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A Buffon-Laplace Type Problem for an Irregular Lattice with Cell Composed by Pentagon + Triangle with Obstacles

G. Caristi, A. Puglisi

Department of Economic Science and Quantitative Method
University of Messina, Italy

M. Stoka

Sciences Accademy of Turin, Italy

Abstract

In same previous papers [1], [2], [3],[4], [5], [6],[7], [8], [9],[10], [11], [12], [13] and [15] the authors studies same Buffon-Laplace problems with different fundamental cells. In this paper we want to compute the probability that a random segment of constant length intersects a side of a lattice with cell represented in fig. 1.

Keywords: Buffon-Laplace type problem; Irregular Lattice

Let $\Re(a;m)$ irregular lattice with the fundamental cell $C_0 = C_{01} \cup C_{02}$ represented in fig.1

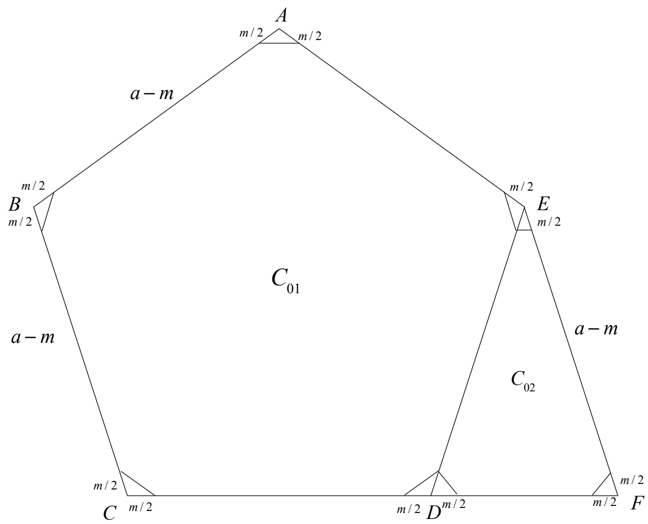

fig.1

By this figure we have

$$|FD| = \frac{a}{2\cos\frac{\pi}{5}}; \qquad |AA_1| = |AA_2| = |BB_1| = |BB_2| =$$

$$|CC_1| = |CC_2| = |DD_1| = |DD_2| = |DD_3| =$$

$$|EE_1| = |EE_2| = |EE_5| = |FF_1| = |FF_2| = \frac{m}{2};$$

$$|A_1A_2| = |B_1B_2| = |C_1C_2| = |D_1D_2| = |E_1E_2| = m\cos\frac{\pi}{5}$$

$$|D_2D_3| = |F_1F_2| = m\sin\frac{\pi}{5}, \qquad |E_2E_5| = \frac{m}{4\cos\frac{\pi}{5}} \tag{1}$$

$$areaAA_1A_2 = areaBB_1B_2 = areaCC_1C_2 =$$

$$areaDD_1D_2 = areaEE_1E_2 = areaDD_2D_3 =$$

$$areaFF_1F_2 = \frac{m^2}{8}\sin\frac{2\pi}{5}$$

$$areaEE_2E_5 = \frac{m^2}{8}\sin\frac{\pi}{5};$$

$$areaC_0 = \frac{a^2}{2}\sin\frac{\pi}{5}\left(2\cos\frac{\pi}{5}+1\right)^2 - \frac{m^2}{8}\sin\frac{\pi}{5}\left(14\cos\frac{\pi}{5}+1\right);$$

We want to compute the probability that a random segment s of constant length l, intersects a side of the lattice $\Re$, i.e. the probability P_{int} that s intersects a side of the fundamental cell $C_{0.}$

The position of the segment s is determinated by center and by the angle φ that is formed with the line CF.

In order to compute P_{int} we consider the limiting positions of the segment s, for a fixed value of φ, in the cell C_{0i}, respectively C_{02} .

We have fig. 2

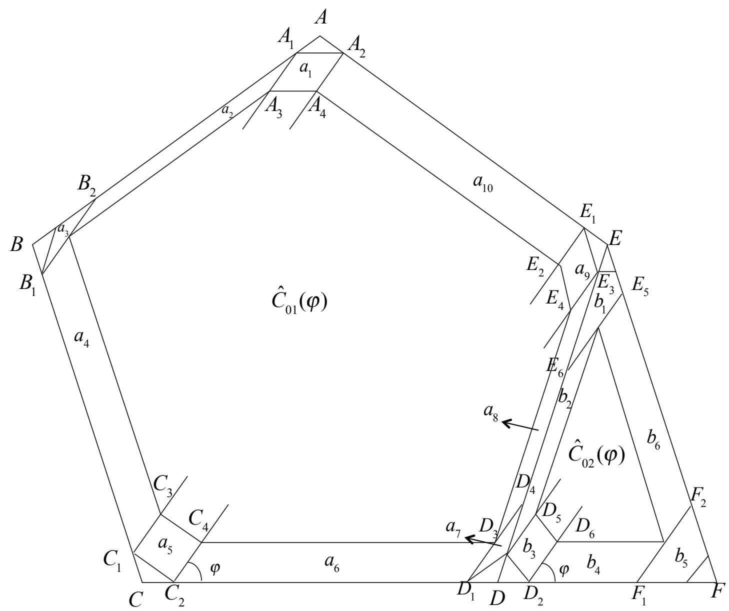

fig.2

and the relations

$$area\widehat{C}_{01}(\varphi) = areaC_{01} - \sum_{j=1}^{10} areaa_j(\varphi), \qquad (2)$$

$$area\widehat{C}_{02}(\varphi) = areaC_{02} - \sum_{j=1}^{6} areab_j(\varphi), \qquad (3)$$

with

$$l < \frac{a}{4\cos\frac{\pi}{5}} - m.$$

To compute $area\widehat{C}_{01}(\varphi)$ we have that:

$$areaa_5(\varphi) = \frac{lm}{2}\cos\frac{\pi}{5}\sin\left(\frac{\pi}{5}+\varphi\right),$$

$$areaa_6(\varphi) = \frac{(a-m)\,l}{2}\sin\varphi,$$

$$areaa_4(\varphi) = \frac{(a-m)\,l}{2}\sin\left(\frac{2\pi}{5}+\varphi\right),$$

$$areaa_3(\varphi) = \frac{lm}{2}\cos\frac{\pi}{5}\sin\left(\frac{2\pi}{5}-\varphi\right),$$

$$areaa_2(\varphi) = \left[a-m-\frac{l\sin\left(\frac{2\pi}{5}-\varphi\right)}{\sin\frac{\pi}{5}}\right]\frac{l}{2}\sin\left(\varphi-\frac{\pi}{5}\right),$$

$$areaa_1(\varphi) = \frac{lm}{2}\cos\frac{\pi}{5}\sin\varphi,$$

$$areaa_7(\varphi) = \frac{lm}{4}\sin\varphi - \frac{m^2}{8}\sin\frac{2\pi}{5},$$

$$areaa_8(\varphi) = \left(a-\frac{m}{2}-\frac{l\sin\varphi}{\sin\frac{2\pi}{5}}\right)\frac{l}{2}\sin\left(\frac{2\pi}{5}-\varphi\right),$$

$$areaa_9(\varphi) = \frac{lm}{2}\cos\frac{\pi}{5}\sin\left(\frac{2\pi}{5}+\varphi\right),$$

$$areaa_{10}(\varphi) = \frac{(a-m)\,l}{2}\sin\left(\frac{\pi}{5}+\varphi\right).$$

All these relations give us

$$A_1(\varphi) = \sum_{j=1}^{10} a_j(\varphi) = \frac{al}{2}$$
$$\left[\left(2\cos\frac{\pi}{5}+1\right)\sin\varphi + 2\sin\frac{2\pi}{5}\cos\varphi\right] + \frac{lm}{2}$$
$$\left\{\cos\frac{\pi}{5}\left[2\cos\frac{\pi}{5}\cos\left(\varphi-\frac{\pi}{5}\right)-\sin\varphi\right] - \sin\frac{2\pi}{5}\cos\varphi -\right.$$
$$\left.\left(\cos\frac{2\pi}{5}+\frac{1}{2}\right)\sin\varphi\right\} - \frac{l^2\sin\left(\frac{2\pi}{5}-\varphi\right)}{\sin\frac{2\pi}{5}}$$
$$\left[\left(\cos\frac{2\pi}{5}+2\right)\sin\varphi - \sin\frac{2\pi}{5}\cos\varphi\right] - \frac{m^2}{8}\sin\frac{2\pi}{5} \qquad (4)$$

and the relation (2) becomes

$$area\widehat{C}_{01}\left(\varphi\right)=areaC_{01}-A_{1}\left(\varphi\right) \tag{5}$$

To compute $area\widehat{C}_{02}(\varphi)$ we have that:

$$areab_{1}\left(\varphi\right)=\frac{lm}{2}\sin\frac{\pi}{5}\sin\left(\frac{3\pi}{10}+\varphi\right),$$

$$areab_{2}\left(\varphi\right)=\left[\frac{a}{2\cos\frac{\pi}{5}}-\frac{m}{2}-\frac{l\sin\left(\frac{2\pi}{5}+\varphi\right)}{\sin\frac{2\pi}{5}}\right]\frac{l}{2}\sin\varphi,$$

$$areab_{5}\left(\varphi\right)=\frac{l^{2}}{\sin\frac{\pi}{5}}\sin\left(\frac{2\pi}{5}+\varphi\right)\sin\left(\frac{2\pi}{5}-\varphi\right)-\frac{m^{2}}{8}\sin\frac{\pi}{5},$$

$$areab_{6}\left(\varphi\right)=\left[a-\frac{m}{2}-\frac{l\sin\left(\frac{2\pi}{5}+\varphi\right)}{\sin\frac{\pi}{5}}\right].\frac{l}{2}\sin\left(\frac{2\pi}{5}-\varphi\right),$$

$$areab_{4}\left(\varphi\right)=\left[a-\frac{l\sin\left(\frac{2\pi}{5}-\varphi\right)}{\sin\frac{\pi}{5}}-\frac{l\sin\varphi}{\sin\frac{2\pi}{5}}\right]\frac{l}{2}\sin\left(\frac{2\pi}{5}+\varphi\right).$$

All these relations give us

$$A_{2}\left(\varphi\right)=\sum_{j=1}^{6}b_{j}\left(\varphi\right)=\frac{al}{2}\left(\frac{\sin\varphi}{2\sin\frac{\pi}{5}}+2\sin\frac{2\pi}{5}\cos\varphi\right)-$$

$$\frac{lm}{2}.\sin\frac{\pi}{5}\cos\left(\varphi-\frac{\pi}{5}\right)-\frac{l^{2}}{4\sin\frac{\pi}{5}}$$

$$\left[\left(1-\frac{\cos\frac{2\pi}{5}}{2\cos\frac{\pi}{5}}\right)\cos2\varphi+\sin\frac{\pi}{5}\sin2\varphi+\frac{\cos\frac{2\pi}{5}}{2\cos\frac{\pi}{5}}\right]-$$

$$\frac{m^{2}}{8}\sin\frac{\pi}{5}\left(1+2\cos\frac{\pi}{5}\right) \tag{6}$$

and the relation (3) becomes

$$area\widehat{C}_{02}\left(\varphi\right)=areaC_{02}-A_{2}\left(\varphi\right). \tag{7}$$

Denoting with M_i,$(i=1,2)$ the set of all segments s that have their center in the cell C_{0i} and with N_i the set of all segments s completely contained in C_{0i}we have [16]:

$$P_{int}=1-\frac{\mu\left(N_{1}\right)+\mu\left(N_{2}\right)}{\mu\left(M_{1}\right)+\mu\left(M_{2}\right)}, \tag{8}$$

where μ is the Lebesgue measure in the euclidean plane.

The measure $\mu(M_i)$ and $\mu(N_i)$ we use the kinematic measure of Poincaré [14]:

$$dK = dx \wedge dy \wedge d\varphi$$

where x, y are the coordinates of center of s and φ the fixed angle.

We can write

$$\mu(M_i) = \int_{\pi/5}^{2\pi/5} dy \int\int_{\{(x,y)\epsilon C_{01}\}} dxdy = \int_{\pi/5}^{2\pi/5} (areaC_{01})\, dy = \frac{\pi}{5} areaC_{0i},$$

then

$$\mu(M_1) + \mu(M_2) = \frac{\pi}{5} areaC_0. \tag{9}$$

In the same way, we have

$$\mu(N_i) = \int_{\pi/5}^{2\pi/5} dy \int\int_{\left\{(x,y)\epsilon \widehat{C}_{0i}\right\}} dxdy = \int_{\pi/5}^{2\pi/5} \left[areaC\widehat{C}_{0i}(\varphi)\right] dy$$

and

$$\mu(N_1) + \mu(N_2) = \frac{\pi}{5} areaC_0 - \int_{\pi/5}^{2\pi/5} \left[A_1(\varphi) + A_2(\varphi)\right] dy \tag{10}$$

Then we have that

$$A_1(\varphi) + A_2(\varphi) = \frac{al}{2}$$

$$\left[\left(2\cos\frac{\pi}{5} + 1 + \frac{1}{2\sin\frac{\pi}{5}}\right)\sin\varphi + 4\sin\frac{2\pi}{5}\cos\varphi\right] +$$

$$\frac{lm}{2}\left[\left(\cos\frac{2\pi}{5} - \sin\frac{\pi}{5} + 1\right)\cos\left(\varphi - \frac{\pi}{10}\right) -\right.$$

$$\left.\sin\frac{2\pi}{5}\cos\varphi - \left(\cos\frac{2\pi}{5} + \cos\frac{\pi}{5} + \frac{1}{2}\right)\sin\varphi\right] -$$

$$\frac{l^2}{4\sin\frac{2\pi}{5}}\left[\left(5\cos\frac{2\pi}{5} - 3\cos\frac{\pi}{5} - \sin\frac{\pi}{5} + 1\right)\right.$$

$$\cos 2\varphi + \left(3\sin\frac{2\pi}{5} - \cos\frac{\pi}{5} + \sin\frac{\pi}{5} + 1\right)$$

$$\sin 2\varphi - 5\cos\frac{2\pi}{5} + \cos\frac{\pi}{5} - \sin\frac{\pi}{5} - 1\Big] - \frac{m^2}{8}\sin\frac{\pi}{5}\left(4\cos\frac{\pi}{5}+1\right). \qquad (11)$$

and

$$\mu(N_1) + \mu(N_2) = \frac{\pi}{5} areaC_0 - \left\{\frac{al}{2}\left(1 - 2\sin\frac{\pi}{5} - \cos\frac{\pi}{5}\right) - \frac{lm}{2}\right.$$

$$\left(\sin\frac{2\pi}{5}\sin\frac{\pi}{10} + 1\right) - \frac{l^2}{4\sin\frac{2\pi}{5}}\left[6\cos\frac{\pi}{10} + \cos\frac{2\pi}{5} - 2\sin\frac{2\pi}{5} - 1 - \frac{2\pi}{5}\right.$$

$$\left.\left.\left(5\cos\frac{2\pi}{5} - \cos\frac{\pi}{5} + \sin\frac{\pi}{5} + 1\right) - \frac{\pi m^2}{40}\sin\frac{\pi}{5}\left(4\cos\frac{\pi}{5}+1\right)\right\}. \qquad (12)$$

Then P_{int} become:

$$P_{int} = \frac{5/\pi}{\frac{a^2}{2}\sin\frac{\pi}{5}\left[4\cos\frac{\pi}{5}\left(1+\cos\frac{\pi}{5}\right)+1\right] - \frac{m^2}{8}\sin\frac{\pi}{5}\left(14\cos\frac{\pi}{5}+1\right)}$$

$$\left\{\frac{al}{2}\left(1 - 2\sin\frac{\pi}{5} - \cos\frac{\pi}{5}\right) - \frac{lm}{2}\left(\sin\frac{2\pi}{5}\sin\frac{\pi}{10}+1\right) - \frac{l^2}{4\sin\frac{2\pi}{5}}\right.$$

$$\left[6\cos\frac{\pi}{10} + \cos\frac{2\pi}{5} - 2\sin\frac{2\pi}{5} - 1 - \frac{2\pi}{5}\left(5\cos\frac{2\pi}{5} - \cos\frac{\pi}{5} + \sin\frac{\pi}{5} + 1\right)\right] -$$

$$\left.\frac{\pi m^2}{40}\sin\frac{\pi}{5}\left(4\cos\frac{\pi}{5}+1\right)\right\}.$$

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