Sample separation and the sensitivity of investment to cash flow:

Is the monotonicity condition empirically satisfied?

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Abstract

This paper studies whether the monotonicity condition of the investment-cash flow sensitivity is satisfied empirically. We show that if this condition holds, then the point of sample separation does not affect the monotonic relationship between the sensitivities of any two complementary classes of observations. Our test, based upon observable averages of the investment-cash flow sensitivity, rejects the monotonicity condition for any common metric of financing constraints we use. The testing procedure we propose reconciles the conflicting findings of the literature about the shape of the investment-cash flow sensitivity.

The behaviour of the investment-cash flow sensitivity with respect to metrics of financing constraints continues to be a controversial issue in the corporate finance literature. Some studies document the existence of an increasing investment-cash flow sensitivity (ICFS) with respect to the degree of financing constraints, while others provide evidence of a decreasing or non-monotonic sensitivity.[1]

Carefully reading Kaplan and Zingales (1997) suggests that such conflicting findings should not be surprising, given that the monotonicity condition – i.e. the assumption that the ICFS increases monotonically as financing constraints tighten – is not well-grounded theoretically; nor, despite its importance, has monotonicity been explicitly tested for. Moreover, results, indicating an ICFS inversely or non-monotonically correlated with the degree of financing constraints (as in Kaplan and Zingales, 1997; Almeida and Campello, 2007; Cleary, Povel and Raith, 2007), rest upon the sorting scheme that may or may not be valid. Hence, like the results that indicate monotonicity, the above results are potentially questionable. A sorting scheme may be arguable because the true degree of financing constraints is unobservable (Hubbard, 1998) and, therefore, it is not clear either which metric to adopt for measuring financing constraints or whether the severity of financing constraints increases or decreases under that metric (Fazzari, Hubbard and Petersen, 2000; Kaplan and Zingales, 2000). A sorting scheme may be questioned also because it depends on number of classes of observations facing different degrees of financing constraints and location of the sample separation points. The selection of the splitting points is an important decision because the shape of the ICFS curve itself is sensitive to "whether the point of sample separation is successfully determined" (Hovakimian, 2009:163).

The above compounds the uncertainty in testing for the monotonicity condition. This paper proposes a testing strategy for the monotonicity condition, based on observable averages of the true ICFS, which sidesteps the uncertainties above. We show that if the monotonicity condition holds,

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[1]Among many, see Fazzari, Hubbard and Petersen (1988), Kaplan and Zingales (1997), Almeida and Campello (2007), Cleary, Povel and Raith (2007), Hennessy and Whited (2007), Lyandres (2007), Hovakimian (2009), Hadlock and Pierce (2010), Bond and Söderbom (2013), Mclean and Zhao (2014), Erel, Jang and Weisbach (2015), Lewellen and Lewellen (2016) and Ağca and Mozumdar (2017).

then the sample separation point does not affect the monotonic relationship between the sensitivities of two complementary classes of observations. This property of the monotonicity does not depend on whether the degree of financing constraints is decreasing or increasing with the sorting metric, nor on the true unknown number of classes and class size. This gives an advantage to our testing procedure, because the ICFS of the two complementary classes are observable, whereas the true underlying sensitivity is hard to observe, unless one correctly identifies number and position of the sample separation points. If, instead, the partition of the sample were not successfully identified, the estimate of the ICFS for the classes would be some unknown combination of the underlying sensitivities, and the conclusions concerning monotonicity might be at risk.

Secondly, we design an empirical strategy for testing the monotonicity condition by comparing the ICFS of complementary classes of observations, using a given sorting metric, a range of sample separation points and a large sample of non-financial US firms from 1990 to 2013. We sort the sample using the index of financing constraints developed by Hadlock and Pierce (2010), and we use the bottom 5% and the top 95% of the ordered observations to construct the two complementary classes. We assign to each class a dummy variable, interact the cash flow with these dummy variables to obtain an otherwise standard model of investment as in Eq. (14). Finally, we test the hypothesis of equality of the two ICFS parameters. We repeat the exercise using the bottom 10% and the top 90%, the bottom 15% and the top 85%, and so on, up to the bottom 95% and the top 5% subsamples, checking whether rejection of the null hypothesis depends on the sample separation point. If this is the case, we conclude that monotonicity does not hold.

Empirical results in Section 3 show that the rejection of the null hypothesis does depend on the position of the sample separation point. To be precise, if the splitting point, s, is at the left tail of the ordered sample, the null hypothesis of equality of parameters is rejected, and the ICFS of the class of observations $i \leq s$ is lower than the ICFS of the complementary class of observations $i > s$. For splitting points located around the centre of the ordered sample, the null hypothesis is not rejected.

However, if the splitting point, s, is at the right tail of the ordered sample, the null hypothesis of equality of parameters is rejected again, and the ICFS of the class of observations $i \leq s$ is higher than the ICFS of the complementary class of observations $i > s$. This conclusion is robust to the specification of the estimating model, the estimation framework, potential measurement errors in the Tobin's Q, the macroeconomic cycle, and the quality of the sample firms.

Our finding of non-monotonicity of ICFS might raise questions about the suitability of the Hadlock and Pierce (2010) index as a metric of financing constraints. While there are alternative metrics of financing constraints, considerable debate still exists with respect to their merits (Hennessy and Whited, 2007; Hadlock and Pierce, 2010; Hoberg and Maksimovic, 2015; Farre-Mensa and Ljungqvist, 2016; Buehlmaier and Whited, 2018). Determining the best metric of financing constraints is beyond the scope of our paper but, as an additional contribution, we provide three pieces of supporting evidence. In the first exercise we provide corroboratory evidence rejecting the monotonicity condition using alternative metrics of financing constraints, such as the Kaplan and Zingales (1997) and Whited and Wu (2006) indexes. Since our analysis of the rejection of monotonicity suggests that the ICFS depends on the Hadlock and Pierce (2010) index, in the second exercise we test the hypothesis of a well-behaving non-monotonic relationship between the ICFS and the Hadlock and Pierce (2010) index. In particular, we generalize the investment model by allowing for different types of non-monotonicity. Estimation results confirm that the magnitude of the ICFS depends on the Hadlock and Pierce (2010) index, following an inverted U-shaped curve. This result helps to rule out theoretical models that do not predict an inverted U-shaped ICFS. On the other hand, it requires the presence of a monotonic correlation between the Hadlock and Pierce (2010) index and the degree of financing constraints. This correlation will be the third and final piece of evidence we will search for. We are not going to investigate whether the firm is more or less financially constrained: in our framework, the rejection of the monotonicity condition is independent of whether the degree of financing constraints is increasing or decreasing with respect to the sorting metric

(Fazzari, Hubbard and Petersen, 2000; Kaplan and Zingales, 2000). Moreover, we do not need to determine the strength of the correlation between the metric and the financing constraint (Hoberg and Maksimovic, 2015; Farre-Mensa and Ljungqvist, 2016). We simply need evidence to support the hypothesis that the sorting metric is monotonically correlated with the degree of financing constraints. In presence of such correlation, we can conclude that the violation of monotonicity is engendered by a non-monotonic relation between the ICFS and the degree of financing constraints, rather than by possible non-monotonicity of the sorting metric with respect to the financing constraint. The results from this analysis show that the Hadlock and Pierce (2010) index sorts monotonically all other metrics of financing constraints: firms that are more (or less) tightly constrained according to the Hadlock and Pierce (2010) index are also more (or less) constrained according to the Kaplan and Zingales (1997) index, the Whited and Wu (2006) index, and several financial characteristics commonly used to proxy for financing constraints.

In sum, besides introducing the test for monotonicity of IFCS, we find support both for the Almeida and Campello (2007) prediction of an inverted U-shaped ICFS curve and for the hypothesis that the Hadlock and Pierce (2010) index is monotonically correlated with the degree of financing constraints.

1. Data and motivation

1.1 Data

We use a large and heterogeneous sample of US corporations from 1989 to 2013, starting with all US Compustat firms. From this dataset, we eliminate financial firms (SIC Codes 6020-6799) and regulated utilities (SIC Codes 4011-4991). Our resulting sample is well diversified by sector, as measured by primary SIC code. It comprises firms in agriculture, mining, forestry, fishing and construction (SIC codes 100-1731); manufacturing (SIC codes 2000-3990); retail and wholesale trade (SIC codes 5000-5990); and services (SIC codes 7000-8900). Observations from 1989 were used only to construct variables including lagged terms, and were not used in the regressions. Firm-year

observations where total assets or sales are not positive are deleted.

Like Cleary, Povel and Raith (2007) and Lyandres (2007), we work with the unbalanced panel of firm-year observations. The use of firm-year observations allows firms' financial status to be reclassified every year, and class composition to vary over time, so as not to "neglect […] the information that the financial constraints may be binding for the same firm in some years but not in others. It would be more advisable in these cases to allow firms to transit between different financial states" (Schiantarelli, 1996:78).

Three key firm variables, in our analysis, are gross investment (*Compustat data item 128*), cash flow (*data item 14 + data item 18*), and market-to-book ratio (*data item 6 - data item 60 - data item 74 + data item 199 × data item 25, all divided by data item 6).* To control for endogeneity, we use operating cash flow rather than free cash flow, since the operating cash flow is not affected by financing or investment decisions. To control for heteroscedasticity due to differences in firm size, we scale both investment and cash flow by beginning-of-period net fixed assets (*data item 8*), which are at constant 2013 prices (as are total assets). Age is the number of years preceding the observation year that the firm has a non-missing stock price on the Compustat file. Size is the log of total assets. Firm sales growth is the latest annual change in the firm's inflation-adjusted sales, while industry sales growth is the most recent annual change in three-digit industry inflation-adjusted sales. Cash is defined as cash plus marketable securities (*data item 1*). Dividends are total annual dividend payments (*data item 21 + data item 19*). Total long-term debt is long-term debt plus debt in current liabilities (*data item 9 + data item 34*). Coverage ratio is beginning-of-period operating income after depreciation (*data item 178*) over beginning-of-period interest and related expenses (*data item 15*). R&D is defined as research and development spending over total assets (*data item 46 divided by data item 6*). Tangibility is measured as beginning-of-period net fixed assets. Return on assets (ROA) is calculated as operating income before depreciation (*data item 13*) divided by total assets; total common equity is common/ordinary equity (*data item 60*) divided by total assets, and free cash flow

is defined as cash flow minus investment. To control for outliers due to erroneous data input, we winsorize observations at the 1st and 99th percentiles for cash flow, investment, market-to-book ratio, size and age.

Cleary, Povel and Raith (2007) suggest that the ICFS differs between positive and negative cash flow observations; 29 percent of our firm-year observations show negative cash flow. In the analysis we impose cash flow > 0: including observations with cash flow ≤ 0, the ICFS is more likely to be non-monotonic (Cleary, Povel and Raith, 2007). In eliminating observations with negative cash flow, we also follow a less aggressive approach by working with the unbalanced panel of firm-year observations with cash flow ≥ -1 (Hadlock and Pierce, 2010).

What metric to use to capture the degree of financing constraints has been the subject of intense debate (Hennessy and Whited, 2007; Hadlock and Pierce, 2010; Hoberg and Maksimovic, 2015; Farre-Mensa and Ljungquist, 2016). Hadlock and Pierce (2010), in advocating their index of financing constraints, cite its "many advantages over other approaches, including its intuitive appeal, its independence from various theoretical assumptions, and the presence of corroborating evidence from an alternative approach" (Hadlock and Pierce, 2010: 1912). Moreover, their index is robustly correlated with qualitative indicators of financing constraints, corroborating the evidence presented by Hennessy and Whited (2007). Hoberg and Maksimovic (2015) also offer evidence in support of Hadlock and Pierce (2010), finding that smaller and younger firms are more likely to be equity-constrained. We accordingly adopt the Hadlock and Pierce (2010) index to sort firm-year observations, drawing additional supporting evidence, where needed, from the other two common metrics: the Kaplan and Zingales (1997) and Whited and Wu (2006) indexes. By construction, the three metrics of financing constraints, KZ, WW and HP, given in Eq. (1) - (3) below, increase as the firm's financing constraint tightens:

(1) $Kaplan\ and\ Zingales\ (1997) = 3.13919 total\ long\ term\ debt - 1.001909 cash\ flow - 1.314759 cash - 39.36780 dividend + 0.2826389 market - to - book,$

(2) $Whited\ and\ Wu\ (2006) = \ 0.021 total\ long\ term\ debt - 0.091 cash\ flow -$

$$0.044 size - 0.062 dividend\ positive - 0.035 growth\ sales +$$
$$0.102 Industry\ growth\ sales,$$

(3) $Hadlock\ and\ Pierce\ (2010) = -0.737 size + 0.043\ size^2 - 0.040\ age.$

Table 1 reports the mean value of the main variables used in our analysis. Column (a) displays the means for the unbalanced sample of firm-year observations with cash flow ≥ -1. Column (b) refers to the unbalanced sample with cash flow > 0 and Column (c) pertains to the balanced sample with cash flow > 0 and dividends > 0, following Fazzari, Hubbard and Petersen (1988) and Cleary, Povel and Raith (2007).

Insert Table 1

Not surprisingly, the firms in Column (c), with positive cash flow and positive dividends, form the financially healthiest sample, with the highest net fixed assets, total assets, sales, capital expenditure, market-to-book ratio and dividends. Their lower cash flow, cash stock and total long-term debt suggest that these firms have borrowing capacity and do not need to accumulate cash. Their HP, KZ and WW indexes of financing constraints show them to be less financially constrained than the other two unbalanced samples in Columns (a) and (b). Similarly, the net fixed assets, total assets, sales and capital expenditure of the firms with cash flow > 0 are all greater than those of firms with cash flow ≥ -1, and their indexes of financing constraints are lower.

Table 2 reports the correlations across the three indexes, HP, KZ and WW, when we use the observations with cash flow > 0 and cash flow ≥ -1. The correlations are strong. In line with Hadlock and Pierce (2010) and Farre-Mensa and Ljungqvist (2016), HP index is positively correlated with WW and negatively correlated with KZ. This is because the HP index, which loads heavily on size, is more likely to identify as constrained firms that face high external financing costs, whereas KZ, which isolates firms with low cash stock, low cash flow and high debt, is more likely to identify as constrained firms that are less likely to face a high cost of external financing but do generally have a strong need for finance (Cleary, Povel and Raith, 2007; Hennessy and Whited, 2007). A firm facing

high external financing cost but with relatively little need for finance, thus, might be classified as constrained according to HP index and not according to KZ. However, for observations with cash flow ≥ -1, the sign of the correlation between HP and KZ is positive, as the coefficient of cash flow is negative.

<center>Insert Table 2</center>

1.2 Motivation

To study the ICFS across classes of firms potentially facing different degrees of financing constraints, the researcher sorts the sample of firm-year observations using a given metric of financing constraints, and estimates the ICFS for each class of financing constraints according to the following standard investment model:

$$(4) \quad \left(\frac{I}{K}\right)_{ij,t} = \alpha_j Q_{ij,t} + \beta_j \left(\frac{Cash\ flow}{K}\right)_{ij,t} + \mu_{ij} + \tau_t + \varepsilon_{ij,t},$$

where $i=1,...,n$ are the firms (n being the cross section dimension of the sample); $t=1,...,T$ are the years (T being the time dimension of the sample); $j=1,...,J$ are the number of classes; I is investment; Q is Tobin's Q; α_j is the sensitivity of investment to Q for the j-th class; *Cash flow* is the firm's cash flow; β_j is the ICFS for the j-th class; K is a scaling variable; μ_{ij} and τ_t are respectively individual and year fixed effects and, finally, $\varepsilon_{ij,t}$ is a white noise disturbance term. Comparing the β_J parameters across classes, one reaches a conclusion about the shape of the ICFS.

This estimation approach requires that the sorting scheme is appropriate. The sorting scheme entails the selection of (1) the metric of financing constraints to sort the sample; (2) the number of classes, *J*, and (3) the position of *J-1* sample separation points. Taken together, these decisions determine the shape of the estimated ICFS. The choice of the metric has been heavily debated in the literature, pointing out the relative ability of the various alternative metrics to capture the true degree of financing constraints. This is not surprising: since the empirical model in Eq. (4) is the same in

<center>9</center>

most studies while the empirical conclusion differs, the debate has moved from the analysis of the shape of the ICFS given the sorting metric to the analysis of the behavior of the ICFS under different metrics. The discussion has concluded that the shape of the ICFS may vary depending on the metric, as the different metrics capture different dimensions of constraints (Fazzari, Hubbard and Petersen, 2000; Kaplan and Zingales, 2000; Moyen, 2004; Cleary, Povel and Raith, 2007; Hennessy and Whited, 2007; Hadlock and Pierce, 2010).

Conversely, the literature has had little to say either about the appropriate number of classes J to generate or about their size. These are problematic issues, in that one has to identify the true partition without knowing whether monotonicity holds, noting that the shape of the ICFS is sensitive to "whether the point of sample separation is successfully determined. This may be one of the reasons for the conflicting findings in the previous literature, especially if the relationship between financial constraints and investment-cash flow sensitivity is non-monotonic" (Hovakimian, 2009:163).

In this regard there are clearly some reasons to keep the number of classes J low: fewer classes means larger classes, hence a more efficient estimator of the ICFS and fewer stable parameters to compare. But if J is lower than the unknown true number of classes, then the ICFS estimated for the *j-th* class will be the average of the true underlying ICFS parameters; and if J is higher than the true number of classes, then some of the estimated ICFS parameters will be equal, as they may belong to the same true group of observations. However, the smaller each class, the less efficient the estimator and the greater the number of volatile estimated parameters to compare. This implies that the observed differences in the ICFS between classes may be due either to differences in their true degree of financing constraints or to the lack of precision of the estimator. Deciding the number of classes is further complicated by the need to determine where to locate the sample separation points that, in turn, determine the size of each class.

For the above reasons, it is not easy to distinguish between genuine violations of the monotonicity condition and random fluctuations of the ICFS parameter due to a priori selected

splitting points. To demonstrate how changing splitting points - i.e. class size - may lead to different shapes of the ICFS, we generate three different sorting schemes. Each sorting scheme is based on the HP metric of financing constraints, three classes but different splitting points. We then estimate Eq. (4) for the three classes of each sorting scheme. Table 3 reports the estimation results.

Insert Table 3

The results corresponding to the sorting scheme 1, with classes $HP \leq -3.147$, $-3.147 < HP \leq -2.927$, and $HP > -2.927$, support findings by Fazzari, Hubbard and Petersen (1988) suggesting that the ICFS is increasing. The results obtained from sorting scheme 2, with classes $HP \leq -2.706$, $-2.706 < HP \leq -2.616$, and $HP > -2.616$, are in line with Kaplan and Zingales (1997), as the ICFS is decreasing. Finally, the results obtained using sorting scheme 3, with classes $HP \leq -3.325$, $-3.325 < HP \leq -2.747$, and $HP > -2.747$, yield a non-monotonic ICFS, as in Lyandres (2007), Hovakimian (2009) and Hadlock and Pierce (2010).

Estimation results in Table 3 indicate that differences in the shape of the ICFS may arise not only from different sorting metrics, as previous studies have concluded, but also from differences in the position of the splitting points. In short, if the monotonicity condition of the ICFS is under question, the decision concerning sample separation points, given the sorting metric and the number of classes, is crucially important. In addition, our argument suggests that comparing parameters estimated across classes is not necessarily informative to conclude whether monotonicity holds, both because the estimates of the ICFS depend on the number and position of the splitting points and because they may not be statistically different. To evaluate evidence for or against the monotonicity condition, in Section 2 we provide a testing strategy that is agnostic with respect to the number and position of the splitting points.

2. Methodology

2.1 The monotonicity condition of the averaged ICFS

Our argument is based on the fact that, if monotonicity holds, the position of the sample separation point does not affect the direction of the inequality between the average ICFS for the first s observations and that of the remaining n-s observations. To demonstrate the latter, let the true ICFS parameter, β_i, be monotonically increasing across the firm observations $i = 1, ..., n$:

(5) $\quad \beta_1 \leq \cdots \leq \beta_i \leq \cdots \leq \beta_n,$

(6) $\quad Let \ 1 \leq s \leq n$ be the splitting point.

Eq. (5) implies that:

(7) $\quad \dfrac{\beta_1 + \cdots + \beta_s}{s} \leq \dfrac{\beta_{s+1} + \cdots + \beta_n}{n-s},$

independently of the splitting point s. Note that β_s is the upper bound for the average value of parameters β_i located in the first s positions. If Eq. (5) holds, then β_s cannot be greater than β_{s+1}, as the latter is the lower bound for the average value of parameters β_i in the remaining n-s positions:

(8) $\quad \dfrac{\beta_1 + \cdots + \beta_s}{s} \leq \dfrac{s\beta_s}{s} = \beta_s \leq \beta_{s+1} = \dfrac{(n-s)\beta_{s+1}}{(n-s)} \leq \dfrac{\beta_{s+1} + \cdots + \beta_n}{n-s}.$

For the equality in Eq. (7) to hold, all the β_i parameters must be equal:

(9) $\quad \dfrac{\beta_1 + \cdots + \beta_s}{s} = \dfrac{\beta_{s+1} + \cdots + \beta_n}{n-s},$

regardless of where s is located, while for strict inequality, it must hold $\beta_1 < \beta_n$:

(10) $\quad \dfrac{\beta_1 + \cdots + \beta_s}{s} < \dfrac{\beta_{s+1} + \cdots + \beta_n}{n-s},$

no matter where s is located.

Similarly, if the true ICFS parameter, β_i, is monotonically decreasing across the i observations:

(11) $\quad \beta_1 \geq \cdots \geq \beta_i \geq \cdots \geq \beta_n,$

for any splitting point, s, the average of parameters β_i for the first s observations is greater than or equal to the average of parameters for the remaining n-s observations:

$$(12) \quad \frac{\beta_1 + \cdots + \beta_s}{s} \geq \frac{\beta_{s+1} + \cdots + \beta_n}{n - s}.$$

Clearly, Eq. (7) and Eq. (12) hold if, instead, there are J classes for which the ICFS parameter β_i is the same within the class but different across the J classes:

$$(13) \quad \underbrace{\beta_1 \ldots \beta_1}_{n_1} < \cdots < \underbrace{\beta_j \ldots \beta_j}_{n_j} < \cdots < \underbrace{\beta_J \ldots \beta_J}_{n_J},$$

where n_j is the number of observations having the same ICFS parameter. Notice that the strict inequality in Eq. (13) implies strict inequality in Eq. (10).

2.2 Empirical strategy

The foregoing result provides the groundwork of our empirical strategy for testing the monotonicity/convexity of the ICFS. If Eq. (9) holds regardless of the position of the splitting point, s, we conclude that all observations have the same ICFS parameter β_i. If the equality in Eq. (9) is rejected for at least one s, we conclude that there exist at least two different ICFS parameters in the sample. Moreover, if the direction of the inequality between the average value of the ICFS parameter for the first s and the remaining n-s observations varies with the position of s, then the equality in both Eq. (5) and Eq. (11) is unambiguously rejected.

Our testing procedure for monotonicity consists of the following steps:

1. Use the HP index of financing constraints to order the sample of firm-year observations.

2. Select the splitting point, s, corresponding to the bottom 5 percent of the ordered sample of firm-year observations, and estimate the average ICFS parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ using the following model:

$$(14) \quad \left(\frac{I}{K}\right)_{i,t} = \alpha Q_{i,t} + \gamma_s D_s + \bar{\beta}_s D_s \left(\frac{Cash\ flow}{K}\right)_{i,t} + \bar{\beta}_{n-s} D_{n-s} \left(\frac{Cash\ flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t},$$

where D_s and D_{n-s} are the corresponding dummy variables for the two classes, s and n-s.

3. Test the null hypothesis $H_0: \bar{\beta}_s = \bar{\beta}_{n-s}$ vs $H_A: \bar{\beta}_s \neq \bar{\beta}_{n-s}$.

4. Perform the same testing repeatedly, with splitting points, s, corresponding to 10, 15, 20, …, 95 percent of firm-year observations. The test of the null hypothesis at nineteen splitting points is illustrated in Figure 1.

5. If the direction of the inequality between $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ is preserved when the splitting point, s, changes, the monotonicity condition of ICFS holds; otherwise, the monotonicity condition is rejected.

Insert Figure 1

This testing strategy has several advantages. The first advantage is that in order to test for monotonicity we do not need to know whether the degree of financing constraints is increasing or decreasing with respect to the sorting metric. This is because, independently of whether monotonicity as in Eq. (5) or Eq. (11) holds, both would be rejected if changes in the location of the splitting point affect the direction of the inequality. Secondly, our testing strategy is built on averages of the true underlying sensitivity parameters and does not require determining the true number of classes and the true position of the sample separation points. If the monotonicity condition is imposed at the outset, misidentification of the number and positions of the splitting points can affect the shape of the ICFS. More specifically, if the true ICFS is non-monotonic, depending on the sample separation point the observed ICFS may prove to be either monotonically increasing, monotonically decreasing, or non-monotonic. Finally, as we use the same entire sample, the same sorting metric, estimating model and

estimator, different conclusions about rejection of the null depend solely on the position of the splitting point.

Nevertheless, our testing framework also has two shortcomings. First, if the monotonicity condition is rejected, it provides no information about the shape of the ICFS that determines the rejection. This information is important, because the theoretical models that posit the non-monotonicity of the ICFS with respect to financing constraints disagree on the shape of the ICFS. Since the violation of the monotonicity condition, we observe, depends on the level of the metric, we exploit this information, in Section 3.2, to determine the shape of the ICFS. Secondly, we do not know how far the rejection of monotonicity depends on the sorting metric's being non-monotonically correlated with the degree of financing constraints. Although we do not need to know exactly how well the metric captures financing constraints, we do need evidence that it is to some extent monotonically correlated with constraints. In Section 3.3, we will look for such evidence.

3. Results

3.1 Does the monotonicity condition hold empirically?

Table 4 reports the estimation results of the model in Eq. (14). The upper panel displays the estimates for the full unbalanced sample of 77,086 firm-year observations with cash flow > 0, as in Fazzari, Hubbard and Petersen (1988), Kaplan and Zingales (1997) and Cleary, Povel and Raith (2007). The lower panel reports the estimates for the full unbalanced sample of 93,107 firm-year observations with cash flow $\geq$ -1, as in Hadlock and Pierce (2010). Like previous studies, we use fixed effects estimation, "which maintains separate intercepts for each firm and for each year, to account for unobserved relationships between investment and the independent variables, and to capture business-cycle influences" (Cleary, 1999: 683-684).

Column (a) reports the point of sample separation, s, as a percentage of firm-year observations. Column (b) shows the coefficient γ_s associated with the class of observations $i \leq s$, and Column (c) the coefficient α associated with the market-to-book ratio. Columns (d) and (e) report the investment

cash-flow sensitivity coefficients $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ for the classes of observations $i \leq s$ and $i > s$, respectively. Heteroscedasticity-consistent standard errors are reported in parenthesis. Column (f) shows the adjusted R^2; Column (g) displays the F-statistic for the null hypothesis of equality of the parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$.

For sake of space, we report the results only for a limited number of points of sample separation. The coefficient, α, for the market-to-book ratio is positive and significant in all classes, and the adjusted R^2 is stable over classes. The upper panel shows that if the splitting point is at 5% and 30% of the sample, the test rejects the equality of the parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$; and thus, we conclude that Eq. (9) does not hold. For splitting points at 40% and 80% of the sample, the test does not reject the equality of the parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$. In short, the monotonicity condition is empirically rejected.

Furthermore, the conclusion of non-monotonicity is reinforced by the finding that when the splitting point is at 85% and 95% of the sample, the average ICFS of the lower class, $\bar{\beta}_s$, is statistically greater than that of the upper class, $\bar{\beta}_{n-s}$. The same conclusion holds for the observations with cash flow $\geq$ -1 (lower panel).

Insert Table 4

The conclusion against monotonicity is conditional on several assumptions imposed at the outset. In what follows, we check whether it depends on the specification of the model, the estimation framework, the average financial health of the firms, the time period, and the sorting metric.

Let us start with re-examining the estimated model. The model in Eq. (14) includes the parameter γ_s which allows for differences between the average investment value of class s and that of its complement, n-s. If the average investment of class s is equal to that of n-s, the investment model can be specified as

$$(15) \quad \left(\frac{I}{K}\right)_{i,t} = \alpha Q_{i,t} + \bar{\beta}_s D_s \left(\frac{Cash\ flow}{K}\right)_{i,t} + \bar{\beta}_{n-s} D_{n-s} \left(\frac{Cash\ flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t}.$$

In addition, if the impact of Q is different across class s and n-s, the investment model becomes

$$(16) \quad \left(\frac{I}{K}\right)_{i,t} = \gamma_s D_s + \bar{\alpha}_s D_s Q_{i,t} + \bar{\alpha}_{n-s} D_{n-s} Q_{i,t} + \bar{\beta}_s D_s \left(\frac{Cash\ flow}{K}\right)_{i,t}$$

$$+ \bar{\beta}_{n-s} D_{n-s} \left(\frac{Cash\ flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t}.$$

The estimation results for the models in Eq. (15) and Eq. (16) are reported in the upper and lower panels of Table 5. They again confirm the violation of the monotonicity condition, because the direction of the inequality between the ICFS parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ changes with the splitting point.

Insert Table 5

The ICFS literature generally splits the sample into classes of financing constraints and estimates the ICFS for each class. Then, for each class, the model in Eq. (14) simplifies to the model in Eq. (4). We accordingly test for monotonicity fitting the model in Eq. (4) for different classes, but adding a test for difference in parameters that allows us to draw inferences concerning the shape of the ICFS. Estimation results are given in Table 6. The χ^2 test for equality of the ICFS parameters rejects the monotonicity of the ICFS parameter. In this approach the adjusted R^2 varies between classes, while using our model in Eq. (14), the adjusted R^2 does not depend on the class size.

Insert Table 6

Poterba (1988) points out that, since measurement error in the Tobin's Q can lead to spurious correlation between investment and cash flow, one might find insignificant ICFS parameters once this measurement error is taken into account. Indeed, Erickson and Whited (2000, 2002), by using a GMM estimator based on the higher-order moments of the regression variables, show that cash flow does not matter for investment when the measurement error in Tobin's Q is addressed. Cummins, Hassett and Oliner (2006) support this finding by using a GMM estimator and an analysts-forecasts based measure of Q as superior proxy for the Tobin's Q. Agca and Mozumdar (2017) challenge these studies: by using both the Cummins, Hassett and Oliner (2006) and the Erickson and Withed (2000,

2002) methodology, they find a significant ICFS parameter. Additionally, they find that the ICFS is higher for financially constrained firms, irrespective of the metric of financing constraints used.

In the light of the above, we check whether the non-monotonicity result we have documented is robust to measurement errors in the Tobin's Q, by using a GMM estimator for the model in Eq. (14). We use the standard stock-marked based measure of Q in line with the evidence that an analyst-forecasts based measure of Q is not superior to a stock-marked based measure of Q (Agca and Mozumdar, 2017). Our results, reported in Table 7, show that a GMM estimator, with finite number of lags of Q and cash flow as instruments, yields a non-monotonic ICFS parameter.

Insert Table 7

It is of interest to investigate whether the non-monotonicity of ICFS is robust to the sample period. There is evidence of a change in the sensitivity of investment to cash flow during the financial crisis of 2007-2009, but no consensus on the direction of this change. Mclean and Zhao (2014) find that the sensitivity increased during the crisis, which exacerbated financial constraints. Yet, Chen and Chen (2012) find that the ICFS practically disappeared during the crisis, regardless of the firm's financial strength. More relevant to our analysis of monotonicity is the Allayannis and Mozumdar, (2004) hypothesis that, if the impact of financing constraints on firm investment declines over time, then the ICFS may be almost the same across different classes of constraints. Since our sample period includes the financial crisis, we perform separate analyses for the pre-crisis (1990-2007) and post-crisis (2008-2013) periods. The results, reported in Table 8, continue to reject monotonicity for both periods.

Insert Table 8

Independently of the macroeconomic cycle, the shape of the ICFS parameter may be affected by the financial health of the sample firms (Cleary, Povel and Raith, 2007). Previous studies have used unbalanced and balanced samples, positive and negative cash-flow observations, and dividend-

paying and non-paying firms. The conclusions reached by these studies are affected by the choice of the quality of the observations (see for example, Fazzari, Hubbard and Petersen, 1988; Kaplan and Zingales, 1997; Cleary, 1999; Cleary, Povel and Raith, 2007). Taking this problem into account, we study whether the monotonicity condition holds both for a sample of healthy firms and less healthy firms. As in previous studies, the sample of healthy firms is the balanced sample of observations for dividend payers, while the less healthy sample is the remaining unbalanced sample. The estimation results of the model in Eq. (14), reported in Table 9, provide evidence of non-monotonicity in both samples.

Insert Table 9

Our final robustness check aims to investigate to what extent the violation of monotonicity depends on the choice of the metric of financing constraints. We re-estimate the model in Eq. (14) using the KZ and the WW metrics of financing constraints. Table 10 reports results for the full unbalanced samples of firm-year observations with positive cash flow (upper panel) and with cash flow $\geq$ -1 (lower panel), respectively. Estimation results indicate that the non-monotonicity property is independent of the sorting metric.

Insert Table 10

3.2 Further analysis of the shape of the ICFS

Having found that in the empirical data the monotonicity condition of the sensitivity parameter does not hold, we now want to examine its shape. Various theoretical models that predict a non-monotonic relationship between ICFS and financing constraints do not reach a consensus on the shape. Cleary, Povel and Raith (2007) and Lyandres (2007) find the ICFS curve to be U-shaped; while Almeida and Campello (2007) identify an inverted U-shaped ICFS. Our results of non-monotonicity are consistent with alternative non-monotonic shapes of the ICFS. To bring more clarity on the shape of the ICFS, which determines the violation of the monotonicity condition, we shall use the evidence we

documented that the ICFS depends on the level of the HP index of financing constraints by generalizing the baseline investment model as:

(18) $\quad \left(\frac{I}{K}\right)_{i,t} = \alpha Q_{i,t} + \sum_{r=0}^{R} \beta_r HP_{i,t}^r \times \left(\frac{Cash\ flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t},$

where $R=0,...,3$. If $R=0$, the above model reduces to the baseline investment model in Eq. (4). If $R=1$, we augment this baseline model with the interaction between cash flow and HP, and with $R=2$ we include HP^2 interacted with cash flow to investigate the non-monotonicity of the ICFS. With $R=3$, we test the hypothesis that the ICFS may assume shapes not explored in the literature.

Table 11 presents the estimation results of the model in Eq. (18). Panel 1 reports the estimates for the unbalanced sample of observations with positive cash flow. When $R=1$, the F-statistic of the test for significance of the coefficient β_1 of the HP index interacted with cash flow confirms that the ICFS parameter depends on the HP index. When $R=2$, the F-statistic of the test for significance of β_1 and β_2 doubles, and the adjusted R^2 increases. However, when HP^3 interacted with cash flow is added to the set of regressors, the F-statistic of the test for significance of β_1, β_2 and β_3 declines and the adjusted R^2 does not improve. Therefore Eq. (18) with $R=2$ yields the best specification of the model.

To investigate the shape of the ICFS, we use the fitted quadratic model

(19) $\quad \hat{I} = 0.02808 \times Q - 0.00485 \times CF - 0.06135 \times CF \times HP - 0.01143 \times CF \times HP^2.$

It implies that the ICFS is the following derivative:

$ICFS = \frac{d\hat{I}}{dCF} = -0.01143 \times HP^2 - 0.06135 \times HP - 0.00485,$

which is the second order polynomial in HP. It equals to zero for HP = -5.28 and -0.08. These values are outside the range of the HP index values we observe, suggesting that all firms are exposed to some positive degree of financial constraints. In addition, the polynomial yields an inverted U-shaped ICFS parameter, with the positive peak at HP = -2.68. This inverted U-shape is robust to the type of firm-year observations under analysis. Indeed, by using the same procedure for the sample of observations

with cash flow ≥ -1, we find that the ICFS equals to zero for HP = -5.31 and 0.09, and has a positive

peak at HP = -2.61.

To address potential model misspecification due to the possible omitted variable, HP, we also

estimate the model in Eq. (18) augmented by the HP index:

$$(20) \quad \left(\frac{I}{K}\right)_{i,t} = \alpha Q_{i,t} + \sum_{r=0}^{R} \beta_r HP_{i,t}^r \times \left(\frac{Cash\ flow}{K}\right)_{i,t} + \theta HP_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t}.$$

The estimates of this model are reported in Panel 2 of Table 11. Here, for the sample of observations

with positive cash flow, ICFS equals to zero at HP = -5.14 and 0.12 and has a positive peak at HP =

-2.52. Similarly, for the sample of observations with cash flow ≥ -1, ICFS equals to zero at HP = -

5.14 and 0.28, and has a positive peak at HP = -2.43. Figure 2 depicts the ICFS curve for the sample

of observations with positive cash flow.

<div align="center">

Insert Table 11

Insert Figure 2

</div>

3.3 A closer look at the HP index of financing constraints

Our empirical analysis shows that the three most common metrics of financing constraint, HP,

WW and KZ, are mutually correlated. It also shows that the monotonicity condition of the ICFS is

violated under all of them, and under the HP metric the ICFS parameter has an inverted U-shape.

There is an ongoing debate on "how well" the HP metric correlates with financing constraints.

The finding that the behaviour of the ICFS parameter is not random suggests that the sorting metric

is correlated with the degree of financing constraints. Our framework does not require knowledge of

the degree of correlation. What we need is some evidence of a monotonic correlation between the HP

index and financing constraints. If this condition is not satisfied it would be hard to distinguish

whether the violation of the monotonicity condition is due to the non-monotonicity of the ICFS in the

degree of financing constraints or to a non-monotonic relation between financing constraints and its

empirical proxy, HP. In presence of such correlation, instead, we can definitely conclude that,

regardless of the strength of the correlation between the HP index and the degree of financing constraints, the ICFS is not monotonic.

To determine whether the HP index is monotonically related to the degree of financing constraints, we check whether it sorts the other common metrics of financing constraints monotonically. For that, first we sort firm-year observations by the HP index. We then compute the mean values of the KZ, WW index and a set of firm characteristics commonly used as metrics of financing constraints. The means of these metrics are calculated for the bottom tercile of the HP index, for the top tercile, and for the 30 percent around the median. We then test whether the means of the bottom tercile are statistically different from those of the middle tercile and whether the latter are different from those of the top tercile.

A consistent picture emerges from the results in Table 13. The HP index does sort KZ and WW metrics monotonically: the WW index increases and the KZ index decreases with HP. As also found in Hadlock and Pierce (2010) and Farre-Mensa and Ljungqvist (2016), this comes as no surprise since the HP index is more likely to identify as financially constrained the firms that face high costs of external finance, while the KZ index those with a strong need for finance, as pointed out by Cleary, Povel and Raith (2007) and Hennessy and Whited (2007). Examining also some firm characteristics commonly used to infer financial constraints[2], we find that as HP increases, investment, R&D, sales growth, cash flow and cash stock all increase monotonically, while tangibility, coverage ratio, ROA, debt, equity and dividends decrease monotonically.

In line with the financing constraints literature[3], the firms with the highest HP index are smaller and younger, with more cash flow and cash stock, fewer tangible assets, lower coverage ratios, and lower ROA.

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[2] Fazzari, Hubbard and Petersen (1988), Hoshi Kashiap and Scharfstein (1991), Schaller (1993); Calomiris and Hubbard (1995), Kaplan and Zingales (1997), Almeida and Campello (2007); Hennessy and Whited (2007), Hadlock and Pierce (2010).
[3] Fazzari, Hubbard and Petersen (1988); Almeida and Campello (2007); Cleary, Povel and Raith (2007); Hennessy and Whited (2007); Hadlock and Pierce (2010); Hoberg and Maksimovic (2015); Farre-Mensa and Ljungqvist (2016).

Insert Table 13

Farre-Mensa and Ljungqvist (2016) argue that the HP metric does not actually capture financing constraints but rather reflect differences in growth and financing policies at different stages of the firm's life cycle. The conclusion that high values of HP index capture firms in their fast-growth stage finds partial support in our evidence. True, the firms in the top tercile of HP do invest more, spend more on R&D, and record faster sales growth. However, in contrast to Farre-Mensa and Ljungqvist (2016), these firms show characteristics that are commonly attributed to financing constraints, e.g., lower long-term debt, lower common equity and lower dividend payments. This, together with the inverted U-shaped ICFS, suggests that to some extent the HP index does capture financing constraints, which bolsters confidence in our results. Admittedly, the question "how well" the HP index correlates with the degree of financing constraints is still unsettled (Hoberg and Maksimovic, 2015; Farre-Mensa and Ljungqvist, 2016), and resolving it is beyond the scope of our paper.

Taken , our findings on the shape of the ICFS are in line with Almeida and Campello (2007), showing that "constrained firms' investment-cash flow sensitivities are increasing in asset tangibility, while unconstrained firms' sensitivities show no or little response (often in the opposite direction) to tangibility" (Almeida and Campello, 2007: 1448). Indeed, our results suggest that as the HP metric decreases, tangibility tends to increase along with the ICFS. Once the latter reaches its maximum, any further increase in tangibility tends to reduce the ICFS because higher tangibility makes the firm more likely to be unconstrained.

4. Conclusion

To investigate the monotonicity of the ICFS empirically, we study the relationship between the observed averages of the true underlying ICFS. We show that a necessary and sufficient condition for monotonicity is preserving the direction of the inequality between the averages of two

complementary classes of observations, independently of the point of sample separation. Our test rejects the validity of this condition.

The rejection of the monotonicity based on the averages of the true ICFS strengthens the argument against the monotonicity of the true underlying ICFS. The results support Hovakimian's claim that the conflicting findings in the previous literature arise because the relationship between financial constraints and investment-cash flow sensitivity is non-monotonic (Hovakimian, 2009). Then, depending on the point of sample separation, one may find the ICFS to be monotonically increasing,[4] monotonically decreasing,[5] or even non-monotonic.[6]

Our results leave some unresolved issues for future work. As in previous studies, the relationship of the HP metric with the true underlying degree of financing constraints is still an open question. Hoberg and Maksimovic (2015), for instance, conclude that the conventional measures of financing constraints do not fully capture informational asymmetry issues. Similarly, Farre-Mensa and Ljungqvist (2016) question whether these metrics are adequately correlated with financing constraints. Our results, like those of earlier studies, are still subject to the uncertainty concerning "how well" the proxies used actually capture financing constraints. In testing for monotonicity, to address this uncertainty a joint null hypothesis should be tested, combining the monotonicity of the ICFS with the monotonicity of the sorting metric in the degree of financing constraints. This requires a somewhat different testing methodology, taking into account the uncertainty surrounding the correlation between the sorting metric and the true financing constraint.

Finally, since the behavior of the ICFS with respect to the sorting metric is systematic rather than random, this is an unequivocal indication that the HP metric may capture some regular characteristics of the firm. The relation of these characteristics with the firm's life cycle or with the

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[4] As in Fazzari, Hubbard and Petersen (1988); Calomiris and Hubbard (1995); Gilchrist and Himmelberg (1995); Bond and Mans-Soderbom (2013); Adelino, Lewellen and Sundaram (2015); Erel, Jang and Weisbach (2015); Lewellen and Lewellen (2016) and Ağca and Mozumdar (2017).
[5] As in Kaplan and Zingales (1997); Cleary (1999) and Hadlock and Pierce (2010).
[6] As in Almeida and Campello (2007); Cleary, Povel and Raith (2007); Lyandres (2007); Hovakimian (2009) and Hadlock and Pierce (2010).

degree of financing constraints is an issue for further investigation.

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Table 1
Descriptive Statistics

This table displays the mean value of the main variables used in our empirical analysis. Column (a), (b) and (c) report the means for the full unbalanced samples with *cash flow ≥ -1, cash flow > 0* and with *cash flow > 0* and *dividends > 0,* respectively. The construction of the variables using Compustat data items is as follows: Net fixed assets is data item 8; Total assets is data item 6; Net fixed assets and total assets statistics are in million of inflation-adjusted to year 2013 dollars; Sales is data item 12 inflation adjusted; Capital expenditures is data item 128; Market-to-book ratio is data items 6 - data item 60 - data item 74 + data item 199 times 25, all divided by item 6; Cash flow is data items 14 + data item 18 divided by data item 8 (lagged); Cash holdings is data item 1 divided by data item 6; Dividend is data items 19 + data item 21 + data item 26 divided by data item 6; Total long term debt is data item 9 + data item 34 divided by data item 6. See Section 1 for the definition of Kaplan and Zingales (1997), Whited and Wu (2006) and Hadlock and Pierce (2010) metrics of financing constraints.

Variables	(a) Cash flow ≥ -1	(b) Cash flow > 0	(c) Cash flow > 0, Balanced, and Dividend > 0
Net fixed assets	1085.93	1261.33	4219.43
Total assets	3132.77	3633.39	11621.92
Sales	2915.55	3387.75	11850.20
Capital expenditure	166.64	194.47	555.39
Market-to-Book	1.41	1.41	1.48
Cash flow	0.64	0.84	0.45
Cash	0.12	0.12	0.07
Dividend	0.01	0.01	0.02
Total long term debt	0.20	0.17	0.16
Hadlock and Pierce (2010)	-2.89	-2.97	-3.46
Kaplan and Zingales (1997)	-0.17	-0.49	-0.54
Whited and Wu (2006)	0.09	0.01	-0.11
# Obs.	93107	77086	3720

Table 2

Correlation Analysis

This table reports the correlations between the metrics of financing constraint, due to Kaplan and Zingales (1997), Whited and Wu (2006) and Hadlock and Pierce (2010), for the full unbalanced samples of firm-year observations with *cash flow > 0* and with *cash flow ≥ -1,* respectively. * indicates statistical significance at the 1% level.

	(a)	(b)
	Hadlock and Pierce (2010)	Kaplan and Zingales (1997)
Cash flow > 0. Number of observations = 77,086		
Kaplan and Zingales (1997)	-0.0521*	
Whited and Wu (2006)	0.0666*	0.4955*
Cash flow ≥ -1. Number of observations = 93,107		
Kaplan and Zingales (1997)	0.0266*	
Whited and Wu (2006)	0.0382*	0.9629*

Table 3
Preliminary Evidence

This table presents results from estimating the investment model in Eq. (4) under three different sorting schemes. All coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 77,086 firm-year observations with cash flow>0. The sample period is from 1990 to 2013. The dependent variable is investment. Observations are sorted by the Hadlock and Pierce (2010) index of financing constraint, whose value is reported in Column (a) where, under sorting scheme 1: $HP \leq -3.147$, $-3.147 < HP \leq -2.927$, and $HP > -2.927$; under sorting scheme 2: $HP \leq -2.706$, $-2.706 < HP \leq -2.616$, and $HP > -2.616$ and, under sorting scheme 3: $HP \leq -3.325$, $-3.325 < HP \leq -2.747$, and $HP > -2.747$. Column (b) and (c) report the estimated coefficients associated with the market-to-book ratio and cash flow, respectively. Column (d) and (e) display the number of firms within each class and the adjusted R^2, respectively. Heteroskedasticity-consistent standard errors are in parenthesis.

(a)	(b)	(c)	(d)	(e)
Points of sample separation	α	β	# Firms	Adjusted R^2
Sorting scheme 1				
$HP \leq -3.147$	0.0281 (0.0027)	0.0630 (0.0042)	4410	52.83%
$-3.147 < HP \leq -2.927$	0.0415 (0.0084)	0.0731 (0.0080)	4634	59.23%
$HP > -2.927$	0.0136 (0.0039)	0.0738 (0.0032)	7727	38.31%
Sorting scheme 2				
$HP \leq -2.706$	0.0364 (0.0026)	0.0723 (0.0035)	7264	49.62%
$-2.706 < HP \leq -2.616$	0.0164 (0.0243)	0.0706 (0.0186)	2146	56.76%
$HP > -2.616$	0.0071 (0.0048)	0.0684 (0.0038)	5284	35.64%
Sorting scheme 3				
$HP \leq -3.325$	0.0259 (0.0031)	0.0599 (0.0050)	3121	55.23%
$-3.325 < HP \leq -2.747$	0.0379 (0.0041)	0.0780 (0.0051)	6920	50.65%
$HP > -2.747$	0.0097 (0.0044)	0.0708 (0.0036)	6280	36.35%

Figure 1

Alternative Splitting Points

This figure shows, on the vertical axis, the alternative 19 splitting points s we use in testing for the monotonicity condition. The full sample of firm-year observations (Column ALL) is sorted by the HP index of financing constraints, so that the mean level of financing constraints is progressively increasing both across classes of observations $i > s$ and $i \leq s$. The test for difference in means, reported below, indicates that the mean HP of the lower class is always statistically different from that of the upper class, for any splitting point.

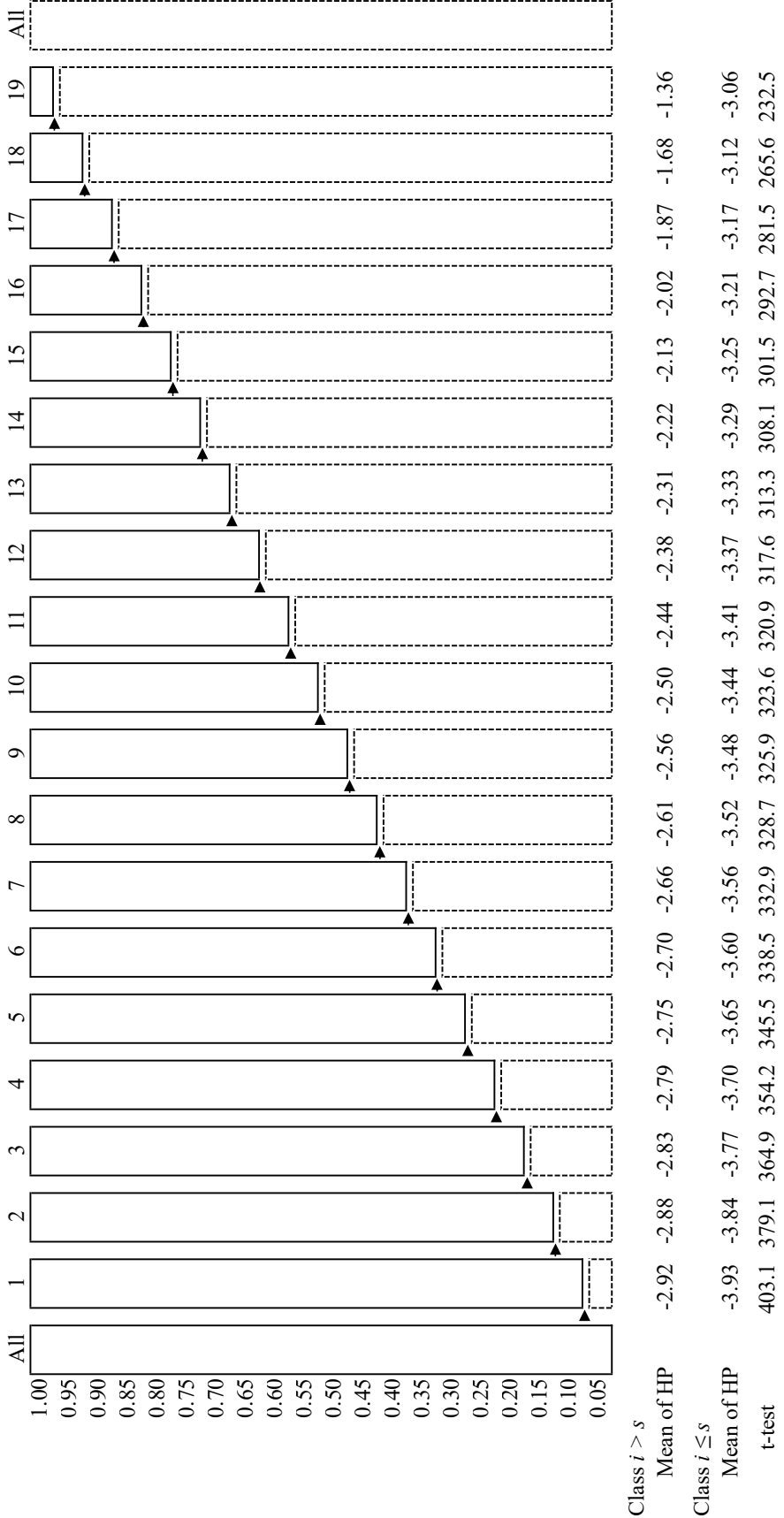|	All	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Class $i > s$																				
Mean of HP		-2.92	-2.88	-2.83	-2.79	-2.75	-2.70	-2.66	-2.61	-2.56	-2.50	-2.44	-2.38	-2.31	-2.22	-2.13	-2.02	-1.87	-1.68	-1.36
Class $i \leq s$																				
Mean of HP		-3.93	-3.84	-3.77	-3.70	-3.65	-3.60	-3.56	-3.52	-3.48	-3.44	-3.41	-3.37	-3.33	-3.29	-3.25	-3.21	-3.17	-3.12	-3.06
t-test		403.1	379.1	364.9	354.2	345.5	338.5	332.9	328.7	325.9	323.6	320.9	317.6	313.3	308.1	301.5	292.7	281.5	265.6	232.5

Table 4
Monotonicity Condition and Sample Separation Points

This table presents results from estimating the investment model in Eq. (14). Coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 77,086 firm-year observations with *cash flow > 0* (upper panel) and for the unbalanced sample of 93,107 firm-year observations with *cash flow ≥ -1* (lower panel). The sample period is from 1990 to 2013. Investment is the dependent variable. Observations are sorted by Hadlock and Pierce (2010) index of financing constraint. Column (a) reports the point of sample separation. Column (b) reports the coefficient associated to the class of observations $i \leq s$. Column (c) reports the coefficient associated with the market-to-book ratio. Columns (d) and (e) report the investment cash-flow sensitivity coefficient for the classes of observations $i \leq s$ and $i > s$, respectively. Column (f) reports the adjusted R^2. Column (g) reports the F-statistic associated to the null hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent standard errors are reported in parenthesis.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic
Cash flow > 0						
5	0.04544 (0.0056)	0.02637 (0.0024)	0.04636 (0.0060)	0.07118 (0.0024)	42.70%	16.25***
30	0.00291 (0.0040)	0.02652 (0.0024)	0.06482 (0.0038)	0.07180 (0.0026)	42.66%	2.89*
40	-0.00128 (0.0039)	0.02656 (0.0024)	0.06771 (0.0035)	0.07136 (0.0027)	42.65%	0.84
80	0.01440 (0.0061)	0.02688 (0.0024)	0.07274 (0.0028)	0.06714 (0.0034)	42.68%	1.99
85	0.02378 (0.0070)	0.02707 (0.0024)	0.07302 (0.0028)	0.06503 (0.0038)	42.72%	3.43*
95	0.04234 (0.0112)	0.02745 (0.0023)	0.07297 (0.0025)	0.05236 (0.0058)	42.82%	11.44***
Cash flow ≥ -1						
5	0.04178 (0.0049)	0.02220 (0.0019)	0.04552 (0.0051)	0.06884 (0.0021)	38.64%	19.60***
30	0.00130 (0.0037)	0.02232 (0.0019)	0.06096 (0.0035)	0.06985 (0.0022)	38.61%	5.15**
40	-0.00405 (0.0037)	0.02232 (0.0019)	0.06552 (0.0032)	0.06903 (0.0023)	38.59%	0.97
80	0.03675 (0.0056)	0.02305 (0.0019)	0.07006 (0.0025)	0.06442 (0.0031)	38.69%	2.33
85	0.04884 (0.0061)	0.02329 (0.0018)	0.07076 (0.0023)	0.06029 (0.0034)	38.78%	7.39***
95	0.07954 (0.0094)	0.02390 (0.0018)	0.06963 (0.0021)	0.05383 (0.0057)	38.80%	7.19***

Table 5

Monotonicity Condition and Estimating Model

In the upper panel, we report results from estimating the investment model in Eq. (15). In the lower panel, instead, we report results from estimating the investment model in Eq. (16). Coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 77,086 firm-year observations with *cash flow > 0,* and for the unbalanced sample of 93,107 firm-year observations with *cash flow ≥ -1,* respectively. The sample period is from 1990 to 2013. The dependent variable is investment. Observations are sorted by Hadlock and Pierce (2010) index of financing constraint. Column (a) reports the point of sample separation. Column (c) reports the coefficient associated with the market-to-book ratio. Columns (d) and (e) report the investment cash-flow sensitivity coefficient for the classes of observations $i \leq s$ and $i > s$, respectively. Column (f) reports the adjusted R^2. Column (g) reports the F-statistic associated with the hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent standard errors are in parenthesis.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)		α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic
Cash flow > 0						
15		0.02660 (0.0024)	0.06161 (0.0040)	0.07140 (0.0025)	42.66%	5.48**
55		0.02656 (0.0024)	0.06778 (0.0029)	0.07186 (0.0028)	42.65%	1.37
90		0.02686 (0.0024)	0.07441 (0.0026)	0.05696 (0.0041)	42.75%	15.89***
Cash flow ≥ -1						
15		0.02237 (0.0019)	0.05599 (0.0039)	0.06948 (0.0021)	38.63%	11.31***
45		0.02237 (0.0019)	0.06588 (0.0029)	0.06909 (0.0023)	38.59%	0.93
90		0.02248 (0.0018)	0.07196 (0.0022)	0.05232 (0.0039)	38.70%	21.18***

Position of s (%)	γ_s	$\bar{\alpha}_s$	$\bar{\alpha}_{n-s}$	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic
Cash flow > 0							
15	0.05090 (0.0059)	0.01454 (0.0030)	0.02753 (0.0026)	0.05818 (0.0045)	0.07194 (0.0025)	42.74%	8.13***
55	-0.01076 (0.0060)	0.02849 (0.0028)	0.02518 (0.0032)	0.06802 (0.0031)	0.07172 (0.0029)	42.65%	0.91
90	-0.00690 (0.0103)	0.03510 (0.0023)	0.00327 (0.0046)	0.07208 (0.0026)	0.06312 (0.0043)	42.95%	3.59*
Cash flow ≥ -1							
15	0.04333 (0.0053)	0.01309 (0.0027)	0.02299 (0.0020)	0.05289 (0.0042)	0.06997 (0.0021)	38.68%	14.43***
45	-0.01222 (0.0051)	0.02548 (0.0026)	0.02125 (0.0022)	0.06597 (0.0031)	0.06899 (0.0024)	38.59%	0.71
90	0.01866 (0.0086)	0.03482 (0.0020)	0.00220 (0.0027)	0.06900 (0.0022)	0.05840 (0.0039)	39.12%	6.05**

Table 6
Monotonicity Condition and Splitting Approach

Results in this table are obtained from estimating the investment model in Eq. (4), using the splitting approach proposed by the litertaure. Coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 77,086 firm-year observations with *cash flow > 0* (upper panel) and for the unbalanced sample of 93,107 firm-year observations with *cash flow ≥ -1* (lower panel). The sample period is from 1990 to 2013. The dependent variable is investment. Observations are sorted by Hadlock and Pierce (2010) index of financing constraint. Column (a) reports the point of sample separation. Column (b), (c) and (d) report the coefficients associated with cash flow, market-to-book ratio, and the adjusted-R^2 respectively for the class of observations $i \leq s$. Column (e), (f) and (g) report the corresponding values for the class of observations $i > s$. Column (h) reports the χ2-Statistic associated with the null hypothesis of equality of parameters in Column (b) and (e). Heteroskedasticity-consistent standard errors are in parenthesis.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
	Class $i \leq s$			Class $i > s$			
Position of s (%)	β	α	Adjusted R^2	β	α	Adjusted R^2	χ^2-Statistic
			Cash flow > 0				
10	0.05286 (0.0102)	0.01436 (0.0078)	62.79%	0.07197 (0.0019)	0.02534 (0.0020)	41.92%	3.41*
15	0.05531 (0.0081)	0.01855 (0.0052)	59.97%	0.07248 (0.0020)	0.02534 (0.0021)	41.60%	4.26**
65	0.06935 (0.0030)	0.03542 (0.0023)	50.00%	0.07295 (0.0030)	0.01213 (0.0035)	37.74%	0.72
70	0.06967 (0.0029)	0.03670 (0.0023)	49.49%	0.07080 (0.0032)	0.00972 (0.0038)	36.35%	0.07
75	0.07291 (0.0027)	0.03637 (0.0022)	49.27%	0.06846 (0.0034)	0.00762 (0.0040)	35.96%	1.06
95	0.07278 (0.0020)	0.03441 (0.0019)	44.97%	0.06467 (0.0079)	-0.00492 (0.0076)	23.25%	0.99
			Cash flow ≥ -1				
10	0.04976 (0.0085)	0.01416 (0.0065)	61.29%	0.06959 (0.0017)	0.02122 (0.0015)	37.90%	5.25**
15	0.04622 (0.0071)	0.01746 (0.0040)	54.63%	0.07036 (0.0017)	0.02106 (0.0016)	37.70%	11.04***
45	0.06196 (0.0035)	0.03236 (0.0026)	49.27%	0.06951 (0.0021)	0.01521 (0.0020)	35.81%	3.40*
60	0.06578 (0.0028)	0.03568 (0.0023)	47.25%	0.06889 (0.0024)	0.01200 (0.0021)	34.59%	0.72
85	0.06906 (0.0020)	0.03644 (0.0018)	43.50%	0.06228 (0.0038)	0.00354 (0.0029)	29.20%	2.46
90	0.06991 (0.0019)	0.03481 (0.0017)	42.31%	0.05839 (0.0047)	-0.00048 (0.0033)	23.72%	5.23**

Table 7

The Role of Measurement Errors

Results in this table are obtained from estimating the investment model in Eq. (14), using a GMM estimator. Coefficient estimates are reported for the full unbalanced sample of 77,086 firm-year observations with *cash flow > 0* (upper panel) and for the unbalanced sample of 93,107 firm-year observations with *cash flow ≥ -1* (lower panel). The sample period is from 1990 to 2013. The dependent variable is investment. Observations are sorted by Hadlock and Pierce (2010) index of financing constraint. Column (a) and (b) report the point of sample separation and the coefficient associated with the class of observations $i \leq s$, respectively. Column (c) reports the coefficient associated with the market-to-book ratio. Columns (d) and (e) report the investment cash-flow sensitivity coefficients for the classes of observations $i \leq s$ and $i > s$, respectively. Models are estimated via the two-step differences GMM estimator, including time dummies and using lags (4 - 16) of market-to-book ratio and cash flow as instruments, in the upper panel, and lags (6-9) in the lower panel. Column (f) reports the p-value for the Hansen J-Test of overidentifying restrictions. Column (g) reports the F-statistic associated with the null hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent standard errors are in parenthesis.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	J-Test	F-Statistic
			Cash flow > 0			
5	0.04365	0.04148	0.04198	0.10778	0.339	3.06*
	(0.0426)	(0.0346)	(0.0326)	(0.0262)		
30	0.18391	0.11996	0.05890	0.08669	0.863	0.84
	(0.0754)	(0.0412)	(0.0278)	(0.0253)		
85	-0.24929	0.03352	0.11851	0.01363	0.654	5.18**
	(0.1188)	(0.0306)	(0.0264)	(0.0438)		
			Cash flow ≥ -1			
10	0.28644	0.24594	0.02761	0.11119	0.146	3.73*
	(0.0836)	(0.0815)	(0.0177)	(0.0377)		
20	0.24296	0.18062	0.07328	0.14609	0.347	1.42
	(0.0829)	(0.1023)	(0.0497)	(0.0435)		
85	-1.17821	0.14694	0.19073	-0.09185	0.903	8.39***
	(0.3165)	(0.0940)	(0.0499)	(0.1002)		

Table 8
Monotonicity Condition and Time Period

This table displays results from estimating the investment model in Eq. (14) for two different sample periods: 1990-2007 and 2008-2013. Coefficient estimates are the within fixed firm and year estimates for the firm-year observations with *cash flow > 0* (upper panel) and for the firm-year observations with *cash flow ≥ -1* (lower panel). The dependent variable is investment. Observations are sorted by Hadlock and Pierce (2010) index of financing constraint. Column (a), (b) and (c) report the point of sample separation, the coefficient associated with the class of observations $i \leq s$, and the coefficient associated with the market-to-book ratio, respectively. Columns (d) and (e) report the investment cash-flow sensitivity coefficients for the classes of observations $i \leq s$ and $i > s$, respectively. Column (f) reports the adjusted R^2. Column (g) reports the F-statistic associated with the null hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent standard errors are in parenthesis.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic
			Cash flow > 0			
Year ≤ 2007. # of Obs: 60,529						
15	0.02640 (0.0050)	0.02700 (0.0028)	0.05977 (0.0061)	0.07851 (0.0032)	43.53%	8.88***
30	0.00480 (0.0043)	0.02717 (0.0028)	0.07016 (0.0051)	0.07802 (0.0033)	43.47%	2.17
85	0.03423 (0.0083)	0.02775 (0.0027)	0.08110 (0.0037)	0.06793 (0.0046)	43.63%	5.99**
Year > 2007. # of Obs: 16,557						
20	0.01922 (0.0082)	0.01154 (0.0065)	0.04909 (0.0089)	0.06698 (0.0049)	51.74%	4.00**
35	0.00570 (0.0110)	0.01159 (0.0065)	0.05886 (0.0078)	0.06702 (0.0051)	51.73%	1.05
50	0.02453 (0.0131)	0.01131 (0.0064)	0.07888 (0.0082)	0.06201 (0.0055)	51.90%	3.44*
			Cash flow ≥ -1			
Year ≤ 2007. # of Obs: 73,475						
15	0.02235 (0.0045)	0.02214 (0.0021)	0.05746 (0.0055)	0.07645 (0.0027)	39.36%	11.26***
30	0.00471 (0.0040)	0.02223 (0.0021)	0.07099 (0.0046)	0.07558 (0.0028)	39.29%	0.9
85	0.06171 (0.0071)	0.02312 (0.0021)	0.07855 (0.0031)	0.06426 (0.0040)	39.59%	9.01***
Year > 2007. # of Obs: 19,632						
20	0.02414 (0.0084)	0.01320 (0.0053)	0.04572 (0.0085)	0.06091 (0.0042)	48.01%	3.23*
80	0.01240 (0.0203)	0.01324 (0.0053)	0.05944 (0.0049)	0.06109 (0.0061)	47.98%	0.05
95	0.07248 (0.0305)	0.01358 (0.0052)	0.06246 (0.0044)	0.04177 (0.0107)	48.15%	3.43*

Table 9
Monotonicity Condition and Quality of Firms

Results in this table are obtained from estimating the investment model in Eq. (14) for the balanced subsample of firms paying dividends, and for the complementary unbalanced subsample. Coefficient estimates are the within fixed firm and year estimates for firm-year observations with *cash flow > 0* and for firm-year observations with *cash flow ≥ -1,* respectively. The sample period is from 1990 to 2013. The dependent variable is investment. Observations are sorted by Hadlock and Pierce (2010) index of financing constraint. Column (a), (b) and (c) report the point of sample separation, the coefficient associated with the class of observations $i \leq s$ and the coefficient associated with the market-to-book ratio, respectively. Columns (d) and (e) report the investment cash-flow sensitivity coefficients for the classes of observations $i \leq s$ and $i > s$, respectively. Column (f) reports the adjusted R^2. Column (g) reports the F-statistic associated with the null hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent standard errors are in parenthesis.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic

Cash flow > 0

Balanced and dividend > 0. # of Obs: 3,720

20	0.00004	0.01813	0.07168	0.06223	32.31%	0.20
	(0.0137)	(0.0054)	(0.0167)	(0.0244)		
80	-0.01377	0.01548	0.09120	0.05549	32.74%	4.61**
	(0.0096)	(0.0052)	(0.0136)	(0.0203)		
95	0.01707	0.01803	0.06205	0.09453	32.37%	3.31*
	(0.0283)	(0.0055)	(0.0225)	(0.0129)		

(Balanced and dividend > 0)^C. # of Obs: 73,366

15	0.02908	0.02639	0.05528	0.07241	42.26%	15.76***
	(0.0046)	(0.0025)	(0.0040)	(0.0025)		
35	0.00123	0.02655	0.06635	0.07178	42.19%	1.74
	(0.0041)	(0.0025)	(0.0036)	(0.0027)		
85	0.02472	0.02711	0.07326	0.06423	42.27%	4.07**
	(0.0072)	(0.0024)	(0.0028)	(0.0039)		

Cash flow ≥ -1

Balanced and dividend > 0. # of Obs: 5,040

25	0.00629	0.01989	0.06618	0.07260	31.47%	0.08
	(0.0139)	(0.0051)	(0.0146)	(0.0284)		
70	-0.01610	0.01843	0.08846	0.06423	31.67%	0.7
	(0.0132)	(0.0046)	(0.0099)	(0.0272)		
90	-0.02546	0.01542	0.11160	0.04516	33.09%	4.50**
	(0.0149)	(0.0048)	(0.0140)	(0.0247)		

(Balanced and dividend > 0)^C. # of Obs: 88,067

20	0.01411	0.02219	0.06017	0.06955	38.11%	4.94**
	(0.0042)	(0.0019)	(0.0039)	(0.0022)		
35	-0.00041	0.02220	0.06417	0.06936	38.10%	1.93
	(0.0039)	(0.0019)	(0.0033)	(0.0023)		
90	0.07198	0.02354	0.07050	0.05534	38.38%	11.68***
	(0.0072)	(0.0019)	(0.0022)	(0.0041)		

Table 10

Monotonicity Condition and Different Sorting Metrics

This table presents results from estimating the investment model in Eq. (14) when observations are sorted either by the Kaplan and Zinglaes (1997) index or by the Withed and Wu (2006) index of financing constraint. Coefficients are the within fixed firm and year estimates for the full unbalanced sample of firm-year observations with *cash flow > 0* (upper panel) and for the unbalanced sample of firm-year observations with *cash flow ≥ -1* (lower panel). The sample period is from 1990 to 2013. The dependent variable is investment. Column (a), (b) and (c) report the point of sample separation, the coefficient associated with the class of observations $i \leq s$ and the coefficient associated with the market-to-book ratio, respectively. Columns (d) and (e) report the investment cash-flow sensitivity coefficients for the classes of observations $i \leq s$ and $i > s$, respectively. Column (f) reports the adjusted R^2. Column (g) reports the F-statistic associated with the null hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic

Sample sorted according to the Kaplan and Zingales (1997) index

Cash flow > 0. # of Obs: 76,654

5	0.08946	0.01977	0.06058	0.12566	43.73%	166.67***
	(0.0135)	(0.0024)	(0.0029)	(0.0047)		
60	0.04035	0.02589	0.06870	0.09893	42.91%	2.05
	(0.0063)	(0.0025)	(0.0024)	(0.0213)		
85	0.02306	0.02879	0.06980	0.03453	42.84%	4.16**
	(0.0047)	(0.0024)	(0.0024)	(0.0174)		

Cash flow ≥ -1. # of Obs: 92,588

15	0.06427	0.01986	0.05933	0.09950	38.95%	48.81***
	(0.0060)	(0.0018)	(0.0024)	(0.0053)		
40	0.02769	0.02254	0.06542	0.06897	38.74%	0.22
	(0.0032)	(0.0018)	(0.0022)	(0.0072)		
80	0.02409	0.02442	0.06843	0.01854	38.78%	44.06***
	(0.0030)	(0.0019)	(0.0021)	(0.0072)		

Sample sorted according to the Withed and Wu (2006) index

Cash flow > 0. # of Obs: 76,849

10	0.09024	0.02318	0.05906	0.10370	43.27%	65.39***
	(0.0085)	(0.0025)	(0.0026)	(0.0055)		
65	0.02225	0.02581	0.06947	0.07774	42.79%	0.94
	(0.0040)	(0.0024)	(0.0024)	(0.0087)		
90	0.02048	0.02706	0.07057	0.04698	42.83%	4.90**
	(0.0053)	(0.0024)	(0.0024)	(0.0107)		

Cash flow ≥ -1. # of Obs: 92,817

10	0.06688	0.02053	0.05771	0.08588	38.91%	38.33***
	(0.0070)	(0.0019)	(0.0024)	(0.0042)		
55	0.02435	0.02176	0.06673	0.07026	38.72%	0.34
	(0.0028)	(0.0019)	(0.0021)	(0.0060)		
90	0.03152	0.02298	0.06872	0.03999	38.77%	12.22***
	(0.0040)	(0.0018)	(0.0021)	(0.0081)		

Table 11

The Shape of the Investment Cash-Flow Sensitivity

This table reports results from estimating the investment model in Panel 1 and Panel 2, respectively. For each model, all coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 77,086 firm-year observations with *cash flow > 0* and for the unbalanced sample of 93,107 firm-year observations with *cash flow ≥ -1,* respectively. Observations are sorted by Hadlock and Pierce (2010) index of financing constraint. Heteroskedasticity-consistent standard errors are in parenthesis.

Panel 1

$$\left(\frac{I}{K}\right)_{i,t} = \alpha Q_{i,t} + \sum_{r=0}^{R} \beta_r HP_{i,t}^r \times \left(\frac{Cash\ flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t}$$

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Market-to-Book	Cash flow	Cash flow × HP	Cash flow × HP^2	Cash flow × HP^3	Adjusted R^2	Null Hypothesis	F-Statistic (*p*-value)

| | | | Cash flow > 0 | | | | |

R=0

| 0.02668 | 0.07052 | | | | 42.64% | | |
| (0.0024) | (0.0024) | | | | | | |

R=1

						$H_0^1: \beta_1 = 0\ vs\ H_A^1: not\ H_0^1$	
0.02731	0.04629	-0.00924			42.73%		11.21
(0.0024)	(0.0078)	(0.0028)					(0.0008)

R=2

						$H_0^2: \beta_1 = \beta_2 = 0\ vs\ H_A^2: not\ H_0^2$	
0.02808	-0.00485	-0.06135	-0.01143		42.96%		22.79
(0.0023)	(0.0115)	(0.0093)	(0.0019)				(0.0000)

R=3

						$H_0^3: \beta_1 = \beta_2 = \beta_3 = 0\ vs\ H_A^3: not\ H_0^3$	
0.02795	0.00012	-0.04789	-0.00374	0.00121	42.96%		15.40
(0.0023)	(0.0122)	(0.0171)	(0.0087)	(0.0013)			(0.0000)

			Cash flow ≥ -1, R=2			$H_0^2: \beta_1 = \beta_2 = 0\ vs\ H_A^2: not\ H_0^2$	
0.02308	0.00478	-0.05361	-0.01027		38.84%		22.85
(0.0018)	(0.0096)	(0.0081)	(0.0017)				(0.0000)

Panel 2

$$\left(\frac{I}{K}\right)_{i,t} = \alpha Q_{i,t} + \sum_{r=0}^{R} \beta_r HP_{i,t}^r \times \left(\frac{Cash\ flow}{K}\right)_{i,t} + \theta HP_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t}$$

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Market-to-Book	Cash flow	Cash flow × HP	Cash flow × HP^2	HP	Adjusted R^2	Null Hypothesis	F-Statistic (*p*-value)

			Cash flow > 0. R=2			$H_0^2: \beta_1 = \beta_2 = 0\ vs\ H_A^2: not\ H_0^2$	
0.03002	0.00728	-0.05588	-0.01110	-0.07959	43.15%		18.89
(0.0022)	(0.0114)	(0.0091)	(0.0019)	(0.0091)			(0.0000)

			Cash flow ≥ -1. R=2			$H_0^2: \beta_1 = \beta_2 = 0\ vs\ H_A^2: not\ H_0^2$	
0.02762	0.01459	-0.04907	-0.01010	-0.10319	39.29%		19.67
(0.0018)	(0.0094)	(0.0078)	(0.0016)	(0.0068)			(0.0000)

Figure 2

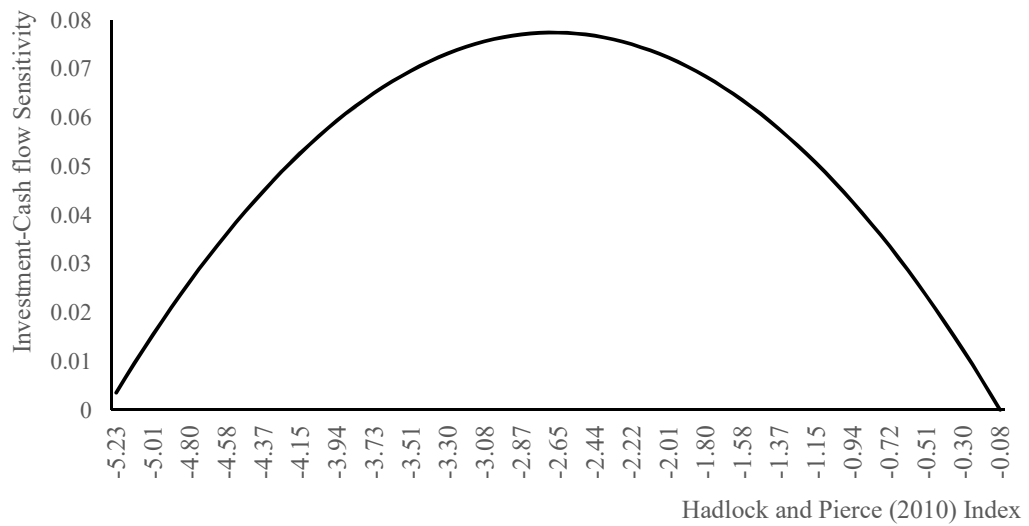The inverted U-shaped ICFS with respect to HP index

Table 12

Monotonicity of Different Sorting Metrics

For the full unbalanced sample of 77,086 firm-year observations with *cash flow > 0* , this table reports the mean value of the three indexes of financing constraint and of a number of firm characteristics commonly used as metrics of financing constraint. The means are calculated for the bottom tercile, the top tercile, and the 30 percent around the median of the Hadlock and Pierce (2010) index. We test the null hypothesis that the means of the bottom tercile are statistically different from those of the middle tercile, and that the latter are statistically different from those of the top tercile. ***, **, * stand for significance at the 1, 5 and 10 % levels, respectively.

(a)	(b)		(c)	(d)		(e)		(f)
Hadlock and	Bottom	-3.602	Middle	-3.070	Tangibility	Bottom	2031.266	Middle	1091.989
Pierce (2010)	Middle	-3.070	Top	-2.224		Middle	1091.989	Top	715.761
	t-stat	348.76***		260.00***		*t*-stat	19.38***		5.09***
Whited and Wu	Bottom	-0.013	Middle	-0.007	Coverage Ratio	Bottom	125.664	Middle	85.951
(2006)	Middle	-0.007	Top	0.049		Middle	85.951	Top	46.388
	t-stat	-1.76*		-4.46***			1.338		4.96***
Kaplan and	Bottom	-0.339	Middle	-0.347	Sales Growth	Bottom	0.081	Middle	0.131
Zinglaes (1997)	Middle	-0.347	Top	-0.788		Middle	0.131	Top	0.142
	t-stat	0.370		10.91***			-21.22***		-3.92***
Size	Bottom	7.741	Middle	6.177	R&D Investment	Bottom	0.030	Middle	0.034
	Middle	6.177	Top	3.571		Middle	0.034	Top	0.071
	t-stat	120.00***		212.28***			-7.11***		-2.97***
Age	Bottom	13.859	Middle	6.030	ROA	Bottom	0.125	Middle	0.107
	Middle	6.030	Top	5.394		Middle	0.107	Top	0.083
	t-stat	170.00***		15.13***			28.81***		2.76***
Cash Flow	Bottom	0.690	Middle	0.731	Total Long Term Debt	Bottom	0.200	Middle	0.167
	Middle	0.731	Top	1.135		Middle	0.167	Top	0.150
	t-stat	-3.17***		-23.25***			21.82***		2.85***
Cash	Bottom	0.106	Middle	0.108	Equity	Bottom	0.393	Middle	0.377
	Middle	0.108	Top	0.135		Middle	0.377	Top	0.328
	t-stat	-1.364		-20.02***			7.85***		3.06***
Market-to-Book	Bottom	1.507	Middle	1.330	Free Cash Flow	Bottom	0.482	Middle	0.477
	Middle	1.330	Top	1.419		Middle	0.477	Top	0.839
	t-stat	18.44***		-7.29***			0.403		-22.02***
Investment	Bottom	0.208	Middle	0.254	Dividend	Bottom	0.014	Middle	0.010
	Middle	0.254	Top	0.296		Middle	0.010	Top	0.009
	t-stat	-20.73***		-14.07***			13.07***		1.079

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