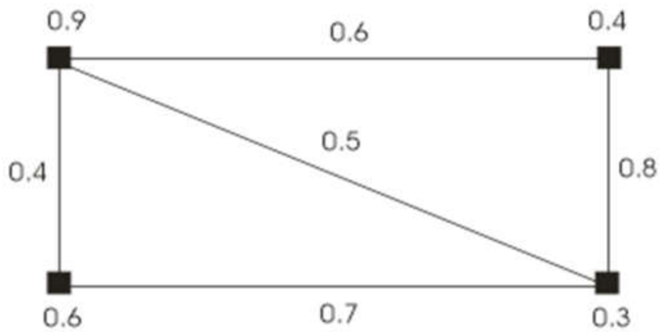


Figure 1. Fuzzy Graph.

In a fuzzy graph, the edges of Figure 1 are represented by fuzzy relations, which are typically defined using a membership function that assigns a degree of membership to each possible relation between two nodes. The nodes themselves can also be represented by fuzzy sets, which can capture uncertainty in the identification of nodes or the ambiguity of their classification.

Definition 4. *The IFG is* $\check{K} = (\tilde{V}, \breve{E})$ *if*

(1) $\tilde{V} = \{\H{u}_1, \H{u}_2, \H{u}_3, \H{u}_4, \ldots \ldots \ldots, \H{u}_n\}$ *is the set of nodes such that* $\check{T}_1: \tilde{V}_1 \to [0,1]$ *and* $\hat{R}_1: \tilde{V}_1 \to [0,1]$ *represents the membership and non-membership grades of the elements* $\H{u}_i \in \tilde{V}$ *respectively with a condition that*

$$0 \leq \check{T}_1(ű_i) + \hat{R}_1(ű_i) \leq 1, for\ all\ ű_i \in \tilde{V}(i \in I).$$

(2) $\check{E} \subseteq \tilde{V} \times \tilde{V}$ *in anywhere* $\check{T}_2: \tilde{V} \times \tilde{V} \to [0,1]$ *and* $\hat{R}_2: \tilde{V} \times \tilde{V} \to [0,1]$ *represents the membership and non-membership grades of the elements* $(ű_i, ű_j) \in \check{E}$ *such that* [14]

$$\check{T}_2(ű_i, ű_j) \leq \min\{(\check{T}_1(ű_i), \check{T}_1(ű_j))\}$$

and

$$\hat{R}_2(ű_i, ű_j) \leq \max\{\hat{R}_1(ű_i), \hat{R}_1(ű_j)\}$$

with a condition

$$0 \leq \check{T}_2(ű_i, ű_j) + \hat{R}_2(ű_i, ű_j) \leq 1, for\ all\ (ű_i, ű_j) \in \check{E}, (i \in I).$$

Example 2. *An IFG in Figure 2,* $\check{K} = (\tilde{V}, \check{E})$ *now* $\tilde{V} = \{ű_1, ű_2, ű_3\}$ *and* $\check{E} = \{ȩ̂_1, ȩ̂_2, ȩ̂_3\}$ *are the set of nodes and edges, respectively.*

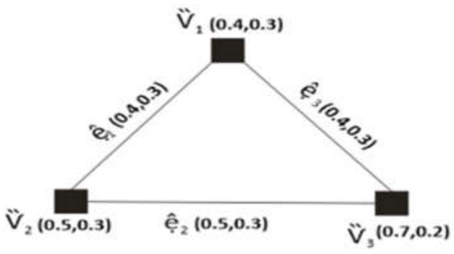


Figure 2. Intuitionistic Fuzzy Graph.

Definition 5. *A pair is the form of PFG* $\check{K} = (\tilde{V}, \check{E})$ *is known as CIFG of an IFG* $\check{K} = (\tilde{V}, \check{E})$ *if* $\check{T}_2(ű_i, ű_j) = \min\{\check{T}_1(ű_i), \check{T}_1(ű_j)\}$ *and* $\hat{R}_2(ű_i, ű_j) = \max\{\hat{R}_1(ű_i), \hat{R}_1(ű_j)\}, for\ all(ű_i, ű_j) \in \check{E}$ [20].

Example 3. *Figure 3 shows a CIFG, since for each node, it is adjacent to all nodes, now* $\tilde{V} = \{ű_1, ű_2, ű_3, ű_4\}$ *nodes and edges* $\check{E} = \{ȩ̂_1, ȩ̂_2, ȩ̂_3, ȩ̂_4\}$

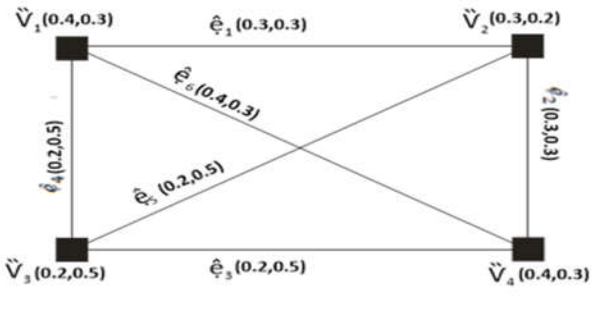


Figure 3. Complete intuitionistic fuzzy graph.

Definition 6. *A pair* $\check{K} = (\tilde{V}, \check{E})$ *is known as PFG if*

(1) $\tilde{V} = \{ű_1, ű_2, ű_3, \ldots\ldots, ű_n\}$ *is the set of nodes in which* $\check{T}_1: \tilde{V} \to [0,1], \hat{R}_1: \tilde{V} \to [0,1]$ *and* $\check{N}_1: \to [0,1]$ *denote the membership grade, neutral grade and grade of non-membership of an element* $ű_j \in \tilde{V}$, *respectively, such that* $0 \leq \check{T}_1(ű_j) + \hat{R}_1(ű_j) + \check{N}_1(ű_j) \leq 1, for\ all\ ű_j \in \tilde{V}, j \in \{1,2,3,\ldots,n\}$.

(2) $\check{E} \subseteq \tilde{V} \times \tilde{V}$ *in anywhere* $\check{T}_2: \tilde{V} \times \tilde{V} \to [0,1]$, $\hat{R}_2: \tilde{V} \times \tilde{V} \to [0,1]$ *and* $\check{N}_2: \tilde{V} \times \tilde{V} \to [0,1]$ *denote the membership grade, neutral and non-membership grades of an element* $(ű_i, ű_j) \in \check{E}$, *respectively such that*

$$\check{T}_2(ű_i, ű_j) \leq \min\{\check{T}_1(ű_i), \check{T}_1(ű_j)\}$$

$$\hat{R}_2(ű_i, ű_j) \leq \min\{\hat{R}_1(ű_i), \hat{R}_1(ű_j)\}$$

$$\check{N}_2(ű_i, ű_j) \leq \max\{\check{N}_1(ű_i), \check{N}_1(ű_j)\}$$

with a condition

$$0 \leq \check{T}_2(ű_i, ű_j) + \hat{R}_2(ű_i, ű_j) + \check{N}_2(ű_i, ű_j) \leq 1, for\ all(ű_i, ű_j) \in \check{E}, i, j \in \{1,2,3, \ldots, n\}.$$

The $1 - \check{T}_2(ű_i, ű_j) + \hat{R}_2(ű_i, ű_j) + \check{N}_2(ű_i, ű_j)$ *is known as a refusal grade of* $(ű_i, ű_j) \in \check{E}$.

Example 4. *In the Figure 4, A pair* $\check{K} = (\tilde{V}, \check{E})$ *in the form of PFG, in the nodes of* $\tilde{V} = (ű_1, ű_2, ű_3, ű_4)$ *and the edge is* $\check{E} = (ê_1, ê_2, ê_3, ê_4)$.

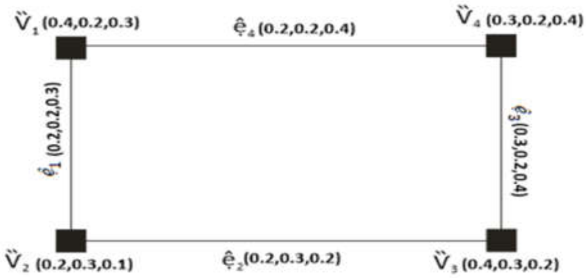


Figure 4. Picture Fuzzy Graph.

In the above picture fuzzy graph (Figure 4), the vertices are represented the value of membership, abstinence, and non-membership and the edges are represented by fuzzy relationships between membership, abstinence, and non-membership.

3. Domination on Picture Fuzzy Graph

In this section, we present the notions of strength, order, degree of nodes and edges, cardinality and completeness for PFG and introduce the definition of domination in PFG, also providing some examples.

Definition 7. *A PFG is the form of* $\check{K} = (\tilde{V}, \check{E})$ *is said to be node cardinality of* $\tilde{V}$ *in PFG is characterized by*

$$|\tilde{V}| = \sum_{ű_i \in \tilde{V}} \frac{1 + \check{T}_1(ű_i) - \hat{R}_1(ű_i) - \check{N}_1(ű_i)}{3}, for\ all\ ű_i \in \tilde{V}.$$

Definition 8. *A PFG in the form of* $\check{K} = (\tilde{V}, \check{E})$ *is said to be edge cardinality of* $\check{E}$ *is represented and explained by*

$$|\check{E}| = \sum_{(ű_i, ű_i) \in \tilde{V}} \frac{1 + \check{T}_2(ű_i, ű_j) - \hat{R}_2(ű_i, ű_j) - \check{N}_2(ű_i, ű_j)}{3},\ for\ all(ű_i, ű_j) \in \check{E}.$$

Definition 9. *Let* $\check{K}$ *be a PFG in the form of* $\check{K} = (\tilde{V}, \check{E})$, *then the cardinality of* $\check{K}$ *is represented and characterized by*

$$|\check{K}| = |\,|\tilde{V}| + |\check{E}|\,|$$

$$|\check{K}| = |\sum_{ű_i \in \tilde{V}} \frac{1 + \check{T}_1(ű_i) - \hat{R}_1(ű_i) - \check{N}_1(ű_i)}{3} + \sum_{(ű_i, ű_i) \in \tilde{V}} \frac{1 + \check{T}_2(ű_i, ű_j) - \hat{R}_2(ű_i, ű_j) - \check{N}_2(ű_i, ű_j)}{3}.$$

Definition 10. *A PFG* $\check{K} = (\tilde{V}, \check{E})$ *the grade of node* ű *is known as "the sum of the edges incident at* ű*" is* $d_{\check{K}}(ű)$. *The grades are minimum and maximum of* $\check{K}$ *is* $\delta(\check{K}) = \min\{d_{\check{K}}(ű) : ű \in \tilde{V}\}$ *and* $\Delta(\check{K}) = \max\{d_{\check{K}}(ű) : ű \in \tilde{V}\}$, *respectively.*

Definition 11. *The number of nodes in a PFG of the form of* $\check{K} = (\tilde{V}, \check{E})$ *is known as order of PFG is denoted* $o(\check{K})$ *and the quantity of edges in a PFG* $\check{K}$ *characterized as size of PFG is denoted by* $(\check{K})$.

Example 5. *In the Figure 5,* $\check{K}$ *is a PFG in the nodes of* $\tilde{V} = (ű_1, ű_2, ű_3)$ *and* $\check{E} = (ȩ_1, ȩ_2, ȩ_3)$ *are edges. The cardinality is represented by* $|\tilde{V}| = 1.5$, $|\check{E}| = 0.874$ *and* $|\check{K}| = 2.37$. *The grade of* $\check{K}$ *are* $d_{\check{K}}(ű_1) = (0.5, 0.2, 0.6), d_{\check{K}}(ű_2) = (0.4, 0.2, 0.6)$ *and* $d_{\check{K}}(ű_3) = (0.5, 0.2, 0.4)$. *The order and size of PFG* $\check{K}$ *are* $O(\check{K}) = 3, S(\check{K}) = 3$.

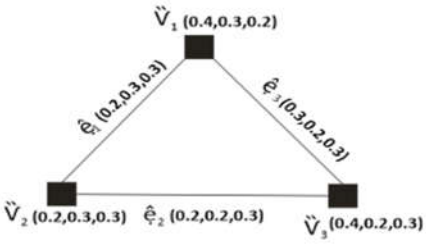


Figure 5. Picture Fuzzy Graph of Cardinality.

Definition 12. *Let* $\check{K} = (\tilde{V}, \check{E})$ *be a form of PFG. Two nodes* $ű_i$ *and* $ű_j$ *are known as neighbors in PFG if the following conditions are satisfied.*

1. $\check{T}_2(ű_i, ű_j) > 0, \hat{R}_2(ű_i, ű_j) > 0, \check{N}_2(ű_i, ű_j) > 0$
2. $\check{T}_2(ű_i, ű_j) = 0, \hat{R}_2(ű_i, ű_j) > 0, \check{N}_2(ű_i, ű_j) > 0$
3. $\check{T}_2(ű_i, ű_j) > 0, \hat{R}_2(ű_i, ű_j) = 0, \check{N}_2(ű_i, ű_j) > 0$
4. $\check{T}_2(ű_i, ű_j) = 0, \hat{R}_2(ű_i, ű_j) = 0, \check{N}_2(ű_i, ű_j) > 0$
5. $\check{T}_2(ű_i, ű_j) > 0, \hat{R}_2(ű_i, ű_j) = 0, \check{N}_2(ű_i, ű_j) = 0$
6. $\check{T}_2(ű_i, ű_j) = 0, \hat{R}_2(ű_i, ű_j) > 0, \check{N}_2(ű_i, ű_j) = 0, ű_i, ű_j \in \tilde{V}$.

Definition 13. *In a PFG, the form of* $\check{K} = (\tilde{V}, \check{E})$ *is a series of different nodes* $\{ű_1, ű_2, ű_3, \ldots, ű_n\}$ *such that, for some* i *and* j, *they are called a path if these conditions hold:*

1. $\check{T}_2(ű_i, ű_j) > 0, \hat{R}_2(ű_i, ű_j) > 0, \check{N}_2(ű_i, ű_j) > 0$
2. $\check{T}_2(ű_i, ű_j) = 0, \hat{R}_2(ű_i, ű_j) > 0, \check{N}_2(ű_i, ű_j) > 0$
3. $\check{T}_2(ű_i, ű_j) > 0, \hat{R}_2(ű_i, ű_j) = 0, \check{N}_2(ű_i, ű_j) > 0$
4. $\check{T}_2(ű_i, ű_j) > 0, \hat{R}_2(ű_i, ű_j) > 0, \check{N}_2(ű_i, ű_j) = 0$
5. $\check{T}_2(ű_i, ű_j) = 0, \hat{R}_2(ű_i, ű_j) = 0, \check{N}_2(ű_i, ű_j) > 0$
6. $\check{T}_2(ű_i, ű_j) > 0, \hat{R}_2(ű_i, ű_j) = 0, \check{N}_2(ű_i, ű_j) = 0$
7. $\check{T}_2(ű_i, ű_j) = 0, \hat{R}_2(ű_i, ű_j) > 0, \check{N}_2(ű_i, ű_j) = 0$

   The strength of a path $\acute{P} = ű_1, ű_2, ű_3, \ldots, ű_{n+1} (n > 0)$ *is* n.

Definition 14. *For any two nodes* $ű_i$ *and* $ű_j$ *in a PFG in the form of* $\check{K} = (\tilde{V}, \check{E})$ *are path connected, then the path strength in anywhere* $\min_{i.j} \check{T}_{2ij}$ *is characteristic* $\check{T}$*-strength of the feeble arc,* $\min_{i,j} \hat{R}_{2ij}$ *is the* $\hat{R}$*- characteristic strength of the most weakest arc and* $\max_{i,j} \check{N}_{2ij}$ *is the* $\check{N}$*-strength of the strongest arc presented as*

$$\left(\min_{i.j} \check{T}_{2ij}, \min_{i,j} \hat{R}_{2ij}, \max_{i,j} \check{N}_{2ij}\right)$$

Definition 15: *In a PFG in the form of* $\check{K} = (\tilde{V}, \breve{E})$ *for two nodes* $ű_i, ű_j \in \tilde{V} \subseteq \check{K}$ *then* $\check{T}$-*strength of connectedness between* $ű_i$ *and* $ű_j$ *is* $\check{T}_2^{\propto}(ű_i, ű_j) = \sup\{\check{T}_2^{z}(ű_i, ű_j): z = 1,2,3,\ldots,n\}$, $\hat{R}$ *-strength of connectedness between* $ű_i$ *and* $ű_j$ *is* $\hat{R}_2^{\propto}(ű_i, ű_j) = \inf\{\hat{R}_2^{z}(ű_i, ű_j): z = 1,2,3,\ldots,n\}$ *and* $\check{N}$-*strength of contact between* $ű_i$ *and* $ű_j$ *is* $\check{N}_2^{\propto}(ű_i, ű_j) = \inf\{\check{N}_2^{z}(ű_i, ű_j): z = 1,2,3,\ldots,n\}$. *If* $ű$ *and* $ŭ$ *are connected by way of path of length* z *then* $\check{T}_2^{z}(ŭ, ű)$ *is characterized by*

$$\sup\{\check{T}_2(ŭ, ű_1) \wedge \check{T}_2(ű_1, ű_2) \wedge \check{T}_2(ű_2, ű_3) \ldots \wedge \check{T}_2(ű_{z-1}, ű): ŭ, ű_1, ű_2, \ldots, ű_{z-1}, ű \in \tilde{V}\}$$

$\hat{R}_2^{z}(ŭ, ű)$ *is characterized by*

$$\inf\{\hat{R}_2(ŭ, ű_1) < \vee\ \hat{R}_2(ű_1, ű_2) \vee \hat{R}_2(ű_2, ű_3), \ldots\ldots\ldots \vee\ \hat{R}_2(ű_{z-1}, ű): ŭ, ű_1, ű_2, \ldots., ű_{z-1}, ű \in \tilde{V}$$

and $\check{N}_2^{z}(ŭ, ű)$ *is characterized by*

$$\inf\{\check{N}_2(ŭ, ű_1) \vee\ \check{N}_2(ű_1, ű_2) \vee\ \check{N}_2(ű_2, ű_3), \ldots., \check{N}_2(ű_{z-1}, ű): ŭ, ű_1, ű_2, ű_3, \ldots., ű_{z-1}, ű \in \tilde{V}\}.$$

Definition 16. *Let* $\check{K}$ *be a PFG in the form of* $\check{K} = (\tilde{V}, \breve{E})$ *and* $Ḑ \subseteq \tilde{V}$ *of a graph DS of* $\check{K}$ *if all nodes* $\tilde{V} - Ḑ$ *were adjacent to minimal of one node in* $Ḑ$. *A DS which has the minimum nodes set in* $\check{K}$ *is called minimal DS. The cardinality of minimum DS of* $\check{K}$ *called a dominating number of* $\check{K}$. *For a PFG in the form of* $\check{K} = (\tilde{V}, \breve{E})$, *this arc* $(ŭ, ű)$ *is then known as strong arc if* $\check{T}_2(ŭ, ű) \geq \check{T}_2^{\propto}(ŭ, ű), \hat{R}_2(ŭ, ű) \geq \hat{R}_2^{\propto}(ŭ, ű), \check{N}_2(ŭ, ű) \geq \check{N}_2^{\propto}(ŭ, ű)$.*Then* $ŭ$ *dominates in* $ű$ *if the strong arc exists. A node of neighbor is denoted by* $N(ŭ) = \{ű \in \tilde{V}: (ŭ, ű)\ is\ a\ strong\ arc\}$. *If* $\tilde{V} - Ḑ$ *have a DS* $Ḑ^{-1}$ *in minimal DS of* $\check{K}$.*So,* $Ḑ^{-1}$*is said to be inverse DS of* $\check{K}$ *with respect to* $Ḑ$.

Example 6. *Figure 6 shows a PFG in the form of domination* $\check{K} = (\tilde{V}, \breve{E})$ *in anywhere* $\tilde{V} = \{ű_1, ű_2, ű_3, ű_4, ű_5, ű_6, ű_7\}$ *are nodes and* $\breve{E} = \{ȩ_1, ȩ_2, ȩ_3, ȩ_4, ȩ_5, ȩ_6, ȩ_7\}$ *edges.*

$$Ḑ_1 = \{ű_1, ű_2, ű_5\}$$

$$\tilde{V} - Ḑ_1 = \{ű_2, ű_4, ű_6, ű_7\}$$

Therefore $\tilde{V} - Ḑ_1$ *is adjacent to* $Ḑ_1$, *So* $Ḑ_1$ *is dominating set on* $\tilde{V}$.

$$Ḑ_2 = \{ű_6, ű_7\}$$

$$\tilde{V} - Ḑ_2 = \{ű_1, ű_2, ű_3, ű_4, ű_5\}$$

Thus $\tilde{V} - Ḑ_2$ *is adjacent to* $Ḑ_2$, *So* $Ḑ_2$ *is dominating set on* $\tilde{V}$.

$$Ḑ_3 = \{ű_4, ű_5, ű_6\}$$

$$\tilde{V} - Ḑ_3 = \{ű_1, ű_2, ű_3, ű_7\}$$

Hence $\tilde{V} - Ḑ_3$ *is not adjacent to* $Ḑ_3$, *so* $Ḑ_3$ *is not dominating set on* $\tilde{V}$. *Clearly, the minimum dominating set* $Ḑ_2$ *and the* $F_d(\check{K}) = 1.53$

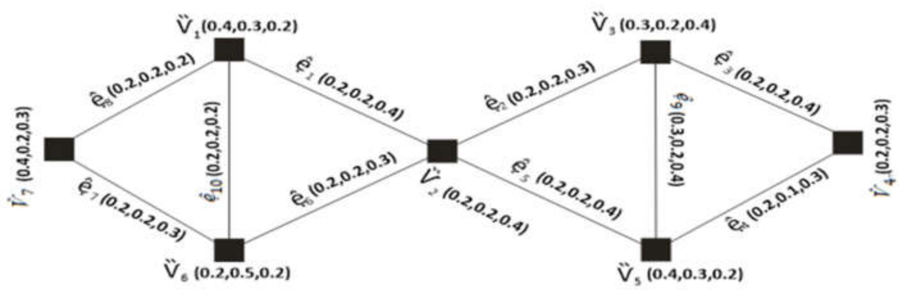


Figure 6. Graph with Domination Number.

4. Double Domination on Picture Fuzzy Graph

In this section we present and study the notion of double domination (DDS) for PFGs with the help of some examples. In particular, we prove some results concerning the cardinality of DDSs.

Definition 17. *A pair* $\check{K} = (\tilde{V}, \breve{E})$ *in the form of PFG is* $D \subseteq \tilde{V}$. *Then* $Ḑ$ *is known as DDS if each end in* $\tilde{V} - Ḑ$ *is ruled by at least two ends in* $Ḑ$. *The minimum cardinality of all the DDSs is called the double dominating number and expressed by* $F_{dd}(\check{K})$.

Example 7. *In the Figure 7, let* $\check{K}$ *be a PFG in the form of* $\check{K} = (\tilde{V}, \breve{E})$, *in anywhere* $\tilde{V} = \{ű_1, ű_2, ű_3, ű_4, ű_5, ű_6\}$ *be the set of nodes and* $\breve{E} = \{ȩ̂_1, ȩ̂_2, ȩ̂_3, ȩ̂_4, ȩ̂_5, ȩ̂_6, ȩ̂_7, ȩ̂_8\}$ *edges.*

$$Ḑ_1 = \{ű_2, ű_3, ű_5, ű_6\}$$

$$\tilde{V} - Ḑ_1 = \{ű_1, ű_4\}$$

Hence, $\{ű_1, ű_4\}$ *are adjacent of two vertices in* $Ḑ_1$. *Therefore,* $Ḑ_1$ *is a DDS on* $\tilde{V}$ *and* $F_{dd}(\check{K}) = 1.6$.

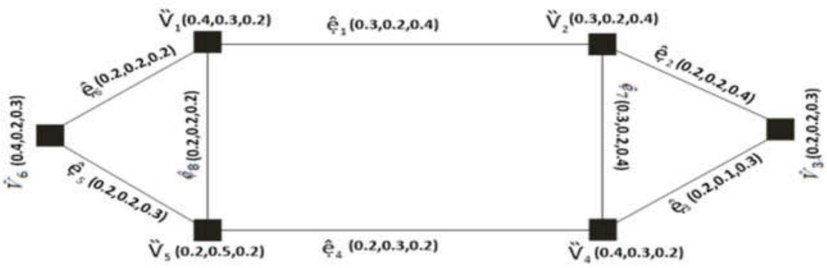


Figure 7. Graph with Double Domination Number.

The above PFG with a double domination number is a type of graph where two sets of vertices are selected such that every vertex in the graph is either in one set or is adjacent to a vertex in both sets (Figure 7).

Theorem 1. *In a PFG* $\check{K}$, *if each vertex in* $\tilde{V} - Đ$ *contains at least two strong neighbors, then DDS exists in* $\check{K}$.

Proof. The DDS in PFG of a node $ű \in \tilde{V} - Ḑ$. If the node $ű \in \tilde{V} - Ḑ$ only one strong neighbor and other nodes in $\tilde{V} - Ḑ$ at least two strong neighbors. For f, $ű \in \tilde{V} - Ḑ$, $\textit{there exists}$ a node $ű \in Ḑ$ such that $Ḑ$ is dominance. This is contrary, so our hypotheses are false. Thus, every node in $\tilde{V} - Ḑ$ must have at least two strong neighbors. □

Example 8. *In a PFG in the form of* $\check{K} = (\tilde{V}, \breve{E})$, *in anywhere* $\tilde{V} = \{ű_1, ű_2, ű_3, ű_4\}$ *be the set of nodes and* $\breve{E} = \{ȩ̂_1, ȩ̂_2, ȩ̂_3, ȩ̂_4\}$ *be the set of edges respectively (Figure 8).*

$$D = \{ű_1, ű_3, ű_4\}$$

$$\tilde{V} - D = \{ű_2\}$$

Hence, $\{ű_1, ű_3, ű_4\}$ *are strong arc and* $\{ű_2\}$ *has at least two strong neighbors.*

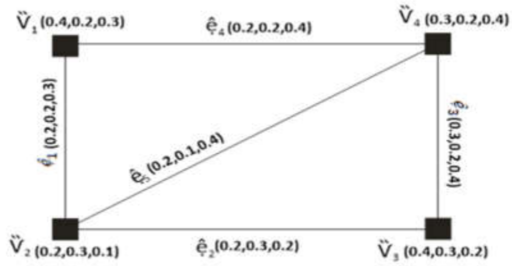


Figure 8. At Least Two Neighbors.

In Figure 8, the vertices and edges of a network are associated with PFSs in a PFGs with at least two neighbours, and each vertices have at least two neighbors. A picture that depicts the degree to which a vertices or edge belongs to the graph is used to represent the degree of membership of a vertices or edge.

Theorem 2. *The PFG of* $\check{K}$ *and* Ḍ *in DDS, then* $|Ḍ| \geq |\tilde{V} - Ḍ|$.

Proof. The DDS in Ḍ of every node ű *in* $\tilde{V} - Ḍ$ required two nodes at least in Ḍ, then each node ű *in* $\tilde{V} - Ḍ$ is a neighborhood. We therefore consider that every node must be strong. So, the more dominant set can be obtained and the adjacent node in Ḍ. Hence, it is proved $|Ḍ| \geq |\tilde{V} - Ḍ|$. □

Example 9. *In the Figure 9, The PFG of* $\check{K} = (\tilde{V}, \check{E})$, *in anywhere* $\tilde{V} = \{ű_1, ű_2, ű_3, ű_4, ű_5, ű_6\}$ *be the set of nodes and edges* $\check{E} = \{ȇ_1, ȇ_2, ȇ_3, ȇ_4, ȇ_5, ȇ_6\}$.

$$Ḍ = \{ű_1, ű_3, ű_5\}$$

So $\tilde{V} - Ḍ = \{ű_2, ű_4\}$, *thus* $|Ḍ| = 2.4$ *and* $|\tilde{V} - Ḍ| = 1.9$. *Hence, proof* $|Ḍ| \geq |\tilde{V} - Ḍ|$.

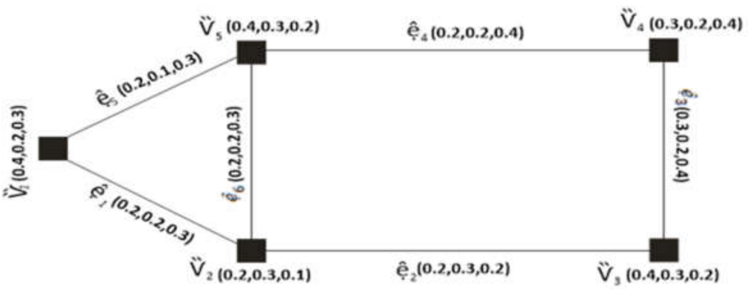


Figure 9. Graph with Cardinality.

The Figure 9 shows picture that accurately conveys the level of a vertices or edge's membership, abstinence, and non-membership in the PFG with cardinality is used to represent the vertices or edge's degree of membership, abstinence, and non-membership respectively. The picture's size corresponds to the set's cardinality, while its intensity corresponds to the degree of membership abstinence, and non-membership respectively.

Theorem 3. *The DDS* Ḍ *is minimal if for two nodes* $\{ŭ, ű\} \in Ḍ$. *These statements hold:*

(1) *there exists a node* $ŭ \in \tilde{V} - Ḍ$ *such that* $N(ŭ) \cap Ḍ \neq \{ŭ, ű\}$
(2) $\tilde{V} - Ḍ$ *is lonely.*

Proof. A minimal DS in the form of PFG Ǩ. We consider ŭ, ű do not verify conditions (1) and (2). Suppose that Ḑ = Ḑ − {ŭ, ű} is a DDS verifying situation (1) and (2). So Ṽ − Ḑ is lonely, which is in contradiction to our supposition as {ŭ, ű ∈ D}.

Conversely, for each node ŭ, ű in a DDS Ḑ which has the conditions (1) and (2) verified. Suppose that minimal DDS is not in Ḑ, we have to write {ŭ, ű} ∈ Ḑ such that Ḑ − {ŭ, ű}. Therefore {ŭ, ű} is at least one node of a strong neighbor in Ḑ − {ŭ, ű}. So, we have nodes ű ∈ Ṽ − Ḑ such that N(ŭ) ∩ Ḑ ≠ {ŭ, ű}; this is contrary. Thus, Ḑ is a minimal DDS. □

Theorem 4. *Let* Ǩ = (Ṽ, Ĕ) *be a PFG if* Ḑ *is minimal DDS then it verifies that*

(1) ŴŤ(Ḑ) ≤ δ(Ǩ) + 2
(2) ŴŤ(Ḑ) ≥ Δ(Ǩ) − 1, *in anywhere* ŴŤ(Ḑ), Δ(Ǩ), δ(Ǩ) *denoted the maximum and minimum grades of weight DDS in* Ǩ *respectively.*

Proof. Consider a minimal DDS of Ḑ

$$1.\hat{W}\check{T}(Ḑ) = (\sum \min_{1\le i\le n}[d_{\check{T}}(ŭ_i)], \sum \min_{1\le i\le n}[d_{\hat{R}}(ŭ_i)], \sum \max_{1\le i\le n}[d_{\check{N}}(ŭ_i)])$$

$$\hat{W}\check{T}(Ḑ) \ge (\min[\sum[d_{\check{T}}(ŭ_i)]], \min[\sum[d_{\hat{R}}(ŭ_i)]], \min[\sum[d_{\check{N}}(ŭ_i)]])$$

$$\hat{W}\check{T}(Ḑ) = (\delta_{\check{T}}(\check{K}), \delta_{\hat{R}}(\check{K}), \delta_{\check{N}}(\check{K}))$$

$$\hat{W}\check{T}(Ḑ) = \delta(\check{K})$$

$$\le \delta(\check{K}) + 2$$

$$2.\hat{W}\check{T}(Ḑ) = (\sum \min_{1\le i\le n}[d_{\check{T}}(ŭ_i)], \sum \min_{1\le i\le n}[d_{\hat{R}}(ŭ_i)], \sum \max_{1\le i\le n}[d_{\check{N}}(ŭ_i)])$$

$$\hat{W}\check{T}(Ḑ) \ge (\max[\sum[d_{\check{T}}(ŭ_i)]], \max[\sum[d_{\hat{R}}(ŭ_i)]], \max[\sum[d_{\check{N}}(ŭ_i)]])$$

$$\hat{W}\check{T}(Ḑ) = (\Delta_{\check{T}}(\check{K}), \Delta_{\hat{R}}(\check{K}), \Delta_{\check{N}}(\check{K}))$$

$$\hat{W}\check{T}(Ḑ) = \Delta(\check{K})$$

$$\hat{W}\check{T}(Ḑ) \ge \Delta(\check{K}) - 1$$

□

Theorem 5. *In the PFG* Ǩ *with only end nodes. Then DDS* Ḑ *does not exist.*

Proof. The PFG Ǩ = (Ṽ, Ĕ) with end nodes. So Ḑ ⊆ Ṽ such that Ǩ end nodes. Therefore, every node in ű ∈ Ṽ − Ḑ*there exist*ű ∈ Ḑ such that Ḑ is a DS. Obviously, Ṽ − Ḑ is none of the nodes dominated by the minimum of both nodes in Ḑ. Thus, it does not exist in any DDS Ḑ. □

Example 10. *In Figure 10,* Ǩ = (Ṽ, Ĕ) *in the form of PFG, in anywhere* Ṽ = {$ű_1, ű_2, ű_3, ű_4, ű_5$} *be the set of nodes and edges* Ĕ = {$ȩ̂_1, ȩ̂_2, ȩ̂_3, ȩ̂_4, ȩ̂_5$}*.We note that strong arcs are* $ű_2$ *and* $ű_4$*, their DDS does not exist, then other strong arcs are required.*

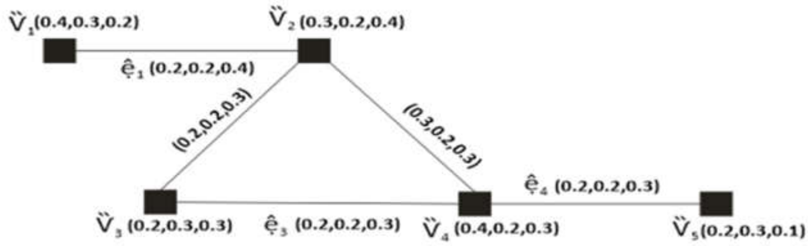


Figure 10. Graph with only End Nodes.

Theorem 6. *For any PFG* $\check{K} = (\tilde{V}, \breve{E})$*,* $F_{dd}(\check{K}) \geq \frac{O(\check{K})}{\Delta_{\hat{R}}(\check{K}) + \Delta_{\check{N}}(\check{K}) + 1}$*, where* $\Delta_{\hat{R}}(\check{K})$ *is a minimum* $\hat{R}$ *grade of* $\check{K}$ *and* $\Delta_{\check{N}}(\check{K})$ *is a maximum* $\check{N}$ *grade of* $\check{K}$ *and* $O(\check{K})$*.*

Proof. We consider that $Ḑ$ in DDS of PFG $\check{K}$ with $|Ḑ| = F_{dd}(\check{K})$, that every node $\tilde{V} - Ḑ$ is adjacent to the same node $Ḑ$.

$$|\tilde{V} - Ḑ| \leq \sum_{i=1}^{n} d(\check{u}_i) \leq F_{dd}(\check{K}).\,\Delta_{\check{T}}(\check{K})$$

$$\Rightarrow O(\check{K}) - F_{dd}(\check{K}) \leq F_{dd}(\check{K})\Delta_1(\check{K})$$

So, $O(\check{K}) - F_{dd}(\check{K}) \leq F_{dd}(\check{K})\Delta_{\hat{R}}(\check{K}) + F_{dd}(\check{K})\Delta_{\check{N}}(\check{K})$

$$O(\check{K}) \leq F_{dd}(\check{K})\Delta_{\hat{R}}(\check{K}) + F_{dd}(\check{K})\Delta_{\check{N}}(\check{K}) + F_{dd}(\check{K})$$

$$there\ exist \leq \left(\Delta_{\hat{R}}(\check{K}) + \Delta_{\check{N}}(\check{K}) + 1\right)F_{dd}(\check{K}).$$

This implies that $F_{dd}(\check{K}) \geq \frac{O(\check{K})}{\Delta_{\hat{R}}(\check{K}) + \Delta_{\check{N}}(\check{K}) + 1}$

Thus, proof is completed. □

Example 11. *In the Figure 11*, *A PFG in the form of* $\check{K} = (\tilde{V}, \breve{E})$*, in anywhere* $\tilde{V} = \{\H{u}_1, \H{u}_2, \H{u}_3, \H{u}_4\}$ *be the set of nodes and edges* $\breve{E} = \{ȩ_1, ȩ_2, ȩ_3, ȩ_4, ȩ_5, ȩ_6\}$*. Note that* $\{\H{u}_1, \H{u}_2, \H{u}_3\}$ *are the strong arcs and* $Ḑ = \{\H{u}_1, \H{u}_2, \H{u}_3\}$*, then* $\tilde{V} - Ḑ = \{\H{u}_2\}$*, therefore* $\{\H{u}_2\}$ *the cut vertex.*

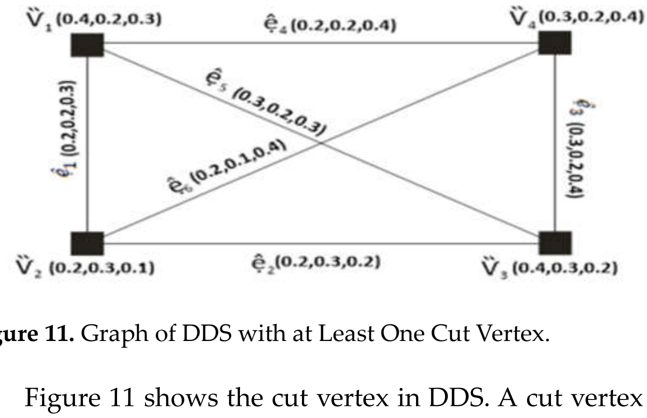


Figure 11. Graph of DDS with at Least One Cut Vertex.

Figure 11 shows the cut vertex in DDS. A cut vertex is a vertex that, if it were to be removed, would divide a graph into two or more separate parts. Finding a double dominance set with a cut vertex assures that the collection of dominant vertices is not limited to a single linked part of the graph, which is why finding such a set is crucial.

Theorem 7. *For any PFG*$\check{K}$*,* $F^{-1}(\check{K}) \leq F_{dd}(\check{K}) \leq |\tilde{V}|$*, in anywhere* $F^{-1}(\check{K})$ *is a DDN.*

Proof. Suppose in a PFG set opposite rule $Ḑ^{-1} \subseteq \tilde{V} - Ḑ$. So, the rule number is twice the opposite rule number by theorem (2). This means that $F^{-1}(\check{K}) \leq F_{dd}(\check{K})$ does not contain all nodes. At least one node $\tilde{V}$ mustbe inside $\tilde{V} - Ḑ$. Hence $\check{K} - \{\tilde{V}\}$ node yields the DDN. Note that $F_{dd}(\check{K}) \leq |\tilde{V}|$. Thus, $F^{-1}(\check{K}) \leq F_{dd}(\check{K}) \leq |\tilde{V}|$. □

Theorem 8. *The PFG of* $\check{K}$ *in DDS* $Ḑ$ *is independent while not in* $\check{K}$.

Proof. The DDS of $Ḑ$ is independent of PFG $\check{K}$, Consider that $\check{K}$ is a complement of $\check{K}$. So $\tilde{V} = \tilde{V}$

$$\check{T}_{1ij} = \min(\check{T}_{1i}, \check{T}_{1j}) - \check{T}_{1ij}$$

$$\hat{R}_{1ij} = \min(\hat{R}_{1i}, \hat{R}_{1j}) - \hat{R}_{1ij}$$

And $\check{N}_{1ij} = \max(\check{N}_{1i}, \check{N}_{1j}) - \check{N}_{1ij}$

Here these change only the values of nodes in $\check{K}$, so the adjacent nodes in $\check{K}$ have a strong neighbor in distinct DDS in $\check{K}$. Thus, the same DDS $Ḑ$ in $\check{K}$ is not independent $\check{K}$. □

5. Application

During election campaigns, political leaders must gather as much support as possible in a relatively short period of time. Obviously, in the case of regional or national general elections, it is impossible for the leader to personally know and persuade all his potential voters, and he must therefore create a hierarchical structure reporting to him and including smaller and smaller groups of people. We can assume that there is a central PFG who can carry out vote persuasion in his group. If a political leader meets a citizen and obtains his or her consent by showing that he or she is interested in his or her issues and problems, then it is very likely that the citizen will also persuade his or her family and friends to vote for that leader.

Applying the PFG theory, it is not necessary for the leader to meet every voter personally, but it will suffice if he meets and convinces the DS citizens who will then undertake to gather the rest of the votes in his favor among the people in his own group.

Example 12. *Let* $\check{K}$ *be a PFG in the form of* $\check{K} = (\tilde{V}, \check{E})$ *in Figure 12 in anywhere we assume 7 voters, denoted the nodes and edges. The node and edge have in PFG* $\tilde{V} = \{ű_1, ű_2, ű_3, ű_4, ű_5, ű_6, ű_7\}$ *is set of all voters and* $\check{E} = \{ȩ̂_1, ȩ̂_2, ȩ̂_3, ȩ̂_4, ȩ̂_5, ȩ̂_6, ȩ̂_7. ȩ̂_8, ȩ̂_9, ȩ̂_{10}, ȩ̂_{11}, ȩ̂_{12}\}$ *denotes the edges. The minimum DS in PFG* $\check{K}$ *is*

$$Ḑ = \{ű_2, ű_3, ű_5, ű_6\}$$

$$\tilde{V} - Ḑ = \{ű_1, ű_7, ű_4\}$$

Now the issue can be solved by using the PF monitor set. In the PFG, the nodes represent voters in any electorate. These voters are connected if there is a connection between electorates. The ratio depends on the relationship between the two electorates. This type of fuzzy value is listed next to the relationship of the two nodes. If people do not have a relationship, they are unrelated.

By using the domination in PFG, there exists a minimum DS in the graph. The politician can just want to meet the minimum DS of citizens. Therefore, all members of citizens and non-citizens of DS are able to ask for votes in DS. They are all members of voters who will transmit their votes to a particular electorate. In Figure 12, we see that the political leader needs to meet only voter's $ű_1, ű_7$ *and* $ű_4$ in order for him to gather enough votes to be elected. The graph below briefly describes the situation.

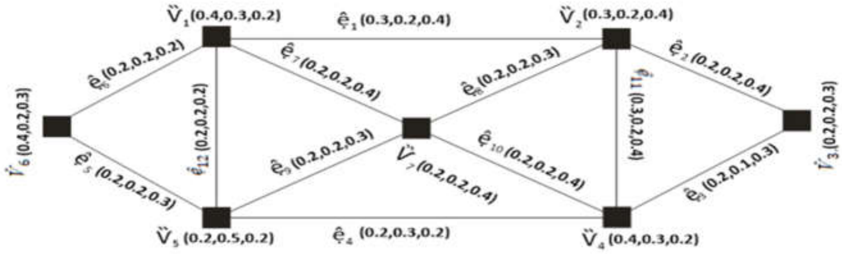
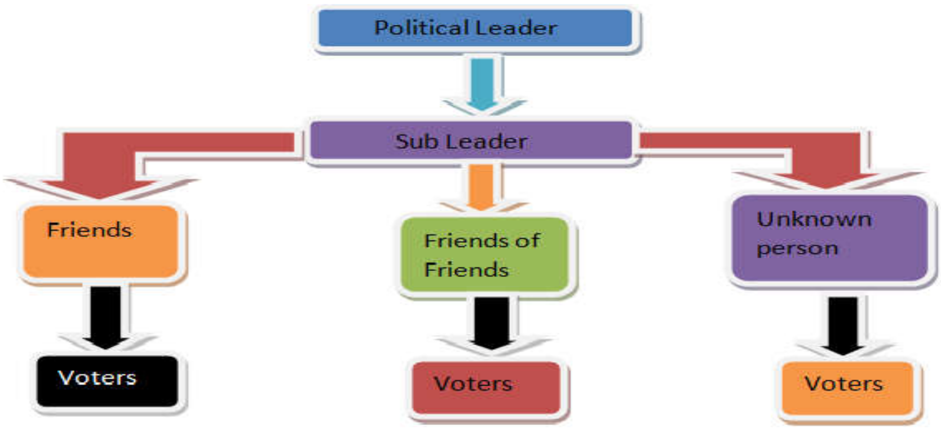


Figure 12. Picture fuzzy graph.

The political leader should have a well-organized and capable team to plan, coordinate, and execute their campaign strategy. The political leader can use sub leaders and friends to reach a wider au-dience and engage with their supporters. The Scheme 1 gave information about any political leader to increase any political leader's supporters.



Scheme 1. Flow chart.

6. Comparative Study and Advantages

In this section, we discuss the idea of DS in PFG. Example (4) is a PFG illustrating nodes and edges in PFN. In this situation, a PFN is better suited to balance the uncertainty case than a fuzzy number or an intuitionistic fuzzy number.

Here we take the example of the area in FG and IFG and show their values. In Figure 13, this type of graph has non-membership and neutrality values of zero, while in Figure 2, the abstinence values are zero. This information shows that both fuzzy and intuitionistic environments can be developed by the PFG and confirms the validity of our generalization.

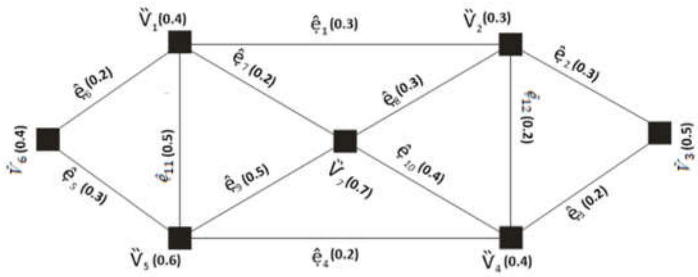


Figure 13. Fuzzy graph.

A vertices or edge's degree of membership indicates how much it is a part of the graph. The strength of a vertices existence in the graph is indicated by its degree of membership, whereas the strength of an edge's relationship between its endpoints is indicated by its degree of membership.

In an intuitionistic fuzzy graph (Figure 14), the vertices and edges of a graph are associated with intuitionistic fuzzy sets. The membership and non-membership degrees of a vertices or an edge reflect the degree to which it belongs to or does not belong to the graph, respectively.

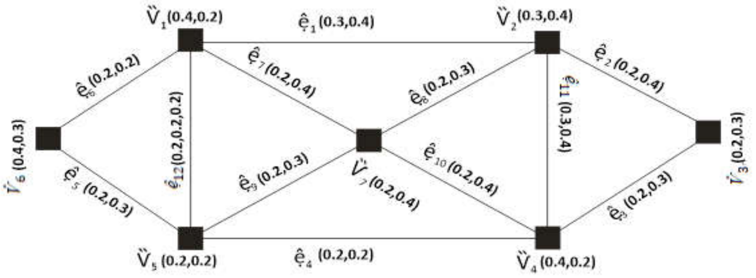


Figure 14. Intuitionistic fuzzy graph.

We are confident that the FG and IFG will not be able to solve this problem above; the reason for this is that these structures were limited only to some types of functions in Table 1. This type of organization does not talk about correction scores and degree negation. If we want to perform this experiment in the condition of PF graph information, it will be unsuccessful because of interference with their structure.

Table 1. FGs, IFGs and PFGs structure**.**

	Membership	Neutral	Non-Membership	Remarks
FGs	✓	×	×	Only discussed membership function
IFGs	✓	×	✓	Discussed membership and non-membership
PFGs	✓	✓	✓	In PFG, we discussed membership, neutral and non-membership function

A comparison of the proposed methods with existing methods is performed in Table 1. The PFGs are superior to all other concepts and methods for dealing with fuzziness. These graphs clearly discuss three different classes, namely membership, abstinence, and non-membership. In the Table 1. "✓ means the membership, neutral, and non-membership exist and × means the membership, neutral, and non-membership not-exist." On the other hand, FGs, CFGs, and IFGs, are fail.

7. Conclusions

The previous test suggests another idea of PFG. This unique idea helps to summarize problematic current ideas such as FG and IFG. This way of thinking forms the foundation for managing four different kinds of information and ideas. Fuzzification in fuzzy logic systems entails converting sharp inputs into fuzzy sets. The type of input variables and the particular application determine the best fuzzification technique to use. Anti-symmetric fuzzy sets are better suited for input variables with a nonlinear relationship than symmetric fuzzy sets, which are frequently employed for input variables with a linear relationship between their linguistic meanings. Additionally, we have suggested and supported the idea of a dual-dominant theoretical authority hypothesis that would summarize all the unresolved impacts of the numerous graph theory plans that are currently in existence. The basic operations are described in terms such as cardinality order, integrity of domination, PFG neighbors, strength and double domination set. Similarly, PFG-related terms are represented along with their properties. The application of the theory shows the extent to which political pioneers may reach a large number of voters anyway. In this particular case, the potential for PF dominance can be very helpful. Equivalent evaluations emphasize that the proposed structure is new and that existing designs at this point help cope with situations where various devices are ignored. The terminology and ideas described in this white paper are arrangements of lengths that are considered PFS and can be considered shifted upward to the extent that they are considered to reduce information loss more effectively than PFS.

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