UNIVERSITY OF MESSINA

PHD IN ECONOMICS, MANAGEMENT AND STATISTICS
XXXIII CYCLE

Department of Economics

Stock Market Volatility and ECB's Unconventional Monetary Policies

Author:
Demetrio LACAVA

Supervisor:
Prof. Edoardo OTRANTO

A.Y. 2019/2020

Declaration of Authorship

I, Demetrio LACAVA, declare that this thesis titled, “Stock Market Volatility and ECB’s Unconventional Monetary Policies” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a the Doctor of Philosophy, in the Department of Economics at University of Messina.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: ____________________________

[Demetrio LACAVA]

Place and date: Messina, December 8, 2020

"If you think education is expensive, try ignorance."

Robert Orben

UNIVERSITY OF MESSINA

Abstract

Department of Economics

Doctor of Philosophy

Stock Market Volatility and ECB's Unconventional Monetary Policies

by Demetrio LACAVA

To address the Great recession, with interest rates close to the zero lower bound, the main central banks, among which the European Central Bank (ECB), resorted to unconventional monetary policy (UMP) measures. These policies mainly consist of asset purchases programmes, which, in the case of the ECB, caused an increase of the Central Bank's balance sheet of about 185% in the period 2009-2019. Since the main aim pursued by ECB through the UMP implementation was to give new stimulus to the real economy, most of the literature analyses their impact on output, inflation as well as on financial stability; however, only a narrow branch of the literature has volatility as a key research objective. Within this PhD thesis, an extended literature review is discussed in *Chapter 1*, whereas in *Chapter 2* we present a new model that, differently to the current literature, allow us to analyse the UMP effects on stock market volatility, by distinguishing the volatility component depending directly on these new monetary policy tools (which accounts for about the 1.4% of the general volatility process). From an empirical application of our model - which focus on four Eurozone stock indexes (CAC40, DAX30, FTSEMIB and IBEX35) - it emerges clearly the UMP effectiveness in stabilising financial markets: therefore, whereas we observe a volatility jump on announcement days, we find a remarkable volatility reduction due to the UMP implementation. Finally, in *Chapter 3*, we extend our analysis within the Markov switching (MS) framework, proving the ECB ability to keep the volatility component depending on UMP within a low volatility regime for a maximum of 53 business days. Moreover, the Model Confidence Set procedure finds out how the forecasting power of our unconventional policy proxies improves after the EAPP announcement, giving also evidence in favour of the better out-of-sample performance of the MS models, at the same time.

Acknowledgements

Attending the PhD course was like a long journey: it was a deep learning period, both as academic and personal level. If I have successfully concluded this course, it is also due to many people that I would like to thank.

First of all I would like to thank my advisor, Prof. Edoardo Otranto, for supporting me with patient and devotion and for his precious time. It was an honour having him as an academic guide because I learnt a lot from his knowledge. His help was crucial to the accomplishment of this PhD thesis.

I would also like to thank Giampiero Gallo for every constructive confrontation, for all his valuable contributions and useful suggestions
Thanks to all the other professors of the PhD course, especially to Matteo Lanzafame, for their advice and their interest in my academic work.
A great thanks to all my colleagues. A special mention to Francesco, Alessandra, Alessia, Carmelo e Luca for supporting each other and for sharing the good times as well as the worries.
A deeper thanks to Valeria for her lovely and constant encouragement, for being my staunchest supporter and for giving me the courage and the strength to continue and conclude this path, especially in the darkest days when, perhaps, giving up was the easiest choice. Thank you because you believe in me more than anyone else.

Last but not least, a huge thanks also to my family for their unfailing support and for their crucial contribution to my education: I owe what I am to you too.

Table of Contents

Abstract iii

Acknowledgements iv

1 ECB's Unconventional Monetary Policies and Literature Review 1

1.1 Introduction 2
1.2 ECB's unconventional monetary policies 3
1.2.1 Monetary policy transmission channels 6
1.3 Literature review 8
1.3.1 A brief review of univariate volatility models 16
1.3.2 Markov Switching volatility models 19
1.4 Conclusion 24

2 Measuring the Effect of Unconventional Policies on Market Volatility 27

2.1 Introduction 28
2.2 The dataset 29
2.2.1 Descriptive statistics 31
2.3 The models 34
2.3.1 The AMEM 34
2.3.2 The Composite AMEM 35
2.4 Estimation results 39
2.4.1 ACM results 45
2.4.2 The Model Confidence Set 59
2.4.3 How long does volatility take to converge to its long-run level? 65
2.5 Conclusion 75

3 Unconventional Monetary Policies and Volatility Regime Shifts 77

3.1 Introduction 78
3.2 Multiplicative Error Models and regime changes 79
3.3 The models 81

3.3.1 ACM with switching in ς process 81
3.3.2 ACM with switching in ξ process 83
3.3.3 Time Varying Transition Probability MS-AMEM 84
3.4 Estimation results 85
3.4.1 MS(ς)-ACM results 89
3.4.2 MS(ξ)-ACM results 96
3.4.3 TVTP-AMEM results 102
3.5 Comparing forecasting performance 106
3.6 Conclusion 109

Appendix Chapter 2 **111**
A.1 Preliminary analysis 111
A.2 Marginal effects 116
A.3 Multi-step forecasting 120

Appendix Chapter 3 **124**
B.1 Hamilton filter and smoother 124

References **134**

List of Figures

1.1 ECB's securities held for monetary policy purposes (millions of Euro). Sample period: June 1, 2009 - December 31, 2019. Source: European Central Bank 5
1.2 ECB balance sheet (Millions of Euro). Sample period: June 1, 2009 - December 31, 2019. Source: European Central Bank . . 6

2.1 France, Germany, Italy and Spain annualized realized kernel volatility. Sample period: June 1, 2009 - December 31, 2019. Observation: 2686 33
2.2 France, Germany, Italy and Spain real GDP growth rate (volume). Sample period: 2009-2019. Source: Eurostat 34
2.3 France, Germany, Italy and Spain Base and Policy volatility component obtained from ACM. Proxy: UMP/TA. Sample period: June 1, 2009 - December 31, 2019 57
2.4 France, Germany, Italy and Spain Base and Policy volatility component obtained from ACM. Proxy: UMP/CMP. Sample period: June 1, 2009 - December 31, 2019 57
2.5 Marginal effects UMP/TA. Model: L-ACM. Sample period: June 1, 2009 - December 31, 2019 58
2.6 Marginal effects UMP/TA. Model: Li-ACM. Sample period: June 1, 2009 - December 31, 2019 59
2.7 Model comparisons 59
2.8 Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 62
2.9 Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 62
2.10 Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 63

2.11 Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 63
2.12 Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 64
2.13 Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 64
2.14 Multi-step forecast. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 66
2.15 Multi-step forecast. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 67
2.16 Multi-step forecast. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 67
2.17 Multi-step forecast. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 68
2.18 Multi-step forecast. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 68
2.19 Multi-step forecast. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 69
2.20 Multi-step forecast. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 69
2.21 Multi-step forecast. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 70
2.22 Multi-step forecast. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 70

2.23 Multi-step forecast assuming 1sd shock. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 . 71
2.24 Multi-step forecast assuming 1sd shock. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 . 71
2.25 Multi-step forecast assuming 1sd shock. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015 . 72
2.26 Multi-step forecast assuming 1sd shock. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 . 72
2.27 Multi-step forecast assuming 1sd shock. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 . 73
2.28 Multi-step forecast assuming 1sd shock. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017 . 73
2.29 Multi-step forecast assuming 1sd shock. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 . 74
2.30 Multi-step forecast assuming 1sd shock. Proxy:UMP/CMP.Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 74
2.31 Multi-step forecast assuming 1sd shock. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019 . 75

3.1 Realized Volatility and high volatility regime probability estimated from MS(ς)-LACM. Sample period: June 1, 2009 - December 31, 2019 95
3.2 Unconventional policy announcements and high regime volatility probability estimated from MS(ς)-LACM. Sample period: June 1, 2009 - December 31, 2019 95
3.3 Weighted average of φ resulting from the MS$(\xi) - LiACM$ and announcement variable. Sample period: June 1, 2009 - December 31, 2019 101
3.4 TVTP-AMEM: RV and High volatility regime. Sample period: June 1, 2009 - December 31, 2019 105

3.5 Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019 107
3.6 Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019 107
3.7 Multi-step forecast. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019 108
3.8 Actual vs forecasted volatility. Model: MS(ξ)-LACM. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019 108
3.9 RV and forecasted high probability regime. Model: MS(ξ)-LACM. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019 109
A.1 UMP/TA, UMP/CMP and TAgrowth. Sample Period: 01/01/2009 – 30/12/2019. 112
A.2 ACF and PACF 113
A.3 ACF and PACF of the UMP/CMP first differences 113
A.4 ACF and PACF from ARIMA(4,1,1). Proxy: UMP/CMP 114
A.5 ACF and PACF from ARMA(4,0,1). Proxy: TAgrowth 115

List of Tables

2.1 Descriptive statistics for France, Germany, Italy and Spain annualized realized kernel volatility. Sample period: June 1, 2009 to August 24, 2018. Observation: 2686 31
2.2 Marginal effects of policy variables on $\mu_{t+\kappa}$. 39
2.3 Model Estimation Results AMEM 41
2.4 Model Estimation Results AMEMX. Proxy: UMP/TA 42
2.5 Model Estimation Results AMEMX. Proxy: UMP/CMP 43
2.6 Model Estimation Results AMEMX. Proxy: TAgrowth 44
2.7 Model Estimation Results ACM. Proxy: UMP/TA 48
2.8 Model Estimation Results ACM. Proxy: UMP/CMP 49
2.9 Model Estimation Results ACM. Proxy: TAgrowth 50
2.10 Model Estimation Results L-ACM. Proxy: UMP/TA 51
2.11 Model Estimation Results L-ACM. Proxy: UMP/CMP 52
2.12 Model Estimation Results L-ACM. Proxy: TAgrowth 53
2.13 Model Estimation Results Li-ACM. Proxy: UMP/TA 54
2.14 Model Estimation Results Li-ACM. Proxy: UMP/CMP 55
2.15 Model Estimation Results Li-ACM. Proxy: TAgrowth 56
2.16 Average marginal effects of policy variables on μ_t – Sample period: June 1, 2009 - December 31, 2019 (parameter estimates as in Tables 2.7–2.15) 58

3.1 MS-AMEM 87
3.2 MS-AMEMX. Proxy: UMP/CMP 88
3.3 MS(ς)-ACM. Proxy: UMP/CMP 90
3.4 MS(ς)-LACM. Proxy: UMP/CMP 91
3.5 MS(ς)-LiACM. Proxy: UMP/CMP 92
3.6 MS(ς) models comparison 94
3.7 MS(ξ)-ACM. Proxy: UMP/CMP 98
3.8 MS(ξ)-LACM. Proxy: UMP/CMP 99
3.9 MS(ξ)-LiACM. Proxy: UMP/CMP 100
3.10 MS(ξ) models comparison 101
3.11 Announcement variable and high volatility regime 102

3.12 TVTP-AMEM. Proxy: UMP/CMP 104
3.13 TVTP-AMEM models comparison. 105
A.1 Unconventional policies proxies' descriptive statistics. Sample period: June 1, 2009 to December 31, 2019. Observation: 2686. 112
A.2 ARIMA (4,1,2). Dependent variable: UMP/CMP. Observations: 2685 114
A.3 ARIMA (4,0,1). Dependent variable: TAgrowth. Observations: 2685 115

Chapter 1

ECB's Unconventional Monetary Policies and Literature Review

Abstract

As a response to the Great Recession, the main Central Banks, such as Federal Reserve (FED), Bank of England (BoE) and the European Central Bank (ECB) resorted to unconventional monetary policies, i.e. central bank's balance sheet expansions. In this chapter we discuss some of the main researches on the effects of this kind of policy both on real economy and financial markets. Generally, results are well expected with unconventional policies boosting GDP and inflation, on the one hand, and contributing to preserve financial stability by reducing bond yields and market volatility, on the other. Finally, we deserve a special attention to the most important volatility models within the univariate framework, with a particular focus on Markov switching models.

Keywords: Unconventional monetary policies, real economy, financial market, volatility models, Markov switching

1.1 Introduction

During the Great Recession, with interest rate close to the zero lower bound, many central banks resorted to unconventional monetary policy measures in order to stimulate the real economy. These policies consist of central bank's balance sheet expansion - generally through Asset Purchases Programmes (APPs) - which affects the real economy by modifying inflation rate expectation during periods in which the so-called liquidity trap makes the conventional policy, i.e. further cuts of interest rate, no longer effective.

Following other central banks such as Federal Reserve (FED) and Bank of England (BoE), the European Central Bank (ECB) established different unconventional monetary measures during the period 2009-2019, with the main objective to address and mitigate the adverse impact of the crisis on the real economy. For this reason, most of the literature focuses on the effects of unconventional policies on output (i.e. Gross Domestic Product, GDP, growth) and inflation. In this regard, several authors have opted for an analysis based on the construction of a counter-factual scenario. This is the case, for example, of the analysis carried out by Kapetanios et al. (2012) and Pesaran and Smith (2016) on the effects of unconventional policies established by BoE, which were crucial to avoid a deeper reduction of GDP as well as of inflation rate; differently, as argued by Chen et al. (2012), in their analysis on the U.S. macroeconomic framework, the effectiveness of these policies requires a credible commitment by Central Bank in keeping interest rate low for a longer period of time. Finally, a positive effect on GDP and inflation was found also by Peersman (2011), who focus on the Eurozone economy.

Even though the main concern of these policies is the real economy, they have also unintended effects on financial markets that are largely studied by recent literature. Among these effects, it is crucial the positive influence that quantitative easing should have on market uncertainty. Thus, while most authors analyse the effects of unconventional policies on bond market (Boeckx et al., 2014; De Santis, 2020; Joyce et al., 2011; Krishnamurthy et al., 2018), some others focus on stock market (Ciarlone and Colabella, 2016, 2018; Georgiadis and Gräb, 2016) emphasizing the role played by the portfolio-rebalancing channel in transmitting monetary policy decisions (Breedon et al., 2012). Clearly, unconventional policies affect market returns and volatility since, by purchasing assets available in the market, the central bank reduces the amount of those assets, giving to private investors an incentive to rebalance their portfolio, opting for a new preferred risk return configuration. In addition, notice how most of the unconventional policies by ECB were established to reduce market uncertainty, which is measured through the expected variance (Rompolis, 2017). Surprisingly, there exists a narrow literature concerning the impact of quantitative easing on volatility as a key research objective (Apostolou and Beirne, 2017; Balatti et al., 2016; Kenourgios et al., 2015; Shogbuyi and Steeley, 2017), modelling volatility mainly through the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) family models (Engle, 1982; Bollerslev, 1986).

In this chapter, after a brief overview of the several unconventional policies established by ECB, we present a literature review on the effects of unconventional monetary polices both on the real economy and on financial markets, with a particular attention on stock market volatility. More in details, the Chapter is structured as follow: section 1.2 examines the unconventional monetary policies established by ECB in the period 2009-2019; the literature review is discussed in section 1.3, in which we also examine the main model used in volatility analysis; finally, section 1.4 concludes with some remarks.

1.2 ECB's unconventional monetary policies

Starting from 2007 three different shocks - the decrease of commodities price, the sub-prime mortgage crisis and the resulting stock market crash - hit the world economy, affecting the real economy with a widespread recession, especially in advanced countries. The crisis broke out in USA and reached Europe immediately, where a new wave of uncertainty - in particular because of fears of unsustainability of the sovereign debt of the so-called PIIGS countries (Portugal, Ireland, Italy, Greece and Spain) - worsened the decrease in GDP observed in the first two years of the financial crisis. Moreover, over the last ten years, the EU economy, despite a deep reduction of interest rates (which were close to zero and sometime even negative), was characterised by low inflation level. With the main purpose of avoiding the danger of deflation, the ECB established many unconventional monetary policies - commonly known as quantitative easing (QE) - that aimed to bring inflation close to the target level of 2%.

The first experience in Europe with unconventional monetary policies dates back to 2008 when - few months later the collapse of Lehman Brothers, which marks the beginning of the financial crisis - the ECB launched the first 12-month Longer Term Refinancing Operations programme (LTRO) which -

by financing credit institutions - clearly aimed to contain the liquidity crisis and the consequent credit crunch the Eurozone was experiencing. It should be specified that generally LTROs have three-month maturity and belong to the conventional monetary policy, the Open Market Operations, in particular. Nevertheless, the extended maturity decided by ECB (up to 3 years) allows us to consider this monetary policy instrument within the unconventional measures.

At the same time, with the main purpose to sustain a particular banks financing channel, the ECB decided on the Covered Bond Purchase Programmes (CBPP1, CBPP2 in November 2011 and CBPP3 in October 2014) which reached a total amount of about €338 billion. Through these programmes, the ECB conducted direct purchases - in both primary and secondary market - of covered bonds with a minimum rating AA and eligible to be used as a collateral for the Euro-system credit operations.

Different unconventional monetary policies were established by ECB to face the sovereign debt crisis mainly caused by an increase in government debt - deriving, among other factors, from the massive public action needed to bail out banks - together with low levels of GDP. These measures include the Security Market Programme (SMP) and the Outright Monetary Transactions (OMT). Through the SMP, the ECB bought more than €200 billion of government bonds on the secondary market in order to achieve a twofold objective of reducing the government bond spreads and restoring the proper functioning of monetary policy transmission channels. It started in May 2010 by purchasing Greece, Ireland and Portugal government bond and it was extended in 2011 to consider also Italy and Spain government bonds. SMP ended in 2012 and it was replaced by the OMT programme, which can be seen as the practical response to the famous "Whatever it takes" declaration by ECB's then-president Mario Draghi, who successfully attempted to reduce the increase in government bond yields caused by the emerging denomination risk. The programme - consisting of outright transactions of government bond with a maturity up to 3 years in the secondary market - was never implemented because of the tight conditions it required. In particular, according to the "conditionality condition", a Eurozone country could have requested for entry in the programme if it had been in serious and blatant macroeconomic distress. In other words, the country had to have received financial support [1] from the Eurozone's bailout institutions, the European

[1] It could consist of both macroeconomic adjustment programme or a precautionary programme.

Financial Stability Facility (EFSF) and/or the European Stability Mechanism (ESM). Importantly, in implementing both SMP and OMT, the ECB ensured that monetary policy stance remained unchanged by carrying out specific sterilization operations.

Lastly, with the aim to adjust the inflation level toward the target level of 2%, the ECB launched the Expanded Asset Purchase Programme (EAPP), which refers to a series of unconventional measures such as the Asset Backed Securities Purchase Programme (ABSPP), the CBPPs and the Corporate [2] and Public Sector [3] Purchase Programme (CSPP and PSPP, respectively), through which the ECB conducted securities purchases up to €80 billion for month: according to official ECB sources, it was of €60 billion per month in the first year; between April 2016 and March 2017 it was incremented up to €80 billion per month and then it came back to the previous level in the following 8 months; finally, during the last year of the programme the invested amount was decreased to €30 billion per month from January to September 2018 and to €15 billion per month between October and December 2018, when the programme ended. However, starting from November 2019 a €20 billion per month Asset Purchases Programme was re-activated.

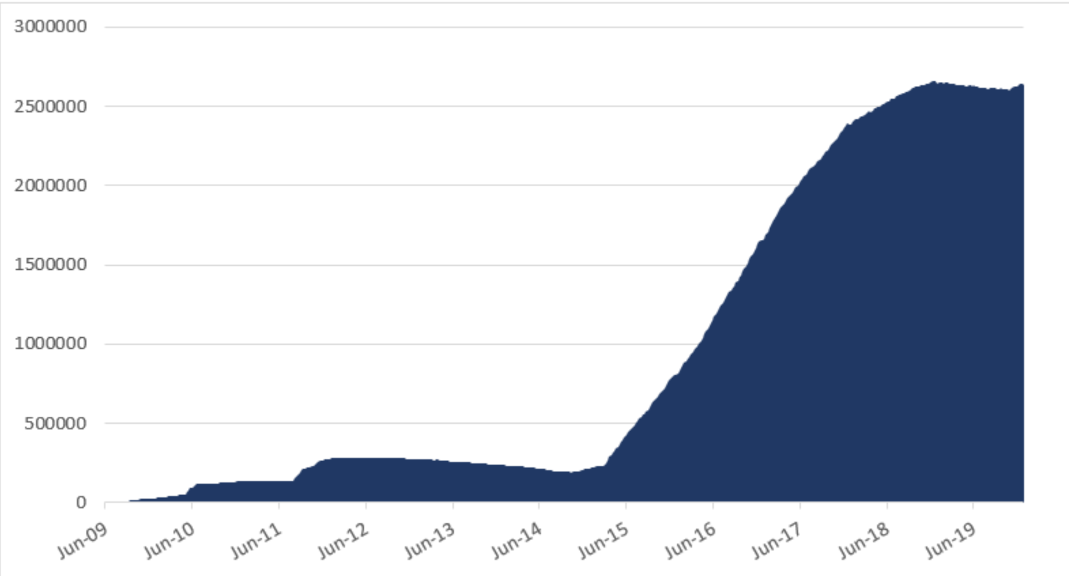

FIGURE 1.1: ECB's securities held for monetary policy purposes (millions of Euro). Sample period: June 1, 2009 - December 31, 2019. Source: European Central Bank

[2]It concerned corporate bonds issued by company different than credit institution with a minimum BBB rating and a remaining maturity between 6 months and 30 years.

[3]Including both central and local government bonds.

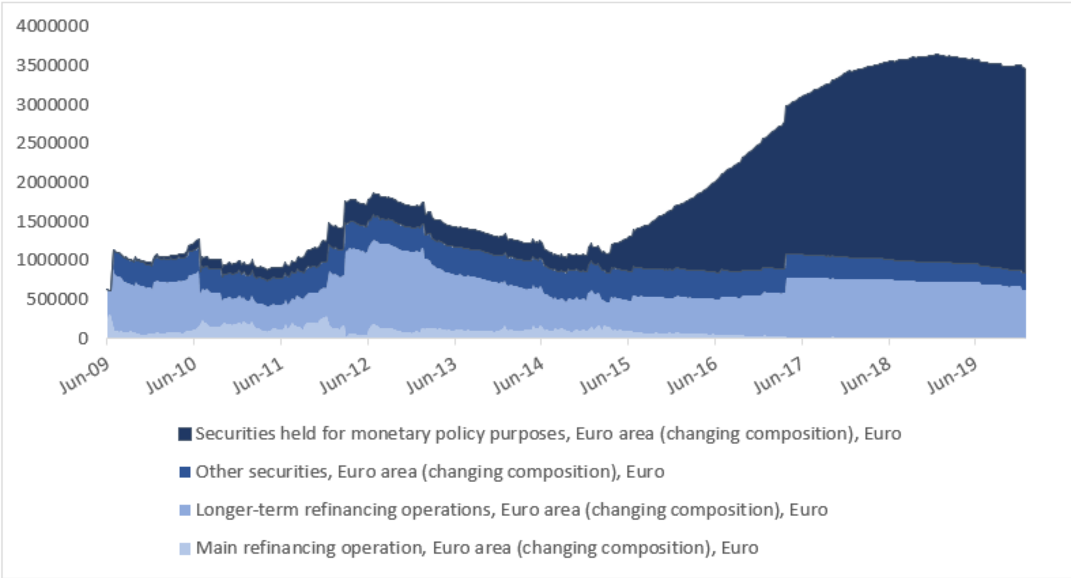

FIGURE 1.2: ECB balance sheet (Millions of Euro). Sample period: June 1, 2009 - December 31, 2019. Source: European Central Bank

Figures 1.1 and 1.2 show the evolution of the amount of securities held by ECB for monetary policy purposes and its size with respect that of other monetary policy instruments, respectively. Not surprisingly, it is evident the positive trend, which starts in 2015, when the ECB announced the EAPP. To give an idea about the size of this measure, suffice it to say that the amount of securities held for unconventional policies purposes over the ECB's total assets, during the period 2009-2018, increased, in average, of about 57% per year; the average increment was of about 65% if we consider the period 2015-2018.

1.2.1 Monetary policy transmission channels

It is a common knowledge that, usually, monetary policy is transmitted into real economy mainly through three different channels. The first one is the *interest rate channel*: basically, an expansionary monetary policy (i.e. a cut of the interest rate) generally leads to a reduction of the interbank interest rate (the Euro Overnight Index Average, EONIA, in the Eurozone), which stimulate loans between banks that should employ the consequent higher liquidity in purchases of assets and in granting an higher level of loans. This mechanism increases both investments and consumer spending and, *ceteris paribus*, the GDP growth.

A further monetary policy transmission channel is the *assets price channel*. By cutting the interest rate, the central bank is generally able to create an upward trend in stock price: as a result, it increases the market values of firms, on the one hand, and the private wealth, on the other hand. Again, this should lead to an increase in both investments and consumptions and, consequently, in GDP growth. This channel relies on the famous Tobin's Q, the ratio between the market value of a firm and its assets' replacement cost. Basically, if the Q index is grater than 1, a firm could issue new shares to finance its investments: it follows that a monetary policy that boosts asset price, stimulates total investments thanks to the lower effort that firms should support to carry out their investment plans.

A third channel, referring to the so-called credit view, is the *credit channel*. The mechanism is quite similar to that of the interest rate channel: a lower interest rate increases banks' reserves which could be used to increase banking loans. Once again, it stimulates GDP growth via higher investments and consumptions. Differently from the interest rate channel, this mechanism affects loans almost directly reducing the threat of the credit crunch, especially during recession periods.

Moreover, a fourth transmission channel for conventional monetary policies is the *exchange rate channel*. Based on the interest rate parity theory, an increase in the monetary base causes a reduction in the national interest rate that become lower than the foreign interest rate. By assuming perfect capital mobility and perfect substitutability of financial assets, it leads to a depreciation of the national currency, which would increase net exportations and, then, the GDP level.

Finally, the conventional monetary policy acts also through the *balance sheet channel* (i.e. more wealth increases the propensity to borrow) and through the *risk taking channel* (a low interest rate gives to investors the incentive to assume more risk in their search for yields).

Cut of interest rates represented the first action took by central banks to face the global financial crisis. However, with the worsened of the crisis, the interest rate reached quickly the zero lower bound, leading central banks to establish new monetary policy measures. The unconventional nature of these new policies creates new channels (in addition to the traditional ones) through which monetary policies deploy their effects into the real economy. Probably, the most important is the *portfolio rebalancing channel*. By purchasing assets available in the market, the central bank reduces the amount of those assets giving to private investors an incentive to rebalance their portfolio, opting for a new preferred risk-return configuration. Therefore, investors are encouraged in investing in other markets in which it will be observed higher prices and lower returns: in other words, purchases of assets by central bank reduce returns not only in the market included in the programme, but in all the other markets it is connected to. Crucial for the effectiveness of this channel is the imperfect substitutability of financial assets, which arises because of asymmetric information and limited commitment, as well as the balance sheet constraints; an important role is also played by the heterogeneity of investors that could have different preferences, different financial constraints and different willingness to take risk. When the purchasing programme involves government bonds the reduction of yields allows government to reduce the interest expenditure, causing a budget constraints easing. Hence, an unconventional monetary policy could activate a sort of *fiscal channel*, through which governments could improve the real economy conditions via expansionary fiscal policies.

Differently, the *signalling channel* relies on the central bank ability to drive market expectations[4]. Therefore, this channel takes action when the central bank announces new unconventional policies or specifies details of programmes that was previously launched: its efficiency is strictly linked to the central bank credibility as well as to the impact of agents' expectation on both macroeconomic and financial market conditions.

Regardless of the transmission channels, it is important to ask how unconventional monetary polices impact the economy. Many authors have tried to answer this question by analysing the unconventional policies effects on several economy components. In the next section we provide a literature review about researches concerning the unconventional monetary policies impact on financial market as well as on macroeconomic variables.

1.3 Literature review

Since unconventional monetary policies were designed to improve output and inflation conditions when the interest rate reaches the zero lower bound - so that it is no possible to boost economy with a further reduction of interest rate, the so-called liquidity trap - most of the researches focus on this macroeconomic indicators.

[4]An important example in that sense is the "What ever it takes" speech.

In this context, an important study refers to Chen et al. (2012) who successfully perform an experiment to find a counter-factual scenario of what would have been the level of inflation and output (among other macroeconomic variables) in absence of the Large-Scale Asset Purchase (LSAP) programmes established by Federal Reserve. They employ a Dynamic Stochastic General Equilibrium model (DSGE) and, based on the preferred habitat concept, they postulate that investors have heterogeneous preferences for assets to account for market segmentation. In particular, they assume that the impact of long-term interest rate on aggregate demand does not depend on short-term interest rate expectations, so that monetary policy could be effective even though it is constrained by the zero lower bound, by impacting directly long term rate; moreover, given the findings in Curdia and Woodford (2011) that unconventional policies involving assets that are viewed as equivalent to reserves (i.e. short term bond) have no effects on macroeconomic conditions, they base their research on long-term bond. Not surprisingly, they find a moderate effect of LSAP programmes in stimulating output and inflation; however, according to them, this effect would have been double if LSAP programmes were joined to a commitment to keep interest rate low for a longer period of time. This suggests that the effectiveness of unconventional policies in boosting GDP and inflation largely depends on market expectations. In particular, in order to have efficacious monetary policies it is necessary that inflation expectations are well anchored, i.e. long term expectations do not depend on temporary shocks. Therefore, it is crucial for policy-makers to establish credible monetary policy measures in order to avoid the threats deriving from de-anchoring expectations. This aspect was investigated in Ciccarelli et al. (2017) in assessing the FED's unconventional monetary policies. They, firstly, measure the degree of anchoring of inflation expectation through a time-varying model and then extend the basic Structural Vector Autoregressive model (SVAR) to account for both expected shocks (i.e. the pace of the balance sheet expansion), on the one hand, and unexpected shocks due, for example, to the specific implementation of the programmes, on the other. Despite a low degree of anchoring during the period 2009-2014, their study suggests that the programmes established by FED adequately reduced the risk of de-anchoring of long-term inflation expectations neutralizing the risk of deflation, as a consequence. Nevertheless the positive effects in stabilizing inflation expectations, they find how unconventional policies had only a limited impact in stimulating inflation and GDP.

The same framework is implemented by Peersman (2011), who focuses on the Euro area and finds a different impact of unconventional policies on output and inflation, with respect to traditional interest rate innovations. As expected, even though both policies have positive effects in rising output and inflation, it emerges how the unconventional policies has a hump shaped effect and that they impact permanently the consumer price index. Moreover, he finds a different transmission channel, with unconventional policies (represented by balance sheet expansion) acting via a banking lending channel.

An important study measuring unconventional policies and their effects on the real economy is the one carried out by Wu and Xia (2016). They develop a new indicator, the shadow rate, that is suitable in proxying unconventional policy measures: starting from the shadow rate term structure model (SRTSM) developed by Black (1995), they present an analytical representation for bond price in a multi-factor SRTSM framework from which they obtain a shadow rate with similar dynamic correlation with some macro variables of interest. By applying the shadow rate they construct, which represents a measure for the monetary policy stance when interest rate is in the zero lower bound, they find how, once the interest rate is in the zero lower bound, because of unconventional policies by FED in the period 2008-2012, the shadow rate reached the level of -1.12%, making the unemployed rate 0.13% lower than it would have been in the absence of unconventional monetary stimuli.

According to Gambacorta et al. (2014) the main weakness of studies concerining the analysis of the effects of unconventional monetary policies is that they focus on samples that, generally, cover also pre-crisis period. Therefore, they argue that this models are not suitable to analysis macroeconomic dynamics and monetary policy transmission during periods of liquidity trap. For this reason they propose to analyse the effects of unconventional policies on the real economy focusing only on the crisis period, enhancing the efficiency of the model by employing a multi-country analysis, through a panel VAR model. According to their research, the unconventional policies have a stronger and more persistent effect on GDP rather than on inflation.

Kapetanios et al. (2012) and Pesaran and Smith (2016) carry out a counterfactual analysis on the effects of unconventional policies on the UK economy, considering a 100 bps reduction in the spread between long and short term government interest rate - as documented by Joyce et al. (2011) - as a baseline scenario. Whereas Pesaran and Smith (2016) estimate a general autoregressive distributed lag (ARDL) model, finding how the effects of unconventional policies on GDP growth are strong but short living (they last less than one year), Kapetanios et al. (2012) prioritize time-varying parameters models. In particular, they estimate a Bayesian VAR (BVAR) model, a Markov switching VAR (MS-VAR)[5] and a time-varying parameters VAR (TVP-VAR), finding how the UK would have witnessed a deeper recession as well as a deeper reduction in inflation if the BoE had not adopted quantitative easing programmes; moreover, the MS-VAR model suggests that the impact on GDP and inflation is directly proportional to the persistence level of the effects of QE on the spread, which passes-through from 80 to 120 bps.

The TVP-VAR is also the model estimated by Gambetti and Musso (2017) to analyse the impact of the APP established in January 2015 on the Euro area real economy. Differently from Kapetanios et al. (2012), They consider the model within the volatility stochastic context and, with some assumptions needed to be sure that only the APP shock is considered[6], they find a short term positive impact on GDP, whereas the positive effects on inflation becomes stronger over time (therefore, it is consistent with the general view that the unconventional policies impact on inflation by affecting inflation expectations).

Andrade et al. (2016) focus on the monetary policy transmission channels to carry out a wide analysis on macroeconomic effects of APP in the Eurozone. In details, they find evidence in favour of: i) the asset valuation channel, which, by means of what they called "banks capital relief", leads to an increase in lending and supports the real economy; ii) a signalling channel, which, together with the iii) re-anchoring expectation channel, stimulates significantly GDP growth as well as inflation conditions.

Even though the main concern of this kind of policy is the real economy, they might have had unintended financial effects that have been largely studied in literature. Most authors analyse the impact of the unconventional monetary policies on both equity and bond returns and volatility. According to the methodology underlying the analysis, the branch of literature concerning the effects of unconventional monetary policies on returns can be divided

[5]They consider four states referring to the different monetary policy regimes adopted by the BoE since the early 1970s.

[6]They impose: 1) an APP shock increases the ECB securities held for unconventional policy purposes and decreases the 10-year government bond yields 2) an APP shock has a lagged positive effect on GDP and inflation and; 3) they consider only the shock that took place in the first quarter of 2015.

into two different categories. The first one is represented by researches based on the event study approach. In this context, authors usually analyse the effect of unconventional monetary policies within a short window around their announcement, generally finding a positive impact on returns. For example, Casiraghi et al. (2013) find that the SMP, which is measured by considering the daily purchases of Italian securities, successfully reduced the Italian sovereign bond yields of about 200 bps. Moreover, through an event study, they find a similar effect also considering the OMT announcement, for which the yields reduction was of about 100 bps. In addition, for what concerns the ECB's unconventional policies effects on bond market, it emerges how default risk and market segmentation were the dominant transmission channels for SMP and OMT (Krishnamurthy et al., 2018) which, together with CBPPs and 3y-LTROs, diminished significantly the borrowing cost for both banks and governments, as argued by Szczerbowicz et al. (2015). He focuses on the policies established by ECB until the end of 2012, finding a sizeable reduction in sovereign bond spread due to the SMP and OMT, ranging between 35 bps for Italy and 476 bps for Greece. As expected CBPPs impacted mostly on the covered bond spreads, while the 3y-LTRO was the only instrument able to reduce money market spread.

With regard to other markets, i.e. stock and exchange rate market, Ciarlone and Colabella (2016) analyse spillovers driven by the ECB's unconventional policies, finding how the APPs by ECB improve significantly nominal exchange rate and stock market returns, in addition to 10-year government bond yields, in CESEE economies. The same authors, Ciarlone and Colabella (2018), extend this analysis by means of a DCC-MGARCH (Dynamic Conditional Correlations-Multivariate GARCH). They proxy APPs through three different variables, and in particular the ECB's holding of securities for monetary policy purpose, finding out a sort of spillover effects into these economies, which decrease stock market and foreign exchange market volatility, while there is a no significant effect for what concerns bond market volatility.

Contrary to Ciarlone and Colabella (2016), Georgiadis and Gräb (2016) find how the EAPP announcement on 22, January 2015 boosts equity price in ASIA and USA, while the positive effects on sovereign bond yields remain confined to Euro area. Finally, in line with Ciarlone and Colabella (2018), they find a depreciation of Euro that is lower if considered against advanced economies than against emerging market economies. Since unconventional policies are generally anticipated by investors, they argued that the event study approach is no longer able to identify the real effect of the announcement. For this reason they focus on the EAPP announcement which, given the huge amount of purchases and the open-ended feature of the programme, surprised investors more than previous APPs. As a result, they find how this announcement had a stronger impact on exchange rate and equity price than OMT and SMP, mainly because of portfolio rebalancing and signalling channels.

In contrast, Altavilla et al. (2015), analysing the same announcement, find how there is a third essential channel, different than portfolio rebalancing channel, in reducing bond yields in Europe: the risk premium channel. More in detail, it consists of three channels, called scarcity channel (based on the preferred habitat theory), credit channel and duration channel. The latter, in particular, played a crucial role also for the effectiveness of SMP, as shown by Eser and Schwaab (2016) through a panel model analysis on PIIGS country.

The event study approach was implemented also in researches regarding FED and BoE unconventional monetary policies. Hattori et al. (2016) find a reduction of uncertainty and risk aversion for both equity and bond market attributable to FED unconventional measures, with a greater effect associated to Forward Guidance than QE announcements. Differently, Joyce et al. (2011) focus on the impact of BoE announcements on specific gilts: in order to consider the immediate reaction of the market to QE announcements they focus on a two-days window, finding a reduction between 70 bps and 150 bps for gilts yields and both investment and non-investment grade corporate bonds, mainly through the portfolio balancing channel (Breedon et al., 2012). Urbschat and Watzka (2019) apply the event study methodology proposed by Joyce et al. (2011) in an analysis concerning the Euro Area. in their event study, they identify 10 events - following the identification method proposed by Fratzscher et al. (2016) - and conclude that the programmes established by the ECB in 2015 and 2016 had a stronger effect at the beginning of the programmes themselves, whereas the marginal effect decreased over time, because of the agents' optimistic expectations about larger interventions by ECB. From this study, it emerges the importance played by the portfolio rebalancing channel, which drove a government bond yields reduction of about 86 bps for Portugal and of about 6 bps for Germany. Instead, Steeley and Matyushkin (2015) focus on the individual gilts volatility finding evidence in favour of what they called Pre-announcement effect, i.e. a decrease in volatility in days preceding the BoE announcements. In addition, in the same study they implement a GARCH(1,1) model from which it derives a reduction in volatility from the QE programmes subsequent to the QE1.

Despite event study approach has the great advantage of investigating the impact of specific events in a short window, it is widely recognized how results largely depend on the size of the window itself. Mainly for this reason, some authors prefer to analyse the unconventional monetary policies effects within the time series analysis framework, mainly using the Vector Autoregressive (VAR) approach. In this context, researches mainly focus on measuring simultaneously the financial and macroeconomic effects of unconventional policies. Altavilla et al. (2014) find that the OMT announcement reduced significantly bond yields and volatility, with a positive effect on consumer price, in the meantime. Differently, Boeckx et al. (2014), measuring unconventional monetary policies through the ECB total assets, find that the policies implemented in the period 2009-2014 stem financial risk by an improvement of lending conditions and a spread reduction between the Eurozone and the German government bonds. The same variable was used to investigate reduction in both market uncertainty and risk aversion, measured through expected variance and variance risk premium, respectively (Rompolis, 2017). Similar results in Fratzscher et al. (2016)[7] who proxy risk aversion through the VIX index and find a positive effect in Eurozone financial market in the period 2007-2012, i.e. an increment in stock returns in core countries and a reduction in peripheral countries' bond yields.

Whereas the above-mentioned studies focus mainly on the announcement effect, some authors analyse the impact of monetary policy shock.
De Santis (2020) tests the impact of ECB's APP on GDP-weighted 10-year Eurozone bond, by estimating a panel error correction model. He states that, since the APP announcement on January 2015 was implicitly communicated to the market in October 2014, the impact of this announcement could be underestimated. For this reason, he constructs a new variable using the number of references to this programme in news stories recorded on Blomberg, finding an average reduction in GDP-weighted 10-year Eurozone bond yields by about 60 bps with higher benefits on peripheral countries. Haitsma et al. (2016) investigate the impact of monetary policy surprise - measured as the spread between German and Italian 10-year government bond (unconventional policies surprise) and the difference in three-month Euribor future spot rate (conventional policies surprise) - in crisis and non-crisis periods .

[7] The novelty feature of this paper relies on the methodology they use to identify the events: in particular they refer to the ECB announcements covered on the front page of the Financial Times.

Although they do not find a difference between conventional and unconventional policies, what emerges is a difference in the sign of these effects in crisis periods with respect to pre-crisis periods. While in the first case they are positive, in quiet periods the coefficient for unconventional surprise is negative, meaning that an unexpected monetary easing leads to an increase in stock returns. No difference between the effect of unconventional and conventional monetary policies emerges also in Rosa (2012), who constructs a surprise variable for measuring the surprise component of LSAP announcement by FED on stock price, after controlling for target shock and news shock[8].

Rogers et al. (2014) provide a comparison between the effectiveness of asset purchase programmes by ECB, FED, BoE and BoJ. They define the monetary policies surprise as the intra-day change in government bond yields around the announcement, finding a no significant effect on financial market only in Japan and a significant appreciation of Euro. Finally, Wright (2012) analyses the surprise effect through a VAR model, identifying the surprise by comparing the variance-covariance matrix of VAR innovations on Federal Open Market Committee (FOMC) announcement days and over non-announcement days. He finds a significant effect on reducing yields for both long-term Treasury and corporate bonds, even though this effect decays in few months.

Although financial market volatility, and in particular financial market stability, was investigated as a part of more extensive analysis in researches above-mentioned, there are relatively few studies concerning financial stability as a main objective. Of course, in most analysis financial market volatility is modelled within the ARCH framework. Shogbuyi and Steeley (2017), through a multivariate GARCH model, find a no significant effect of QE programmes by FED in reducing volatility in US market; despite the increase in market volatility on specific days of QE operations by BoE, QE programmes successfully reduce volatility in UK market, on the one hand, and increase the covariance between UK and US market, on the other hand. A significant effect on US market volatility emerges in Tan and Kohli (2011) in which the VIX index falls significantly during the QE programme and increases when the programme ended.

[8]The surprise variable takes value of 1 for LSAP announcement more restrictive than expected; 0 for no surprise and; -1 for LSAP announcement more expansionary than expected. Regarding target shocks and news shocks, they are defined as the change in current month federal fund future rate in a narrow window around FOMC announcement and the difference between what the FOMC declares and what market expects, respectively.

The GARCH model is also estimated in Apostolou and Beirne (2017) to investigate the volatility spillovers due to unconventional policies by FED and ECB - measured as the change in their balance sheet size - in many emerging economies, in which they record positive volatility spillovers in bond market and negative ones for stock market.

In the same way, Converse (2015), using Realized Volatility (RV) as a proxy for market uncertainty, finds out how FED's announcement concerning the QE3 programme increased bond market volatility during the first year of the programme, while equity volatility was lower during the same period.

Beetsma et al. (2014) focus on Eurozone market finding a no significant impact of monetary policy common news, which becomes significant considering country specific news. Moreover, the considered news decrease correlation between distressed economies and Germany - regardless they are common or country specific - whereas they increase that between distressed countries.

Contrary to the previous literature, Kenourgios et al. (2015) find that the QE announcements affect not only domestic currency but also other currencies. From the Asymmetric Power ARCH (APARCH) model they estimate, it emerges a positive transmission from EUR/USD and JPY/USD exchange rate to other currencies, while they find a negative transmission from GBP to EUR and JPY.

Finally, Balatti et al. (2016) find an inverted V shaped effect: initially the impact of US and UK QE programmes on their own market volatility is positive and becomes negative after five months, in average. According to them, this indicates a spike in market volatility on days immediately following the announcement, while in the long run there would be a quiet period probably because of lower price movements deriving by the unconventional policies implementation.

1.3.1 A brief review of univariate volatility models

The pioneer study in modelling volatility is Engle (1982). What inspired Engle in his study concerning the UK inflation, is the well known stylized fact according to which, even though returns series are characterized by no autocorrelation, positive transformations of returns (i.e. absolute or squared returns) are, instead, autocorrelated. Basing on this consideration, Engle suggests that there is a sort of autoregressive variance for the data generating process, conditional on the available information set: this is the rationale underlying the famous Autoregressive Conditional Heteroskedasticity (ARCH) model. Let the model be:

$$\begin{aligned} &\epsilon_t = \eta_t \sqrt{h_t} \qquad \eta_t|\Psi_{t-1} \sim N(0,1) \;\; and \;\; \epsilon_t|\Psi_{t-1} \sim N(0,h_t) \\ &h_t = Var(r_t|\Psi_{t-1}) \equiv E(r_t - E(r_t|\Psi_{t-1})|\Psi_{t-1})^2 = E(\epsilon_t^2|\Psi_{t-1}) \end{aligned} \tag{1.1}$$

where h_t is the conditional variance and Ψ_{t-1} is the information set at time $t-1$. According to Engle (1982) we could express the conditional variance as a function of the squared lagged innovations, ARCH(p):

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 \tag{1.2}$$

Given that the unconditional variance could be interpreted as the expected value of the conditional one, i.e. $\sigma^2 = E(\epsilon_t^2) = E(var(\epsilon_t|\Psi_t)) = E(h_t)$, and given the process for the conditional variance in (1.2), it could be easily shown that

$$\sigma^2 = \frac{\omega}{1 - \sum_{i=1}^{p} \alpha_i} \tag{1.3}$$

According to the unconditional variance equation in (1.3) the positiveness condition necessary to have a positive unconditional variance is $\omega > 0 \;\; and \;\; \alpha_i \geq 0$ for $i = 1, 2, \ldots, p$, whereas it is required $\sum_{i=1}^{p} \alpha_i < 1$ to guarantee a stationary process (Engle, 1982).

The strength of the ARCH model is that it is able to reproduce the persistence feature which characterizes the volatility process; at the same time, this represents also a crucial weakness, since, often, it requires many innovation's lags (that is, many parameters to be estimated). Bollerslev (1986) overcomes this problem in the famous Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model by introducing conditional variance lags in (1.2). Differently form the ARCH model, here, the simplest GARCH specification, the GARCH(1,1), is sufficient to represent the stylized facts concerning volatility. It simply involves a lagged realization of the innovation (the

ARCH term) and a lagged value of the conditional variance (GARCH term):

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1} \tag{1.4}$$

with $\omega > 0 \;\; and \;\; \alpha, \beta \geq 0$ and $(\alpha + \beta) < 1$, to guarantee positiveness and stationarity, respectively.

A further stylized fact is the so-called "leverage effect". It refers to the empirical regularity that negative returns affect volatility more than positive returns. Nelson (1991) reproduces this empirical finding by introducing the Exponential GARCH (EGARCH), which thanks to the exponential transformation ensure the positiveness of the conditional variance without imposing any parameters constraints. In the simplest form, EGARCH(1,1), the model is specified as:

$$ln(h_t) = \omega + \beta\, ln(h_{t-1}) + \alpha\, ln(\frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}}) + \gamma\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} \tag{1.5}$$

with $0 \leq \beta < 1$ as a sufficient stationarity condition; the asymmetric effect is represented by $\gamma\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}$ and γ is expected to be negative so that we have a higher impact for negative innovations[9].

A simpler model to account for asymmetric effect was developed by Glosten et al. (1993) and it is known as GJR-GARCH:

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1} + \gamma D_{t-1}\epsilon_{t-1}^2 \tag{1.6}$$

where D_{t-1} is a dummy variable taking value of 1 if $\epsilon_{t-1} < 0$ and 0 otherwise: therefore, for positive returns the last term in (1.6) disappears and we obtain the standard GARCH; differently from the EGARCH, now γ is expected to be positive, so that volatility is higher in correspondence of negative returns. Once again, positiveness requires all parameter to be positive, whereas stationarity condition becomes $(\alpha + \beta + \frac{\gamma}{2}) < 1$. Finally, it could be noticed that the GJR-GARCH is actually a special case of the APARCH model (Ding et al., 1993): by defining the APARCH model as $h_t = \omega + \alpha^\star(|\epsilon_{t-1}| - \gamma^\star\epsilon_{t-1})^\delta + \beta h_{t-1}$, the GJR-GARCH could be obtained by simply setting $\delta = 2$; it follows that $\alpha = \alpha^\star(1-\gamma^\star)^2$ and $\gamma = 4\alpha^\star\gamma^\star$ (Ding et al., 1993).

According to Engle and Lee (1999) the volatility persistence could be modelled as the sum of a short-run and a long-run volatility component. Based on the consideration that the long-run volatility estimated by GARCH could

[9]Intuitively, a positive ϵ_{t-1} has a total impact of $\alpha + \gamma < \alpha$, whereas the impact associated to a negative shock, will be $\alpha - \gamma > \alpha$.

be not constant, they replace it with a time varying long-run volatility. Therefore, the model is specified as:

$$\begin{aligned} h_t &= q_t + s_t \\ s_t &= \alpha(\epsilon^2_{t-1} - q_{t-1}) + \delta_2(D_{t-1}\epsilon^2_{t-1} - 0.5q_{t-1}) + \beta(h_{t-1} - q_{t-1}) \\ q_t &= \omega + \rho q_{t-1} + \varphi(\epsilon^2_{t-1} - q_{t-1}) + \delta_1(D_{t-1}\epsilon^2_{t-1} - 0.5h_{t-1}) \end{aligned} \tag{1.7}$$

where q_t and s_t are the long run and the short run components, respectively; 0.5 is a scale factor to account for the mean value of the dummy. Here, s_t represents the short run component, whereas q_t is the long-run volatility; δ_1 and δ_2 measure the long-run and the short-run asymmetric effect. Stationarity condition is different with respect to the GARCH model. Here it is required that both the processes are mean reverting, that is $(\alpha + \beta) < 1$ and $\rho < 1$. In addition, to identify the model the long-run component should have a higher persistence, i.e. $0 < (\alpha + \beta) < \rho < 1$. In their study they apply the model to a large sample of the S&P500, finding a persistence of about 0.85 for the short run component and near to the unit for the long run one. Interestingly, they find out how the asymmetric effect is not present in the long-term.

More recently, thanks to the availability of intra-day data, the literature has witnessed to the development of new volatility models which are based on a new volatility proxy, the Realized Volatility (RV). Although the new framework in modelling volatility refers to the Multiplicative Error Model[10] (MEM, Engle, 2002; Engle and Gallo, 2006), an important model that has been successful over the last decade is the Heterogeneous Autoregressive Model (HAR-RV) developed by Corsi (2009)[11]. In this model, the long memory feature of volatility results from the additive aggregation of three different heterogeneous volatility components: the one day RV (which refers to short term traders), the weekly RV (medium-term investors) and the monthly RV (long-run investors). This simple model is given by:

$$RV_t = \omega + \alpha_d RV_{t-1} + \alpha_w \bar{RV}^{(5)}_{t-1} + \alpha_m \bar{RV}^{(22)}_{t-1} \tag{1.8}$$

where $\bar{RV}^{(5)}_{t-1}$ and $\bar{RV}^{(22)}_{t-1}$ are the weekly and monthly average, respectively. Despite its simplicity, this model is able to reproduce the long memory feature of volatility and it has out-of-sample forecasting performance similar to that of other more complex models.

[10] We analyse it in details in the next chapter.

[11] Here we focus on the simplest version of the model. Other specifications to account for asymmetry and volatility jumps were developed by Corsi and Reno (2009) and Andersen et al. (2007), respectively.

Since the second chapter of this thesis relies on the estimation of multiplicative error models, we discuss this kind of models in next chapter. In what follows, we present some important models which extend ARCH family models within the regime switching framework, in order to reproduce most of the well-known stylized fact, i.e. leptokurtosis, volatility clustering and time-varying correlations.

1.3.2 Markov Switching volatility models

According to several authors, the volatility persistence is actually caused by the presence of regimes which volatility is subject to. In this regard, Lamoureux and Lastrapes (1990), in their analysis on the volatility of 30 CRSP stocks, reach the conclusion that allowing for deterministic shifts in the GARCH constant (i.e. structural breaks), the model provides a remarkably lower persistence level than that of the standard GARCH model.

Most of the recent literature concerning regimes in volatility is based on the presence of a latent variable which follows a Markov Chain: for this reason, this kind of models are known as Markov switching models.
Markov switching model was introduced by Hamilton (1989) who, basing on the Markov switching regression previously implemented by Goldfeld (1973), models changes in parameters of the autoregressive process underlying the US GNP. His novelty approach consists in estimating the likelihood function with respect to population parameters which are subsequently employed to make inference about the unobservable regimes. Briefly, he finds how a model with changes between recession and growth regimes better approximate the US business cycle than a simple AR model.

Almost in parallel, Hamilton and Susmel (1994) and Cai (1994) propose a model with changes in regimes within the ARCH family models.
Hamilton and Susmel (1994) specify a model with changes in regime in the variance process of US weekly stock returns, named SWARCH-L (Markov Switching ARCH model with Leverage effects):

$$
\begin{aligned}
r_t &= \mu_t + u_t \\
u_t &= \sqrt{g_{s_t}}\tilde{u}_t \\
\tilde{u}_t &= h_t v_t \qquad v_t \sim iid(0,1) \\
h_t^2 &= \alpha_0 + \alpha_1 \tilde{u}_{t-1}^2 + \alpha_2 \tilde{u}_{t-2}^2 + \ldots + \alpha_q \tilde{u}_{t-q}^2 + \xi D_{t-1} \tilde{u}_{t-1}^2
\end{aligned}
\tag{1.9}
$$

where r_t is the returns series, whereas $\xi D_{t-1}\tilde{u}_{t-1}^2$ is the term necessary to account for asymmetric effects.

$s_t = 1, 2 \ldots, k$ is a discrete dichotomic latent variable representing the regimes and following a first order Markov chain:

$$\begin{aligned} & Prob(s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots, y_{t-1}, y_{t-2} \ldots) \\ & = Prob(s_t = j | s_{t-1} = i) = p_{ij} \end{aligned} \tag{1.10}$$

Saying differently, considering a two state Markov switching model we get

$$\begin{aligned} & Prob(s_t = 0 | s_{t-1} = 0) = p \\ & Prob(s_t = 1 | s_{t-1} = 0) = 1 - p \\ & Prob(s_t = 1 | s_{t-1} = 1) = q \\ & Prob(s_t = 0 | s_{t-1} = 1) = 1 - q \end{aligned} \tag{1.11}$$

Probabilities are usually collected in the so-called transition probability matrix P

$$P = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

with the constraint that each row sums to 1.
According to Hamilton and Susmel (1994) changes between volatility regimes happen because of changes in the scale factor of the process, g_{s_t}. Some interesting findings, although not entirely unexpected, come from the application of this model, such as: i) the best model specification involves three regimes (low, medium and high volatility regimes); ii) the low volatility regime has a higher duration, whereas high volatility is associated to recessions; iii) the specific regime variance increases going from the low to the high volatility regime.

Differently, Cai (1994) estimates a two states Markov switching ARCH model on monthly Treasury yields, by allowing changes in regime both in returns and volatility process.

$$\begin{aligned} & r_t = c_0 + c_1 S_t + z_t \\ & z_t = \beta_1 z_{t-1} + \cdots + \beta_k z_{t-k} + \epsilon_t \\ & \epsilon_t = u_t \sqrt{h_t} \quad u_t \sim NID(0,1) \\ & h_t = \gamma_0 + \gamma_1 S_t + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 \end{aligned} \tag{1.12}$$

where, as specified by Cai (1994), z_t represents the deviation from the regime mean. Results suggest that there is no a significant difference in the returns of the two regimes (i.e. c_1 is statistically equal to zero), whereas the volatility in the high volatility regime, which corresponds to some important events such as the oil shock (1974) and the change in the FED policy rate (1979-1982), is 10 times higher than the level in the low regime.

Hamilton and Lin (1996) combine the model proposed by Hamilton (1989) together with the model in (1.9) to investigate possible relationships between the industrial production and the stock market volatility, with each series following an own state variable. They hypothesize four different relationships[12]: i) volatility and business cycle regimes are independent each other; ii) on the contrary, volatility and business cycle regimes depend on the same factors; iii) changes in volatility and business cycle regime respond to the same factors, with a different timing; iv) they estimate a model which nests the previous ones (i.e. they impose no restrictions to the state variables). Not surprisingly, they find evidence in favour of the third scenario (results of model iv) go in the same direction), finding a relation between recession and high volatility regimes; in other words, volatility and the business cycle are driven by the same factors, with stock market reacting one month before with respect to the industrial production.

Gray (1996) developed a regime switching model for short term interest rate which allows to consider time varying moments (mean and variance), conditional on the regimes. It is the first study focusing on the MS-GARCH (1,1) model with all the GARCH parameters subject to regime. Intuitively, the problem arising in estimating a MS-GARCH model is that, at each step, the conditional variance (as well as the short term interest rate) depends on the entire regime path (the so-called path-dependence), making the estimation unfeasible. Gray (1996) address this problem by aggregating, at each step, the two possible values of the conditional variance (and the same for the interest rate process). However, this procedure allows an estimation of the variance conditional only on the past information, not on the regimes, and this represents the main weakness of this approach. The model, is specified as in (1.13)

[12]They estimate this models twice, firstly assuming constant variance for the residuals in the industrial production equation, then by assuming it is subject to changes in regime.

$$\begin{aligned} \Delta r_t &= \mu_{s_t,t} + \sqrt{h_{s_t,t}} z_t \quad z_t \sim NID(0,1) \\ \mu_t &= \mu_0 + \mu_1 S_t + (\phi_0 + \phi_1 S_t) r_{t-1} \\ h_t &= \omega_0 + \omega_1 S_t + (\alpha_0 + \alpha_1 S_t)\epsilon_{t-1}^2 + (\beta_0 + \beta_1 S_t) h_{t-1}(S_{t-1}) \end{aligned} \tag{1.13}$$

where s_t takes value 1 and 2; he solves the path dependence problem arising because of $h_{t-1}(S_{t-1})$ by replacing it with

$$\begin{aligned} h_{t-1} = p_{t-1}(\mu_{1,t-1}^2 + h_{1,t-1}) + (1 - p_{t-1})(\mu_{2,t-1}^2 + h_{2,t-1}) + \\ -[p_{t-1}\mu_{1,t-1} + (1 - p_{t-1})\mu_{2,t-1}]^2 \end{aligned}$$

here, p_{t-1} is the probability of regime 1 and it is time-varying, i.e. it depends on the level of the short-term interest rate, representing the idea that high volatility regime is more likely when even the interest rate is high. By applying the model to a sample consisting in 34 years of weekly observations, he finds that both the regimes are persistent (high volatility regime persist less than low volatility regime) with the variance in regime 1 four times higher than that of regime 2; moreover, there are no significant parameters in the mean equation (that is, returns follow a random walk process); finally, he finds a convergence to a long-run mean during periods of high volatility and interest rate.
Dueker (1997) implements four different specifications of the Markov Switching GARCH model to compare their performance in forecasting the implied volatility (VIX): the GARCH-UV model, analogous to Cai (1994), as specified in (1.12) but allowing switching only in the constant variable, γ; the GARCH-NF model inspired from Hamilton and Susmel (1994); further, he modifies the model proposed by Hansen (1992), allowing changes in regime also for the student-t kurtosis parameter (GARCH-K); finally, he considers a model similar to the GARCH-K but assuming the variance equals to $\sigma_t^2 = \frac{h_t n_t}{n_t - 2}$, where n_t refers to the degrees of freedom of the student-t distribution for the GARCH model, as specified in Hansen (1992). Differently from Gray (1996), he estimates the MS-GARCH models by adopting the solution proposed by Kim (1994) to solve the path-dependence. According to Hansen's analysis, only the GARCH-DF and the GARCH-K could predict the VIX better than the standard GARCH model, whereas the GARCH-NF underestimates the volatility after crisis events, because of the too short average duration of the

high volatility regime; finally, the GARCH-UV seems not to be able to capture VIX spikes, due to its constant parameters both of ARCH and GARCH terms.
According to Haas et al. (2004) the main drawback of the Gray's model refers to its analytical intractability, which makes no possible to draw the stationarity conditions of the model. Differently from Dueker (1997) in which the conditional variance depends only on the past information and not on the regime, they generalize the MS-GARCH model in a multi-regime framework accounting for the conditional variance to be dependent on the specific regime GARCH parameters. Moreover, in this setting, the parameters have a clear interpretation, that is $\frac{\alpha_{1j}}{1-\beta_j}$ is the total impact of a shock and β_j reflects the memory parameter of the process in regime j.
Let the model be

$$\begin{aligned} \epsilon_t &= \sigma_t z_t \quad z_t \sim NID(0,1) \\ \sigma_t^2 &= \alpha_0 + \alpha_i \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{1.14}$$

where $\alpha_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{ik}]'$ and $\beta = diag(\beta_1, \ldots, \beta_k)$. Haas et al. (2004) demonstrate that the sufficient condition for covariance stationarity of the process is that $\beta_j < 1$, for $j = 1, \ldots, k$. They apply this model, with two and three regimes, to three different exchange rate series, confirming its good theoretical properties in estimating volatility persistence, kurtosis and overperforming other standard models in the out-of-sample density forecasting.

Markov switching models are particularly suitable also in analysis concerning volatility spillovers. Baele (2005), for example, focuses on volatility spillovers from EU and US markets on several local European financial markets. He proposes a model for time-varying integration among market, i.e. time-varying parameters that measure the sensitive of local market to global shocks. He specifies the returns model as

$$\begin{aligned} & r_{i,t} = \mu_{i,t} + \epsilon_{i,t} \\ & \epsilon_{i,t} = e_{i,t} + \gamma_i^{EU}(S_{i,t}^{EU}) + \hat{e}^E U_t + \gamma_i^{US}(S_{i,t}^{US}) + \hat{e}^U S_t \\ & e_{i,t}|\Psi_{t-1}\ N(0, \sigma_{i,t}^2) \end{aligned} \tag{1.15}$$

where γ_i^{EU} and γ_i^{US} represent the spillover intensities parameter for the EU and US market respectively, each following a proper state variable, $(S_{i,t}^{EU})$ and $(S_{i,t}^{US})$[13]. Results confirm a statistically significance of the spillover intensities parameters: in particular, the one referring to the EU shocks, increases

[13]He considers three different relationships between the two state variable, i.e. a general state (in which he imposes no restrictions), independence and a common state.

significantly starting from the second half of the 1980s probably because of the creation of the Eurozone.

Finally, Ma et al. (2017) extend the HAR model within the Markov switching framework in their analysis of the oil market. They compare twelve specifications of the HAR model with changes in regime, to account for different effects such as leverage and jumps. The analysis suggests that, generally, the presence of regime switching improves significantly the forecasting power of the model. They compare models through the Model Confidence Set (MCS) procedure, from which it emerges that linear models have the worst performance, being systematically excluded from the best set of models.

Markov switching extension was developed also in the multiplicative error model framework, covering several aspects such as, for example, two or multiple regimes and Multi-Chain Markov Switching model. However, we remand the analysis of these models in Chapter 3 of this dissertation, in which it is also provided an empirical analysis based on the estimation of these models.

1.4 Conclusion

In this chapter we present an extended literature review on the effect of unconventional monetary policies both on the real economy and financial markets. This kind of policy represents the main response of Central Banks to the Great Recession. Initially Central Banks tried to address the consequences of the financial crisis through conventional monetary policies, via a cut of the interest rate. However, the worsening of the crisis required repeated actions by Central Banks, with the interest rate reaching the zero lower bound quickly, bringing the economy in the so-called liquidity trap and making a further cuts of interest rate no longer effective. This was the case, in particular, of the ECB, which resorted to unconventional monetary policies starting from 2009, when it established an extended duration (up to 3 years) of the Refinancing operations. Few months later, the ECB decided also on the first asset purchasing programme, the CBPP, whereas the outbreak of the sovereign debt crisis made it necessary the ECB intervention also in the sovereign bond market, through the SMP and the OMT. Finally, to avoid the risk of deflation, in 2015, the ECB established the EAPP, which refers to monthly purchases of asset up to €80 billion per months, within four different purchasing programme (ABS, the CBPP, the CSPP and the PSPP). The extraordinary nature of these measures has prompted researchers to wonder what effects they might have both on the real economy and financial markets. Results are well expected, with unconventional policies that have successfully mitigated the consequences of the crisis, boosting GDP and inflation, on the one hand, and contributing to preserve financial stability, on the other. For example, Kapetanios et al. (2012) analyse the effect of the BoE unconventional policies on the UK real economy, finding how these policies were crucial to avoid a deeper recession and a deeper reduction of inflation. Something different turns out from the analysis carried out by Chen et al. (2012) according to whom a credible commitment to keep interest rate low for a longer period of time by FED would have doubled the effect of unconventional policies on US GDP and inflation.

As said, unconventional monetary policies have also contributed in preserving financial stability. It took place in different ways, such as a reduction in bond returns and market volatility, whereas stock market returns generally increase. The downward effect on bond returns emerges in several analysis, among which Boeckx et al. (2014), who find a general reduction of the spreads between the government bond yields of Eurozone countries and the German government bond, and Joyce et al. (2011), who focus on the UK market finding a general decrease of returns also concerning investment and non-investment grade corporate bond. Similar results were also found by Krishnamurthy et al. (2018) who certify the improvement of lending conditions - because of the unconventional monetary policies established by ECB - both for governments and banks. For what concerns stock market returns most of the search find evidence in favour of a sort of spillover effect (Ciarlone and Colabella, 2016, 2018; Georgiadis and Gräb, 2016; Fratzscher et al., 2016), whereas others find positive effect deriving from unconventional policies surprise (Haitsma et al., 2016; Rosa, 2012; Rogers et al., 2014; Wright, 2012). What is interesting to notice is how results seem not to depend on the applied methodology: indeed, the analysis above-mentioned find similar results each other despite some researchers perform an event study analysis, whereas others opted for the time series framework. Furthermore, we deserved a special attention to studies which have market volatility as a main objective, in which it emerges the crucial role played by unconventional policies. Among these studies, through a multivariate GARCH model, Apostolou and Beirne (2017) find an increase in correlation between UK and US stock market, whereas Shogbuyi and Steeley (2017) find spillover effects in many emerging economies; finally, an interesting results was find by Balatti et al. (2016), who find an inverted V shaped effect, with volatility increasing after the unconventional policies announcements, whereas it decreases after few months. In this respect, in the next two chapters we perform an empirical analysis on the effect of unconventional policies established by ECB on stock market volatility: in Chapter 2, we employ a new model, which allow us to estimate the part of volatility depending directly on these policies, whereas, in Chapter 3, we extend the analysis within the Markov switching framework to test the ECB ability to keep volatility within a low volatility regime.

Chapter 2

Measuring the Effect of Unconventional Policies on Market Volatility[†][⋆]

Abstract

Unconventional monetary policies (UMP), established by European Central Bank (ECB) in response to the Great Recession, had remarkable effects both on real economy and financial markets. Our research aims to analyse the impact of UMP by ECB on stock market volatility in four Eurozone countries within the MEM framework. We propose a model to allow volatility to depend on this kind of policy by distinguishing between the announcement and the implementation effects, measured through a dummy variable and a proxy for securities held for monetary policy purposes, respectively. While we observe an increase in volatility on announcement days, we find a negative implementation effect, which causes a remarkable reduction in volatility in the long term. Finally, the Model Confidence Set approach finds how the forecasting power of the proxy improves significantly after the Expanded Asset Purchases Program (EAPP) announcement.

Keywords: Unconventional monetary policy, Financial market, Realized Volatility, Multiplicative Error Model, Model Confidence Set.

[†]A working paper, based on a revised version of this paper, entitled "Measuring the Effects of Unconventional Policies on Stock Market volatility" (co-authored with G. Gallo and E. Otranto) was published by CRENos (WP 2020_06) and by Italian Statistical Society (available at `https://it.pearson.com/content/dam/region-core/italy/pearson-italy/pdf/Docenti/Universit%C3%A0/Pearson-SIS-2020-atti-convegno.pdf`). A preliminary version was also submitted at the ERFIN-2019 and ITISE-2019 international conferences, as well as at the 60th Annual Conference (RSA) of the Italian Economic Association (SIE).

[⋆]*I would like to thank Giampiero M. Gallo for every constructive confrontation, for all his valuable contributions and useful suggestions for this study.*

2.1 Introduction

Modelling and forecasting volatility is a crucial task both for investors and researchers: for investors because volatility is essential to determine the asset price correctly; for researchers because making inference on the expected value of a variable requires a correct specification of its expected variance. Volatility process, including analysis on the effects of unconventional policies (Apostolou and Beirne, 2017; Shogbuyi and Steeley, 2017), is generally investigated through ARCH family models (Engle, 1982; Bollerslev, 1986). However, despite the effectiveness of the GARCH model, it is proved that it has poor forecasting performance, i.e. the R-squared of the Mincer-Zarnowitz regression of the volatility estimated via GARCH on the daily squared returns (which is generally used as a volatility proxy) is lower than 5%. For this reason, the new frontier in analysing volatility is represented by the Multiplicative Error Model, MEM (Engle, 2002; Engle and Gallo, 2006), in which volatility is the product of a time-varying factor (following a GARCH process) and a positive random variable ensuring positiveness without resorting to logs. Basing on MEM, Brownlees et al. (2012) propose a model - the Composite AMEM (ACM) - in which the conditional variance is the sum of a short–run and a long–run component; similarly, Otranto (2015) proposes a new model to capture spillovers effects in financial markets, by decomposing the mean equation as the sum of two components, both evolving according to a GARCH model. These models could be considered a general framework where inserting the effect of unconventional policies as an unobservable factor, providing its estimate and its weight on the level of volatility. In other terms, we further modify these models to allow volatility to depend on unconventional monetary policies. In particular, in our specification, the first equation composing the mean equation evolves as a GARCH model (capturing the pure volatility mechanism) while the second one follows an autoregressive process with exogenous variables, to capture both the announcement effect and the implementation effect of unconventional measures on volatility.

More precisely, our research aims to analyse the impact of unconventional monetary policies by ECB on stock market volatility in four Eurozone countries (France, Germany, Italy and Spain). We proxy for unconventional policies by using three different variables, relating with the existing literature in using two of those, i.e. the balance sheet size growth (Apostolou and Beirne, 2017; Voutsinas and Werner, 2011) and the ratio between the securities purchased and ECB's total asset (d'Amico et al., 2012; Voutsinas and Werner, 2011).

In carrying out our analysis we employ a realized volatility measure based on high frequency data, which should remove endogeneity arising when monetary policy decisions coincide with a stock price reduction, as argued by Ghysels et al. (2017).

We allow for unconventional policies to affect volatility both in an additive and a multiplicative way and we compare model through information criteria (AIC and BIC) and by using loss function for evaluating the forecasting power (MAE and MSE); finally a Model Confidence Set procedure is performed to compare the out-of-sample forecasting performance of the estimated models.

The paper is organized as follow. Data are described in Section 2.2, while section 2.3 analyses the high frequency methodology employed in our empirical analysis. Section 2.4 presents results. Finally, section 2.5 concludes with some remarks.

2.2 The dataset

In carrying out our time series analysis, we consider a dataset consisting of 2686 daily observations of annualized realized kernel volatility[1] of four market indexes (CAC40 for France, DAX30 for Germany, FTSE MIB for Italy and IBEX35 for Spain – referred to by country in what follows) provided by the Oxford Man Institute[2], from June 1, 2009 to December 31, 2019. We also allow our model to take into account the asymmetric effect, i.e. a higher volatility response to negative returns with respect to the positive ones[3].

To investigate the impact of unconventional monetary policies by ECB we consider two different variables, which refer to the short term and the long term effect on volatility, respectively. The short term effect is measured by means of a dummy variable taking value of 1 on days in which ECB releases a monetary policy announcement and 0 otherwise[4]. To proxy for the long-run

[1] Data are, hence, expressed as percentage annualized RV. That is, $\sqrt{RV * 252} * 100$.

[2] The latest version is available at `https://realized.oxford-man.ox.ac.uk/data/download`.

[3] We obtain returns as the log difference of market index closing price between two consecutive days (data are available at: `https://www.investing.com/`).

[4] We obtain information relative to monetary policy decisions, needed to construct our dummy variable, from the ECB website: `https://www.ecb.europa.eu/press/pr/activities/mopo/html/index.en.html`.

effect we construct three different variables: i) the change in ECB's balance sheet size growth (Apostolou and Beirne, 2017; Voutsinas and Werner, 2011), named TAgrowth; ii) the amount of securities held by ECB for unconventional policy purposes as a fraction of total asset, named UMP/TA (d'Amico et al., 2012; Voutsinas and Werner, 2011) and; iii) the amount of securities held for unconventional policies with respect to that held for conventional measures, UMP/CMP. More precisely, since what matters for the effectiveness of these policies is the balance sheet composition, rather than its size (Curdia and Woodford, 2011), the last proxy should measure the effect on market volatility of the unconventional policies weight - relative to the conventional policies weight - in ECB balance sheet. We obtain daily data on securities held for monetary policy purposes from ECB website[5] and Datastream.

The use of the kernel function is necessary to make realized volatility - which, basically, is the square root of the realized variance - a consistent estimator of volatility in presence of microstructure noise. In particular, realized variance was introduced by Andersen and Bollerslev (1998) as a proxy for Integrated Variance (IV), basing on the common view that both price and returns of financial assets evolve according to a continuous-time process:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t)$$

where dp(t) is the logarithmic price increment, $\mu(t)$ is a drift, $\sigma(t)$ is the spot volatility and W(t) is a standard Brownian motion. From this process, it derives that the one-period daily return is:

$$r_t = p(t) - p(t-1) = \int_{t-1}^{t} \mu(s)ds + \int_{t-1}^{t} \sigma(s)dW(s) \rightarrow r_t \sim N(\int_{t-1}^{t} \mu(s)ds, IV_t)$$

where IV_t is the so-called integrated variance. Since the latter is not computable, Andersen and Bollerslev (1998) suggest using the realized variance, which is simply obtained by summing up the intra-day squared returns:

$$\sum_{i=1}^{M} r_{t,i}^2$$

where $r_{t,i}$ represents market returns, computed as the first difference of log price, whereas M is the number of intervals within a trading day.

The problem with using intra-day returns to compute RV is that, at a very high frequency, returns suffer from autocorrelation, mainly because of microstructure noise - which is defined as a deviation from the fundamental

[5] Available at: https://www.ecb.europa.eu/stats/policy_and_exchange_rates/minimum_reserves/html/index.en.html.

value that is induced by the characteristics of the market under consideration - e.g. bid-ask spread, the discreteness of price changes and asymmetric information. For this reason, the recent literature proxies for IV by means of realized kernel volatility (RVk), which, as shown by Barndorff-Nielsen et al. (2008), is a robust estimator of the volatility, in particular with respect to microstructure noise of the markets.

$$RVk_t = \sum_{h=-H}^{H} k \frac{h}{(H+1)} \gamma_h$$

$$\gamma_h = \sum_{j=|H|+1}^{H} r_{j,t} r_{j-|h|,t}$$

where k is the kernel function and H its the bandwidth (henceforth we refer to RV as the realized kernel volatility).

2.2.1 Descriptive statistics

Looking at the descriptive statistics (Table 2.1) one can notice a similarity between the France and Germany RV series, which have a lower mean value than that of Italy and Spain, as expected. In fact, the latter countries, during the sample period, experienced a deeper recession, which contributes to create uncertainty among investors with a direct impact on financial markets.

TABLE 2.1: Descriptive statistics for France, Germany, Italy and Spain annualized realized kernel volatility. Sample period: June 1, 2009 to August 24, 2018. Observation: 2686

	France	**Germany**	**Italy**	**Spain**
Mean	13.62	13.85	15.28	16.53
Median	12.07	12.4	13.57	14.53
Min	2.28	2.14	1.58	2.98
Max	79.65	88.44	77.72	148.61
St.Dev.	7.49	7.25	7.77	9.01
Skewness	2.18	2.24	2.07	3.14
Kurtosis	9.38	10.6	7.81	24.96

This is also shown by Figure 1, which plots the annualized kernel volatility series for the considered countries. In all the cases it is quite clear how volatility clustering affects all the series: indeed, one can notice how volatility is high, for example, between August and November 2011, whereas it stays at low level for a long period between July 2012 and the end of 2014. In the same Figure, we highlight some important dates (red lines) which give us a first idea on how unconventional policies affect market volatility. More in detail:

- SMP announcement on May 10, 2010. It was designed to manage the spread increase by purchasing government bonds. Initially just bond of Greece, Ireland and Portugal came into the purchase programme, which in 2011 was extended also to Italy and Spain government bond: one can notice, indeed, the significant reduction in volatility observed in August 2011, when Italy and Spain government bond entered the programme.

- "Whatever it takes" declaration by Mario Draghi on July 26, 2012, which served to reassure investors regarding the emerging denomination risk.

- EAPP announcement on January 22, 2015. It was established mainly to improve monetary policy transmission mechanisms, to contrast credit crunch so as to favour conditions for banks to increase financing for the real economy and, finally, contributing to one of the main ECB's objective, that is to adjust the inflation rate toward to the target level of 2%.

- March 10, 2016. The purchased amount of securities within the EAPP was incremented to €80 billion per month. What emerges is an effect caused also by the amount of securities purchased by ECB.

- October 26, 2017. Volatility increases after the announcement through which ECB communicated the cut of the monthly purchases, which reduced to €15 billion.

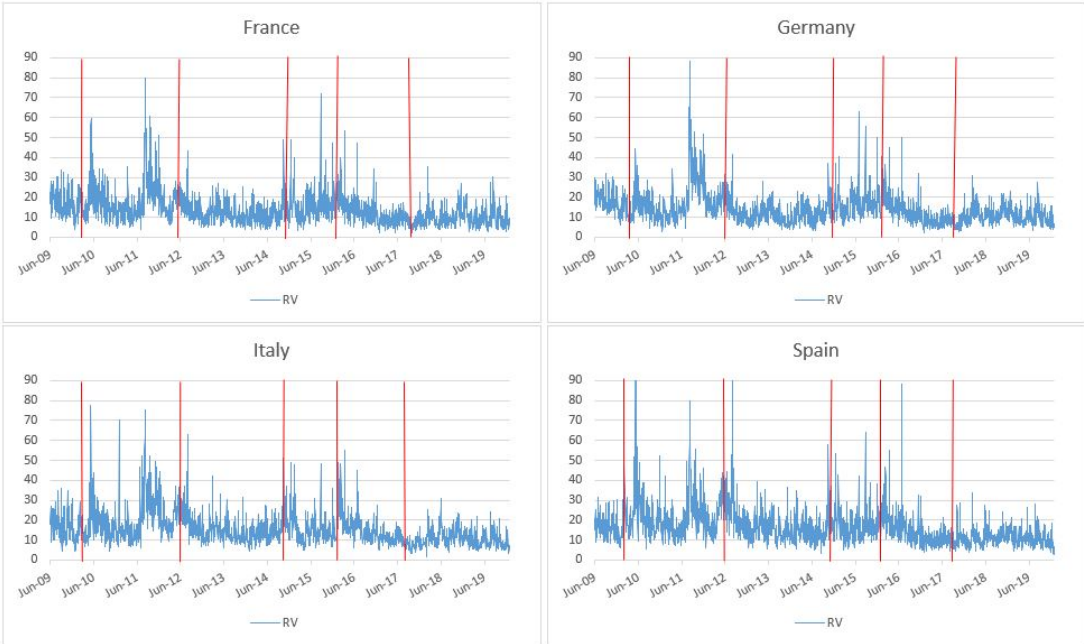

FIGURE 2.1: France, Germany, Italy and Spain annualized realized kernel volatility. Sample period: June 1, 2009 - December 31, 2019. Observation: 2686

A simple visual inspection shows the decrease of volatility in correspondence of the previous events. Our purpose is to quantify the effect and the weight of these events on the level of volatility.

To conclude the analysis of our dataset, it is interesting to look at the evolution of the GDP of the selected countries. Figure 2.2 shows the evolution of the real GDP growth rate (volume[6]) over the sample period. In details, the real GDP growth rate seems to follow a similar pattern in all the considered countries, with a reduction detected in the two-year period 2010-2012, because of the sovereign debt crisis; conversely, for the period 2012-2017 - when the most important unconventional measures (OMT and EAPP) had been implementing - there was an increase in the real GDP growth, which becomes positive in all the cases. Finally, what is important to notice here is the different behaviour among countries, with a remarkable reduction of the real GDP growth rate in Italy and Spain, where it was also negative for large periods of time, whereas it remained always positive in France and Germany; in the light of these considerations, in analysis of volatility we could refer to France and Germany as *core countries*, while we could consider Italy and Spain as *peripheral countries*.

[6]We focus on this measure because of its suitability in allowing comparisons both over time and between different economies.

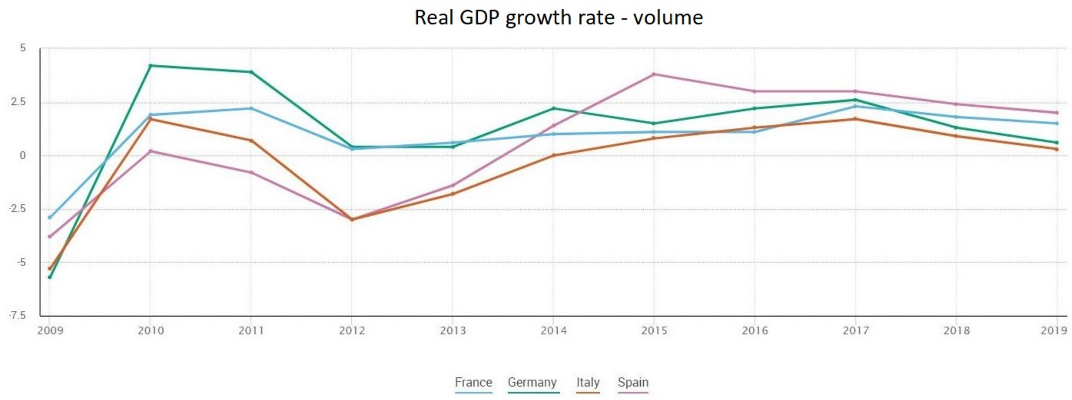

FIGURE 2.2: France, Germany, Italy and Spain real GDP growth rate (volume). Sample period: 2009-2019. Source: Eurostat

2.3 The models

2.3.1 The AMEM

Volatility is generally investigated in the GARCH framework - Engle (1982) for the first definition of ARCH model and Bollerslev (1986) for the GARCH specification - even though in the last two decades the new frontier in modelling market volatility is based on the Multiplicative Error Model (MEM) as defined by Engle (2002) and successively revised by Engle and Gallo (2006) to allow for asymmetric effects (AMEM). Let us call RV_t the realized volatility of a certain asset (index) at time t. Since volatility is the evolution of a non-negative process, Engle (2002) and Engle and Gallo (2006) propose to model it as the product of a time-varying factor μ_t, representing the conditional expectation of the volatility and following a GARCH type dynamics, and a positive random variable, ϵ_t:

$$\begin{aligned} &RV_t = \mu_t \epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta}) \\ &\mu_t = \omega + \alpha RV_{t-1} + \beta\mu_{t-1} + \gamma D_{t-1} RV_{t-1} \end{aligned} \tag{2.1}$$

where Ψ_{t-1} is the information set available at time $t-1$, and D_{t-1} a dummy variable taking value 1 if the return of the asset (index) at time $t-1$ is negative, 0 otherwise. In this model, called Asymmetric Multiplicative Error model (AMEM), the usual constraints for positiveness and stationarity are imposed: $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0, \gamma \geq 0$ and $(\alpha + \beta + \frac{\gamma}{2}) < 1$. Finally, as for the GARCH model, this causes the unconditional mean equal to

$$\mu = \frac{\omega}{1-\alpha-\beta-\frac{\gamma}{2}}$$

Following Engle and Gallo (2006) we consider the Gamma distribution for error term, ϵ_t, from which, due to the unit mean property, it derives a conditional density function given by:

$$f(\epsilon_t|\Psi_{t-1}) = \frac{1}{\Gamma(\vartheta)}\vartheta^{\vartheta}\epsilon_t^{\vartheta-1}$$

Similarly, concerning the conditional density function of RV_t, we get

$$f(RV_t|\Psi_{t-1}) = \frac{1}{\Gamma(\vartheta)}\vartheta^{\vartheta}RV_t^{\vartheta-1}\mu_t^{-\vartheta}\exp(-\vartheta\frac{RV_t}{\mu_t})$$

Consequently, considering the conditional distribution of RV_t, it is simple, given the assumption of a Gamma distribution depending only on a parameter (from which it derives a conditional mean and variance of the error term equal to 1 and $\frac{1}{\vartheta}$), to verify that:

$$E\left(RV_t|\Psi_{t-1}\right) = \mu_t \qquad Var\left(RV_t|\Psi_{t-1}\right) = \mu_t^2/\vartheta.$$

This last property shows that the AMEM possesses a very flexible structure, implying not only a time–varying conditional mean, but also a time–varying conditional variance (volatility of volatility), with the possibility to capture possible clustering in volatility.

2.3.2 The Composite AMEM

The AMEM represents the basis for the model developed by Otranto (2015) to capture volatility spillovers among markets in a univariate framework (the Spillover AMEM, SAMEM). In particular, he decomposes the mean equation μ_t as the sum of two components, both evolving according to a GARCH model. Consequently, the SAMEM is specified as:

$$\begin{aligned}
&RV_t = \mu_t\epsilon_t,\ \ \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta})\\
&\mu_t = \varsigma_t + \xi_t\\
&\varsigma_t = \omega + \sum_{h=1}^{p_0}\alpha_{0,h}RV_{t-h} + \sum_{j=1}^{q_0}\beta_{0,j}\varsigma_{t-j} + \gamma_0 D_{0,t-1}RV_{t-1}\\
&\xi_t = \sum_{h=1}^{p_i}\alpha_{i,h}RV_{t-h} + \sum_{j=1}^{q_i}\beta_{i,j}\xi_{t-j} + \gamma_i D_{0,t-1}RV_{i,t-1}
\end{aligned} \tag{2.2}$$

with $D_{i,t-1}$ equals 1 when the return of market i is negative and 0 otherwise. Stationarity and positiveness conditions are the same as in the original model by Engle (2002): stationarity, in particular, requires the stationarity in covariance of each volatility component. Importantly, the SAMEM nests the simplest AMEM, which is obtained when all the $\beta_{i,j}$ coefficients are constrained to be zero.

Starting from this specification, we develop it to insert the effect of unconventional policies as a latent factor, which affects the dynamics of volatility. Our specification is also similar to the Composite AMEM (ACM) developed by Brownlees et al. (2012), in which the conditional variance is the sum of a short run and a long run component. Actually, the SAMEM and the ACM could be considered similar each other, in the sense that SAMEM could be seen as a particular specification of the ACM, suitable for the analysis of spillover effects. More precisely, our model is based on the decomposition of the volatility level in the sum of two unknown components, representing, respectively, the "proper" volatility of the market and the effect of the unconventional policies. The great advantage of this representation consists in the possibility to quantify the last one and to verify its effect on the global volatility of the market.

More in detail, the model we propose - following Brownlees et al. (2012), we call it Composite AMEM– ACM - consists of four equations:

$$
\begin{aligned}
&RV_t = \mu_t \epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta}) \\
&\mu_t = \varsigma_t + \xi_t \\
&\varsigma_t = \omega + \alpha RV_{t-1} + \beta\varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\
&\xi_t = \delta(E\,(x_t|\Psi_{t-1}) - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi\xi_{t-1}
\end{aligned}
\tag{2.3}
$$

Stationarity and positiveness conditions remain almost unchanged with respect to the SAMEM. Once again, for stationarity it is required that both components are stationary in covariance, that is $(\alpha + \beta + \frac{\gamma}{2}) < 1$ and $\psi < 1$; positiveness, instead, simply require that $(\varsigma_t + \xi_t > 0)$ for each t, which could be ensured even though δ is negative, as we expect.

In this model, ς_t represents the proper volatility of the market, due to its intrinsic dynamics, which evolves as the third equation in (2.3); ξ_t represents the effect due to the unconventional policies and follows an AR(1) process with exogenous variables x_t and Δ_t. Δ_t is a dummy variable, taking value 1 on days characterized by the communication of monetary policy news by ECB, 0 otherwise: it represents the effect of the monetary policy announcements by the Central Bank. The other variable, x_t, represents, in turns, the growth of ECB balance sheet size (TAgrowth), the ratio between the amount

employed in unconventional polices and total asset (UMP/TA) and the ratio between the amount invested for unconventional policies purposes and the amount invested for conventional policies (UMP/CMP). The timing of these exogenous variables deserves particular attention. Because of the Efficient Market Hypothesis and since monetary policy announcements are regularly scheduled, we can consider the current value of the announcement variable, Δ_t. For what concerns the proxies representing the long run effect, since at time t we cannot know their current value, we have to use market expectations on these variables. From a preliminary analysis (cf. Appendix A.1), it emerges how the UMP/TA follows a random walk process, which allows us to measure market expectations on this variable by using its own lagged value. So, the last equation in (2.3), can be written as: $\xi_t = \delta(x_{t-1} - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi\xi_{t-1}$. Differently, concerning the balance sheet size growth and the UMP/CMP proxy, it seems that the data generating process of these variables is an MA(1) and ARIMA (4,1,2) with the second lag as a specific lag for the Moving Average component, respectively; therefore, we proxy for these variables by using the current fitted value, obtained from the estimation of these models.
Moreover, the proxies are net of their own average value in order to ensure a zero unconditional mean for the ξ_t process.

Finally, as argued by Engle and Lee (1999)), the coefficient ψ in ξ_t process, is required to be less than β to ensure the identification of the model (for $\psi = \beta$ the ACM reduces to the AMEM with exogenous variables, AMEMX). The identification condition becomes clearer if we consider the ARMA (1,1) representation of model (2.3):

$$RV_t = [(RV_t - \mu_t) + (\alpha + \beta)RV_{t-1} - \beta(RV_{t-1} - \mu_{t-1})]\epsilon_t$$

$$RV_t = [(RV_t - \varsigma_t - \xi_t) + (\alpha + \beta)RV_{t-1} - \beta(RV_{t-1} - \varsigma_{t-1} - \xi_{t-1})]\epsilon_t$$

$$RV_t = [(RV_t - \varsigma_t) + (\alpha + \beta)RV_{t-1} - \beta(RV_{t-1} - \varsigma_{t-1}) + \beta\xi_{t-1} - \xi_t]\epsilon_t$$

By considering ξ_t as in (2.3) the ARMA representation becomes:

$$RV_t = [(RV_t - \varsigma_t) + (\alpha + \beta)RV_{t-1} - \beta(RV_{t-1} - \varsigma_{t-1}) + \beta\epsilon_t - \delta x_{t-1} - \varphi\Delta_t + \\ +(\beta - \psi)\xi_{t-1}]\epsilon_t$$

Since one of the main aims pursued by ECB by means of unconventional policies is to stabilize financial markets in the short run, we expect an immediate effect of this kind of policy in reducing stock market volatility. In other words, in our model, the part of volatility depending directly on unconventional policies represents the short run component of realized volatility as well as the proper volatility dynamics represents the long run component. Basing on this assumption, following Engle and Lee (1999) we expect the long run component has a higher persistence than the short one, that is $0 < \psi < \beta < 1$. It follows that this condition identifies the model since if it is not the case, the two components would be interchangeable (Engle and Lee (1999)).

It is important to underline that ξ_t is an unobservable signal, with a proper dynamics, which represents the part of the conditional mean of realized volatility due to the unconventional policies. After estimation we will obtain an inference on this signal, so it will be possible to quantify and plot the effect of the ECB unconventional actions on the volatility RV_t. Moreover, we can also measure the weight of the volatility depending directly on unconventional policies with respect to the general level of volatility, which is simply given by $1 - \frac{\varsigma_t}{\mu_t} = \frac{\xi_t}{\mu_t}$.

The estimation procedure is based on the quasi maximum likelihood estimator, so that the estimators of the unknown coefficients in (2.3) are consistent and asymptotically normal, as shown by Engle (2002) for the MEM case. As discussed by Engle and Gallo (2006), even if the Gamma distribution is not appropriate for ϵ_t, this procedure gives us consistent and efficient estimators (given the Quasi-Maximum likelihood interpretation); if θ is unknown (as usual), robust standard errors will shield against the shape of the Gamma distribution.

Importantly, since ξ_t is an unobservable signal, it is impossible to say for sure whether its impact on the general level of volatility should be considered in an additive way, as in (2.3), rather than in a multiplicative way, as specified in Brownlees et al. (2011). If the latter is the case, the identification of the model requires also that the unconditional mean of ξ_t is equal to one: in what follows we discuss two different specifications of the Multiplicative ACM which ensure the compliance with this constraint.

Logistic ACM. In the first specification we allow ξ_t to impact on RV_t through a logistic function - and by means of the delta method. The model, called Logistic-ACM (L-ACM), is specified as in (2.4)

$$
\begin{aligned}
&RV_t = \mu_t \epsilon_t, \;\; \epsilon_t | \Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta}) \\
&\mu_t = \varsigma_t 2 \tfrac{exp(\xi_t)}{1+exp(\xi_t)} \\
&\varsigma_t = \omega + \alpha RV_{t-1} + \beta \varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\
&\xi_t = \delta(E\left(x_t | \Psi_{t-1}\right) - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi \xi_{t-1}
\end{aligned} \tag{2.4}
$$

Linear ACM. Alternatively, we ensure the compliance with the unit mean constraint of ξ_t by adding a constant term in its equation. Within this specification, named Linear-ACM (Li-ACM), our model is given by:

$$
\begin{aligned}
&RV_t = \mu_t \epsilon_t, \;\; \epsilon_t | \Psi_{t-1} \sim Gamma(\vartheta, \tfrac{1}{\vartheta}) \\
&\mu_t = \varsigma_t \xi_t \\
&\varsigma_t = \omega + \alpha RV_{t-1} + \beta \varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\
&\xi_t = (1-\psi) + \delta E\left(x_t | \Psi_{t-1}\right) - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi \xi_{t-1}
\end{aligned} \tag{2.5}
$$

once again, in both cases $\bar{x}$ and $\bar{\Delta}$ represent the mean value of the proxies for the short and long run effect, respectively.

A way to summarize the difference in dynamics with respect to the policy actions across model specifications is to derive the marginal effects of a change in x_{t-1} (respectively, Δ_t) on $\mu_{t+\tau}$ (τ-steps), as done in Table 2.2 (cf. Appendix A.2 for the derivation).

TABLE 2.2: Marginal effects of policy variables on $\mu_{t+\kappa}$.

Model	Marginal effect on $\mu_{t+\kappa}$
AMEMX	$\kappa\beta^{\tau}$
ACM	$\kappa\psi^{\tau}$
L-ACM	$2\varsigma_{t+\tau}\kappa\psi^{\tau}\frac{exp(\xi_{t+\tau})}{(1+exp(\xi_{t+\tau}))^2}$
Li-ACM	$\kappa\psi^{\tau}\varsigma_{t+\tau}$

Note: $\kappa = \delta$ for the marginal effects of x_{t-1}
$\kappa = \varphi$ for the marginal effects of Δ_t

2.4 Estimation results

In this section, we present the estimation results according to the estimated models.

The estimation of the AMEM, which we use as guideline, gives us similar results for all countries, as shown in Table 2.3. For all the models coefficients are highly significant (at 1% level) and there is a high level of persistence, calculated as ($\alpha + \beta + \gamma/2$), ranging between 0.92 (Italy) and 0.94 (France); the impact of the news, represented by the coefficients α and γ, seems strong (around 0.2 with an increase around 0.1 in presence of negative news). In the same table we show the Ljung-Box statistics for lags 1, 5 and 10, showing how the AMEM is able to capture the autoregressive structure of the volatilities of the models (in fact we do not reject the null of uncorrelation at 1% significance, excluding Germany at the higher lag and Spain at lag 5).

In a first analysis concerning the impact of unconventional policies on stock market volatility, we simply estimate an AMEM with exogenous variables (AMEMX). Estimation results are shown in Tables 2.4-2.6.
According to other researches in literature (Bomfim, 2003; Chan and Gray, 2018; Shogbuyi and Steeley, 2017) the coefficient φ of the dummy variable has a positive sign, meaning that, on days in which there is a monetary policy communication, there is an immediate reaction in the market with a clear increase of volatility (Table 2.4) between 1.067 (1.057 if we consider the UMP/CMP proxy, Table 2.5) points for the strongest market (Germany) and 2.059 (2.062) points for Italy, which, together with Spain, represents one of the most sensible market to this kind of policy.

As expected, the long run effect has a negative sign in all the countries, meaning that unconventional policies successfully reduce stock market volatility in the long run. Here again, the effect is stronger for Italy but, in general, the difference between core and peripheral countries is remarkable. A unit increase of the ratio between amount employed for unconventional policies and ECB's total asset leads to a reduction in realized volatility of between 0.454 and 0.636 points for Germany and France, whereas it is around 1.07 and 1.02 points for Italy and Spain. respectively.

Therefore, considering the significance of the coefficients representing the short term and the long term effect of the unconventional policies, a first important result is that unconventional polices have significantly reduced stock market volatility during the sample period.

TABLE 2.3: Model Estimation Results AMEM

a)

	France	**Germany**	**Italy**	**Spain**
ω	0.857 ***	0.957 ***	1.198 ***	1.055 ***
	(0.046)	(0.017)	(0.073)	(0.075)
α	0.171 ***	0.193 ***	0.268 ***	0.224 ***
	(0.017)	(0.01)	(0.018)	(0.013)
β	0.708 ***	0.692 ***	0.608 ***	0.671 ***
	(0.019)	(0.011)	(0.023)	(0.007)
γ	0.113 ***	0.087 ***	0.084 ***	0.078 ***
	(0.008)	(0.006)	(0.005)	(0.021)
θ	7.559 ***	9.46 ***	10.593 ***	9.149 ***
	(0.222)	(0.29)	(0.423)	(0.317)
b)				
Ljung-Box 1 lag	0.155	0.171	0.299	0.01 **
Ljung-Box 5 lag	0.111	0.028 *	0.497	0.009 ***
Ljung-Box 10 lag	0.115	0.002 ***	0.336	0.047 *

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2686

TABLE 2.4: Model Estimation Results AMEMX. Proxy: UMP/TA

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.136 ***	1.081 ***	1.708 ***	1.533 ***
	(0.121)	(0.052)	(0.221)	(0.11)
α	0.165 ***	0.191 ***	0.274 ***	0.205 ***
	(0.020)	(0.011)	(0.032)	(0.014)
β	0.689 ***	0.684 ***	0.568 ***	0.656 ***
	(0.029)	(0.016)	(0.043)	(0.02)
γ	0.120 ***	0.090 ***	0.086 ***	0.087 ***
	(0.009)	(0.011)	(0.011)	(0.007)
δ	-0.636 ***	-0.454 ***	-1.074 ***	-1.025 ***
	(0.075)	(0.035)	(0.161)	(0.086)
φ	1.297 ***	1.067 ***	2.059 ***	1.964 ***
	(0.381)	(0.311)	(0.471)	(0.361)
θ	7.728 ***	9.610 ***	11.030 ***	9.487 ***
	(0.228)	(0.298)	(0.427)	(0.338)
b)				
Ljung-Box 1 lag	0.258	0.262	0.691	0.025 **
Ljung-Box 5 lag	0.167	0.063 *	0.721	0.015 **
Ljung-Box 10 lag	0.137	0.004 ***	0.472	0.086 *

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P-values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2686

TABLE 2.5: Model Estimation Results AMEMX. Proxy: UMP/CMP

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.14 ***	0.993 ***	1.735 ***	1.6 ***
	(0.101)	(0.002)	(0.14)	(0.13)
α	0.157 ***	0.202 ***	0.271 ***	0.2
	(0.016)	(0.012)	(0.023)	(0.013)
β	0.696 ***	0.681 ***	0.569 ***	0.656 ***
	(0.024)	(0.011)	(0.031)	(0.02)
γ	0.122 ***	0.088 ***	0.086 ***	0.089 ***
	(0.009)	(0.006)	(0.008)	(0.008)
δ	-0.232 ***	-0.156 ***	-0.39 ***	-0.391 ***
	(0.024)	(0.011)	(0.038)	(0.036)
φ	1.286 ***	1.057 ***	2.062 ***	1.968 ***
	(0.308)	(0.28)	(0.421)	(0.377)
θ	7.742 ***	9.82 ***	11.043 ***	9.498 ***
	(0.228)	(0.312)	(0.43)	(0.338)
b)				
Ljung-Box 1 lag	0.258	0.575	0.702	0.032 **
Ljung-Box 5 lag	0.178	0.222	0.746	0.017 **
Ljung-Box 10 lag	0.122	0.092 *	0.482	0.087 *

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 2.6: Model Estimation Results AMEMX. Proxy: TAgrowth

a)

	France	**Germany**	**Italy**	**Spain**
ω	0.907 ***	0.9 ***	1.32 ***	1.119 ***
	(0.055)	(0.246)	(0.23)	(0.063)
α	0.166 ***	0.208 ***	0.277 ***	0.214 ***
	(0.013)	(0.03)	(0.034)	(0.021)
β	0.706 ***	0.683 ***	0.591 ***	0.675 ***
	(0.017)	(0.05)	(0.042)	(0.024)
γ	0.118 ***	0.086 ***	0.085 ***	0.082 ***
	(0.008)	(0.008)	(0.021)	(0.006)
δ	-1.427	-1.298	-1.838	-0.874
	(1.185)	(4.878)	(3.296)	(0.64)
φ	1.492 ***	1.224 ***	2.275 ***	2.255 ***
	(0.324)	(0.374)	(0.437)	(0.365)
θ	7.625 ***	9.74 ***	10.779 ***	9.259 ***
	(0.219)	(0.304)	(0.429)	(0.315)
b)				
Ljung-Box 1 lag	0.173	0.455	0.45	0.011 **
Ljung-Box 5 lag	0.153	0.195	0.781	0.012 **
Ljung-Box 10 lag	0.122	0.087 *	0.518	0.063 *

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

2.4.1 ACM results

In Tables 2.7-2.9 estimation results for the ACM are shown. It is interesting to underline as the coefficient β, in general, changes slowly with respect to the AMEM or the AMEMX, whereas α decreases and γ increases; in particular, the maximum decrease of α is of about 14% for Spain, with respect to the AMEM; it is of about 10% for Italy, with respect to the AMEMX.

Our proxies are significant at 1% level and enter the model with the expected sign. In both cases the higher impact is observed for peripheral countries: in average, the short run and the long run effects for Italy is higher with respect to France and Germany of about 25% and 77%, respectively; this increment is even more remarkable when we consider the long term effect for Spain: it is of about 54% and 113% with respect to France and Germany, respectively. In addition to the significance of our proxy, the impact of unconventional policies on stock market volatility is evident also by looking at the weight of the part of volatility depending directly on this kind of policy with respect to the general level of volatility. More specifically, it derives how the reduction of volatility caused by ECB's unconventional policies is between -0.6% and -1.4%, for Germany and Spain, respectively. The signal representing the effect of unconventional policies seems not to have an autoregressive dynamics, since the AR coefficient is never significant at 1% level (it is significant at 10% level only for France).

Similar results are obtained when we estimate the model by using the other proxy, UMP/CMP. The sign and the significance of the dummy variable Δ_t do not change; here too, the lowest parameter is recorded for Germany (2.288) and the highest for Italy (3.454). Even in this case, the proxy for the long run effect enters the model with a negative sign but with a remarkably lower magnitude. The highest effect is now about -1.066, observed for Spain, meaning that the increase of securities held for unconventional policies purpose, relative to that held for conventional purpose, leads to a reduction of the realized volatility of about 1 point. This long run effect can be also seen in Figures 2.3 and 2.4, which plot the evolution of the volatility components (ς, the blue line, and ξ, the red dotted-line). In both cases, this effect is more evident starting from October 2014 - when the ECB implicitly communicated to the market that it would have purchased also private sector bond (as well as government bond) - coming in the form of change in the slope of ξ equation (red line[7]), which lasts for the entire period of the programme. This

[7]As can be seen, the line representing the ξ process is defined in three different level: the lowest depends on the characteristic path of the part of volatility depending directly

is, perhaps, the most interesting result since it should mean that the quantity of securities held for monetary purpose, related to both total asset and securities held for conventional policies purpose, is actually crucial for the effectiveness of these unconventional policies. In addition, in April 2017 it can be observed an increase in this volatility component, which is probably due to the reduction of the amount of securities purchased by ECB, falling down to €60 billion from the previous level of €80 billion.

Lastly, we deserve a special mention to the third proxy, the balance sheet size growth, which is defined as a quantitative proxy (Voutsinas and Werner, 2011). Looking at results (Table 2.9), it is important to notice how it enters the model with a positive sign. Therefore, even though it is never significant, an increase of the ECB's total assets causes a boost in stock market volatility.

Results remain almost unchanged when we consider the multiplicative specifications (Tables 2.10-2.15). Once again the qualitative proxies enter the model with the expected sign and with the highest level of significance. In both cases, coefficients are pretty lower, however it should depend from the fact that here unconventional policies affect volatility in a multiplicative way: in other words, whereas in the additive version of the ACM ξ_t is generally negative - so that higher proxies' coefficients will reduce stock market volatility - within the multiplicative versions ξ_t is always positive, therefore unconventional policies depress volatility if the coefficients associated to the proxies are less than 1. For this reason it could be interesting to make a first comparison between ACM models by contrasting the resulting marginal effects. Marginal effects are equal to the first partial derivative of μ_t with respect to the proxy, $\frac{UMP}{TA}_{t-1}$ (Δ_t). Whether, as shown in Table 2.2, in the additive specification their are constant and equal to the estimated coefficients, in multiplicative specifications we have time varying marginal effects - i.e. they depend on ς and/or on φ.

Looking at the average value (Table 2.16[8]), it emerges how, in both cases, the higher marginal effect derives from the linear version of the multiplicative ACM. More specifically, a unit increase in the UMP/TA proxy leads to a reduction in realized volatility between -1.577 (Germany) and -3.382 (Spain) points; conversely, on announcement days volatility increases, in average, by 2.603 and 4.146 points, for Germany and Spain, respectively. In sum, once

on unconventional policies; the highest, instead, represents volatility spikes due to the announcement effect; finally, the intermediate line, represents the level of volatility on days after a monetary policy announcement, when volatility comes back to its previous level.

[8]It should be noticed that, for what concerns Δ_t, marginal effects are considered only with respect to announcement days. Therefore, before computing the average value, we multiplied the marginal effect of Δ_t by Δ_t itself.

again, it seems unconventional policies have an higher impact on peripheral countries, although the effect is remarkable also in the core ones. The evolution of marginal effects deriving from the two specifications of the multiplicative model is shown in Figures 2.5 and 2.6: for both models and for all the countries, marginal effects have a specular behaviour and it is interesting to notice how they follow the behaviour of the realized volatility measure. In particular, it could be observed how the marginal effect line of the announcement variable has picks in correspondence of volatility spike, whereas the period of lower marginal effects of the qualitative proxy, corresponds to period of low volatility of the market. Of course, it represents a further sign of the validity of our model and of the proxies we use.

Even in the multiplicative specifications, the quantitative proxy is never significant; this allow us to conclude saying that - as argued Curdia and Woodford (2011) - our results support the idea that what matters for unconventional policies to be effective in reducing stock market volatility is the balance sheet composition, not the size.

In statistical terms, the ACM improves the performance in terms of Ljung-Box statistics, so that all the models seem to capture the autoregressive structure of the volatility of the four markets, the only exception, once again, is Germany at the highest lag. However, for what concerns the multiplicative specifications some residuals correlation still remains for Spain at the first lag, meaning that a further adjustment is needed to capture the autoregressive nature of volatility in this country: generally, this is addressed by adding a further lag of the GARCH term (i.e. ς_{t-2} in our model); conversely, we try to overcome this problem through a Markov switching extension of the model, as largely discussed in Chapter 3.

Model comparisons (Figure 2.7) corroborates the intuition that the effectiveness of unconventional policies in reducing stock market volatility largely depends on the balance sheet composition. Comparisons are based on the information criteria (AIC and BIC) and loss functions for evaluating the forecasting capability of the models (MSE and MAE). For all the countries, the preferred model (the one lying within the black box) is the L-ACM, with unconventional policies measured through UMP/CMP.

In the next section we further compare models relying on the out-of-sample forecasting power, which will be evaluated within the Model Confidence Set framework.

TABLE 2.7: Model Estimation Results ACM. Proxy: UMP/TA

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.056 ***	1.034 ***	1.533 ***	1.438 ***
	(0.065)	(0.016)	(0.260)	(0.135)
α	0.154 ***	0.183 ***	0.249 ***	0.197 ***
	(0.015)	(0.014)	(0.033)	(0.024)
β	0.707 ***	0.696 ***	0.604 ***	0.671 ***
	(0.015)	(0.013)	(0.043)	(0.027)
γ	0.117 ***	0.089 ***	0.086 ***	0.085 ***
	(0.011)	(0.008)	(0.019)	(0.015)
δ	-1.836 ***	-1.328 ***	-2.354 ***	-2.823 ***
	(0.326)	(0.283)	(0.519)	(0.528)
φ	2.817 ***	2.317 ***	3.448 ***	3.428 ***
	(0.539)	(0.470)	(0.569)	(0.671)
ψ	0.111 *	0.098	0.051	0.058
	(0.060)	(0.077)	(0.083)	(0.090)
θ	7.817 ***	9.700 ***	11.183 ***	9.580 ***
	(0.231)	(0.301)	(0.436)	(0.349)
b)				
Ljung-Box 1 lag	0.132	0.184	0.3	0.011 **
Ljung-Box 5 lag	0.215	0.056 *	0.8	0.019 **
Ljung-Box 10 lag	0.21	0.003 ***	0.647	0.119

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2686

TABLE 2.8: Model Estimation Results ACM. Proxy: UMP/CMP

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.06 ***	0.948 ***	1.553 ***	1.504 ***
	(0.084)	(0.267)	(0.178)	(0.141)
α	0.147 ***	0.194 ***	0.246 ***	0.192 ***
	(0.018)	(0.025)	(0.024)	(0.022)
β	0.713 ***	0.693 ***	0.606 ***	0.671 ***
	(0.022)	(0.043)	(0.033)	(0.027)
γ	0.119 ***	0.087 ***	0.087 ***	0.087 ***
	(0.011)	(0.009)	(0.009)	(0.009)
δ	-0.664 ***	-0.458 **	-0.849 ***	-1.066 ***
	(0.112)	(0.181)	(0.16)	(0.165)
φ	2.823 ***	2.288 ***	3.454 ***	3.43 ***
	(0.531)	(0.447)	(0.559)	(0.624)
ψ	0.122 *	0.091	0.055	0.062
	(0.063)	(0.406)	(0.084)	(0.076)
θ	7.828 ***	9.916 ***	11.195 ***	9.59 ***
	(0.23)	(0.316)	(0.437)	(0.349)
b)				
Ljung-Box 1 lag	0.145	0.455	0.315	0.014 **
Ljung-Box 5 lag	0.227	0.716	0.842	0.024 **
Ljung-Box 10 lag	0.189	0.099 *	0.678	0.131

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 2.9: Model Estimation Results ACM. Proxy: TAgrowth

a)

	France	**Germany**	**Italy**	**Spain**
ω	0.853 ***	0.864 ***	1.199 ***	1.082 ***
	(0.12)	(0.053)	(0.118)	(0.075)
α	0.159 ***	0.202 ***	0.256 ***	0.212 ***
	(0.02)	(0.017)	(0.022)	(0.025)
β	0.718 ***	0.692 ***	0.619 ***	0.68 ***
	(0.028)	(0.019)	(0.029)	(0.024)
γ	0.116 ***	0.085 ***	0.085 ***	0.08 ***
	(0.011)	(0.011)	(0.006)	(0.013)
δ	-1.675	-1.335	-2.450	0.038
	(1.233)	(2.505)	(2.238)	(0.041)
φ	3.03 ***	2.405 ***	3.708 ***	3.711 ***
	(0.563)	(0.447)	(0.577)	(0.651)
ψ	0.138 **	0.115	0.085	0.088
	(0.065)	(0.081)	(0.062)	(0.073)
θ	7.706 ***	9.825 ***	10.922 ***	9.338 ***
	(0.221)	(0.309)	(0.437)	(0.325)
b)				
Ljung-Box 1 lag	0.122	0.404	0.236	0.011 **
Ljung-Box 5 lag	0.194	0.213	0.79	0.025 **
Ljung-Box 10 lag	0.156	0.082 *	0.586	0.107

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b) P–values for** Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 2.10: Model Estimation Results L-ACM. Proxy: UMP/TA

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.011 ***	1.026 ***	1.480 ***	1.410 **
	(0.007)	(0.059)	(0.239)	(0.621)
α	0.151 ***	0.182 ***	0.248 ***	0.193 ***
	(0.015)	(0.012)	(0.042)	(0.060)
β	0.712 ***	0.696 ***	0.605 ***	0.674 ***
	(0.014)	(0.014)	(0.049)	(0.091)
γ	0.119 ***	0.090 ***	0.089 ***	0.086 ***
	(0.010)	(0.008)	(0.022)	(0.017)
δ	-0.297 ***	-0.219 ***	-0.343 ***	-0.389 ***
	(0.057)	(0.052)	(0.048)	(0.133)
φ	0.464 ***	0.378 ***	0.518 ***	0.501 ***
	(0.093)	(0.073)	(0.084)	(0.087)
ψ	0.194 **	0.175 *	0.089	0.123
	(0.081)	(0.097)	(0.055)	(0.298)
θ	7.827 ***	9.719 ***	11.185 ***	9.626 ***
	(0.230)	(0.301)	(0.437)	(0.355)
b)				
Ljung-Box 1 lag	0.108	0.173	0.287	0.006 ***
Ljung-Box 5 lag	0.192	0.051 *	0.805	0.012 **
Ljung-Box 10 lag	0.201	0.003 ***	0.625	0.089 *

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2686

TABLE 2.11: Model Estimation Results L-ACM. Proxy: UMP/CMP

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.025 ***	0.946 ***	1.522 ***	1.493 ***
	(0.017)	(0.102)	(0.168)	(0.146)
α	0.143 ***	0.192 ***	0.245 ***	0.186 ***
	(0.014)	(0.015)	(0.025)	(0.019)
β	0.717 ***	0.693 ***	0.605 ***	0.674 ***
	(0.013)	(0.019)	(0.033)	(0.025)
γ	0.121 ***	0.088 ***	0.089 ***	0.089 ***
	(0.011)	(0.009)	(0.009)	(0.009)
δ	-0.119 ***	-0.076 ***	-0.133 ***	-0.155 ***
	(0.021)	(0.016)	(0.019)	(0.025)
φ	0.466 ***	0.374 ***	0.518 ***	0.499 ***
	(0.091)	(0.076)	(0.083)	(0.089)
ψ	0.184 **	0.203 ***	0.093	0.136
	(0.082)	(0.078)	(0.058)	(0.091)
θ	7.849 ***	9.947 ***	11.216 ***	9.654 ***
	(0.23)	(0.315)	(0.439)	(0.354)

b)

Ljung-Box 1 lag	0.117	0.441	0.312	0.008 ***
Ljung-Box 5 lag	0.205	0.237	0.85	0.015 **
Ljung-Box 10 lag	0.197	0.097 *	0.66	0.1

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 2.12: Model Estimation Results L-ACM. Proxy: TAgrowth

a)

	France	**Germany**	**Italy**	**Spain**
ω	0.828 ***	0.847 ***	0.176 ***	1.038 ***
	(0.085)	(0.26)	(0.055)	(0.044)
α	0.156 ***	0.199 ***	0.255 ***	0.206 ***
	(0.018)	(0.017)	(0.019)	(0.019)
β	0.723 ***	0.695 ***	0.621 ***	0.688 ***
	(0.024)	(0.038)	(0.021)	(0.016)
γ	0.116 ***	0.085 ***	0.086 ***	0.081 ***
	(0.011)	(0.015)	(0.01)	(0.012)
δ	0.024	-0.07	-0.096	0.203
	(0.026)	(0.389)	(0.213)	(0.127)
φ	0.489 ***	0.387 ***	0.542 ***	0.526 ***
	(0.089)	(0.134)	(0.081)	(0.092)
ψ	0.211 ***	0.234	0.127 **	0.173
	(0.061)	(0.341)	(0.062)	(0.112)
θ	7.716 ***	9.845 ***	10.944 ***	9.385 ***
	(0.22)	(0.307)	(0.44)	(0.323)
b)				
Ljung-Box 1 lag	0.101	0.36	0.224	0.005 ***
Ljung-Box 5 lag	0.174	0.194	0.787	0.014 **
Ljung-Box 10 lag	0.154	0.082 *	0.587	0.08 *

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 2.13: Model Estimation Results Li-ACM. Proxy: UMP/TA

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.025 ***	1.034 **	1.490 ***	1.426 ***
	(0.053)	(0.519)	(0.146)	(0.132)
α	0.153 ***	0.183 ***	0.250 ***	0.195 ***
	(0.017)	(0.055)	(0.022)	(0.023)
β	0.709 ***	0.694 ***	0.603 ***	0.671 ***
	(0.018)	(0.094)	(0.030)	(0.028)
γ	0.119 ***	0.090 ***	0.088 ***	0.086 ***
	(0.012)	(0.008)	(0.008)	(0.009)
δ	-0.161 ***	-0.115 **	-0.178 ***	-0.207 ***
	(0.029)	(0.057)	(0.031)	(0.037)
φ	0.231 ***	0.188 **	0.254 ***	0.247 ***
	(0.045)	(0.073)	(0.038)	(0.042)
ψ	0.134	0.138	0.058	0.072
	(0.083)	(0.204)	(0.073)	(0.112)
θ	7.820 ***	9.714 ***	11.181 ***	9.623 ***
	(0.231)	(0.301)	(0.436)	(0.352)

b)

Ljung-Box 1 lag	0.125	0.194	0.324	0.008 ***
Ljung-Box 5 lag	0.207	0.054 *	0.824	0.015 **
Ljung-Box 10 lag	0.201	0.003 ***	0.624	0.1

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2686

TABLE 2.14: Model Estimation Results Li-ACM. Proxy: UMP/CMP

a)

	France	**Germany**	**Italy**	**Spain**
ω	1.035 ***	0.958 ***	1.534 ***	1.514 ***
	(0.027)	(0.029)	(0.22)	(0.166)
α	0.144 ***	0.195 ***	0.247 ***	0.189 ***
	(0.014)	(0.015)	(0.03)	(0.024)
β	0.715 ***	0.69 ***	0.603 ***	0.67 ***
	(0.012)	(0.015)	(0.041)	(0.03)
γ	0.122 ***	0.087 ***	0.089 ***	0.088 ***
	(0.01)	(0.01)	(0.012)	(0.012)
δ	-0.063 ***	-0.042 ***	-0.069 ***	-0.083 ***
	(0.01)	(0.009)	(0.013)	(0.013)
φ	0.231 ***	0.188 ***	0.254 ***	0.247 ***
	(0.041)	(0.036)	(0.039)	(0.042)
ψ	0.141 **	0.132 *	0.06	0.076
	(0.057)	(0.077)	(0.094)	(0.084)
θ	7.844 ***	9.938 ***	11.213 ***	9.65 ***
	(0.231)	(0.316)	(0.439)	(0.354)
b)				
Ljung-Box 1 lag	0.137	0.498	0.356	0.011 **
Ljung-Box 5 lag	0.22	0.251	0.873	0.019 **
Ljung-Box 10 lag	0.196	0.093 *	0.661	0.117

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 2.15: Model Estimation Results Li-ACM. Proxy: TAgrowth

a)

	France	**Germany**	**Italy**	**Spain**
ω	0.837 ***	0.859 ***	1.187 ***	1.057 ***
	(0.113)	(0.084)	(0.103)	(0.074)
α	0.158 ***	0.202 ***	0.258 ***	0.21 ***
	(0.021)	(0.02)	(0.021)	(0.023)
β	0.72 ***	0.692 ***	0.618 ***	0.683 ***
	(0.03)	(0.024)	(0.027)	(0.023)
γ	0.117 ***	0.085 ***	0.086 ***	0.08 ***
	(0.011)	(0.009)	(0.007)	(0.012)
δ	0.009	-0.03	-0.049	0.108
	(0.021)	(0.033)	(0.164)	(0.08)
φ	0.244 ***	0.197 ***	0.267 ***	0.262 ***
	(0.043)	(0.035)	(0.039)	(0.044)
ψ	0.163 *	0.155 ***	0.088	0.098
	(0.089)	(0.045)	(0.076)	(0.079)
θ	7.708 ***	9.834 ***	10.937 ***	9.377 ***
	(0.221)	(0.308)	(0.439)	(0.324)
b)				
Ljung-Box 1 lag	0.125	0.421	0.272	0.009 ***
Ljung-Box 5 lag	0.193	0.208	0.828	0.02 **
Ljung-Box 10 lag	0.152	0.076 *	0.589	0.1

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** P–values for Ljung-Box statistics. P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

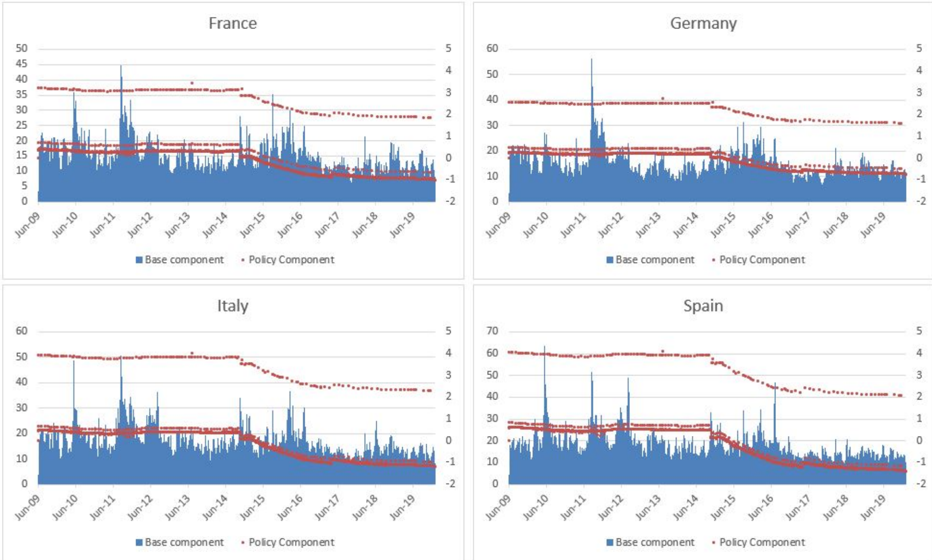

FIGURE 2.3: France, Germany, Italy and Spain Base and Policy volatility component obtained from ACM. Proxy: UMP/TA. Sample period: June 1, 2009 - December 31, 2019

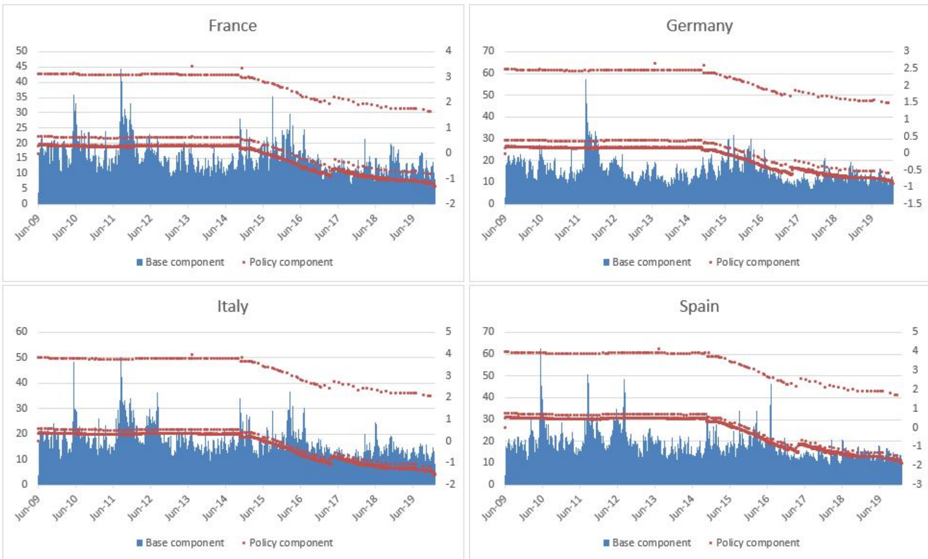

FIGURE 2.4: France, Germany, Italy and Spain Base and Policy volatility component obtained from ACM. Proxy: UMP/CMP. Sample period: June 1, 2009 - December 31, 2019

TABLE 2.16: Average marginal effects of policy variables on μ_t – Sample period: June 1, 2009 - December 31, 2019 (parameter estimates as in Tables 2.7–2.15)

	Marginal effect of $\frac{UMP}{TA}_{t-1}$ on μ_t			Marginal effect of $\Delta_t$on μ_t		
	ACM	L-ACM	Li-ACM	ACM	L-ACM	Li-ACM
France	-1.836	-1.571	-2.172	2.817	2.236	3.177
Germany	-1.328	-1.158	-1.577	2.317	1.821	2.603
Italy	-2.354	-2.033	-2.693	3.448	2.785	3.953
Spain	-2.823	-2.489	-3.382	3.428	2.912	4.146

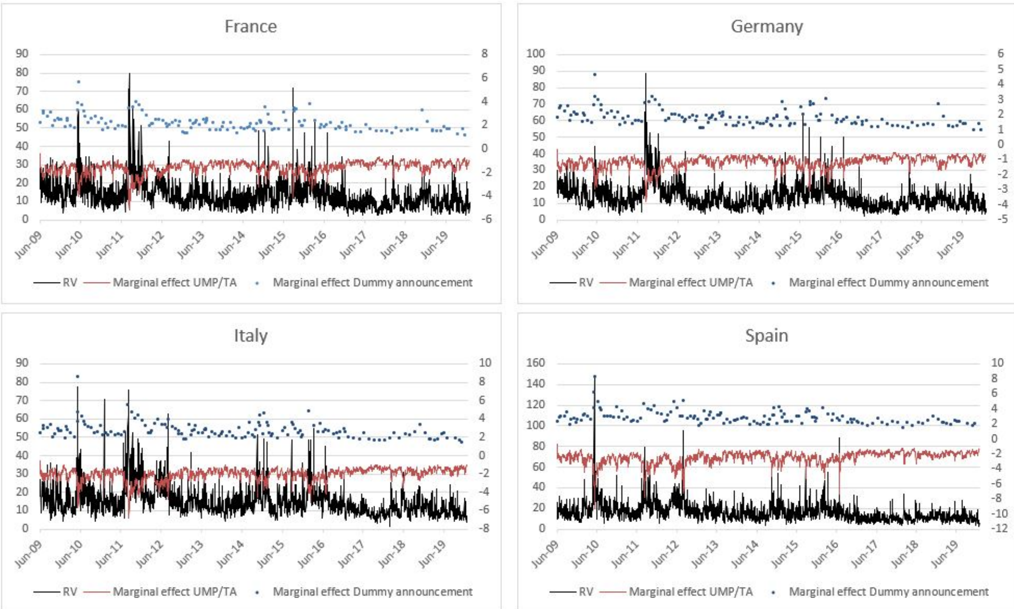

FIGURE 2.5: Marginal effects UMP/TA. Model: L-ACM. Sample period: June 1, 2009 - December 31, 2019

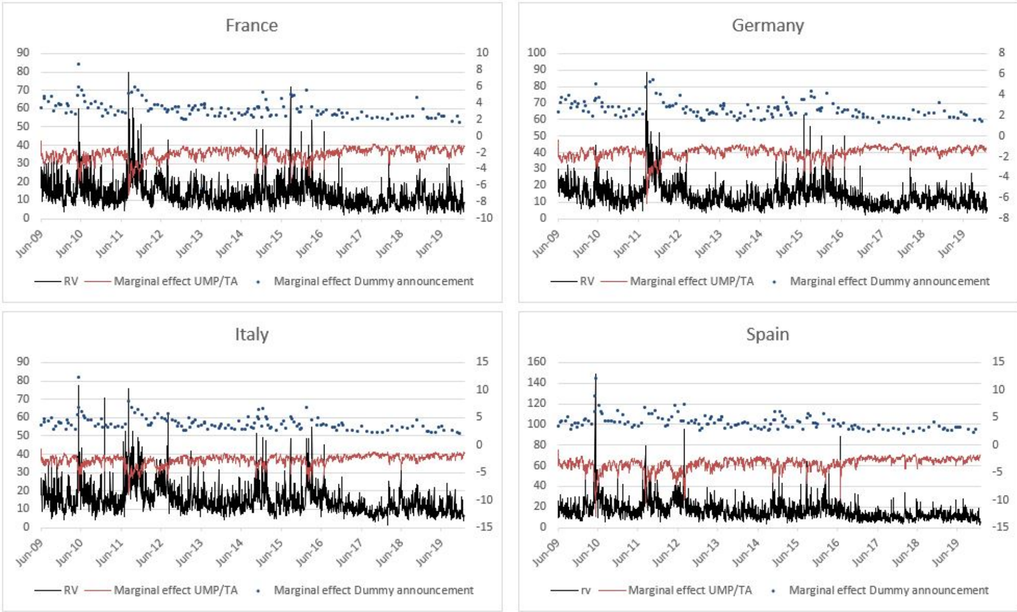

FIGURE 2.6: Marginal effects UMP/TA. Model: Li-ACM. Sample period: June 1, 2009 - December 31, 2019

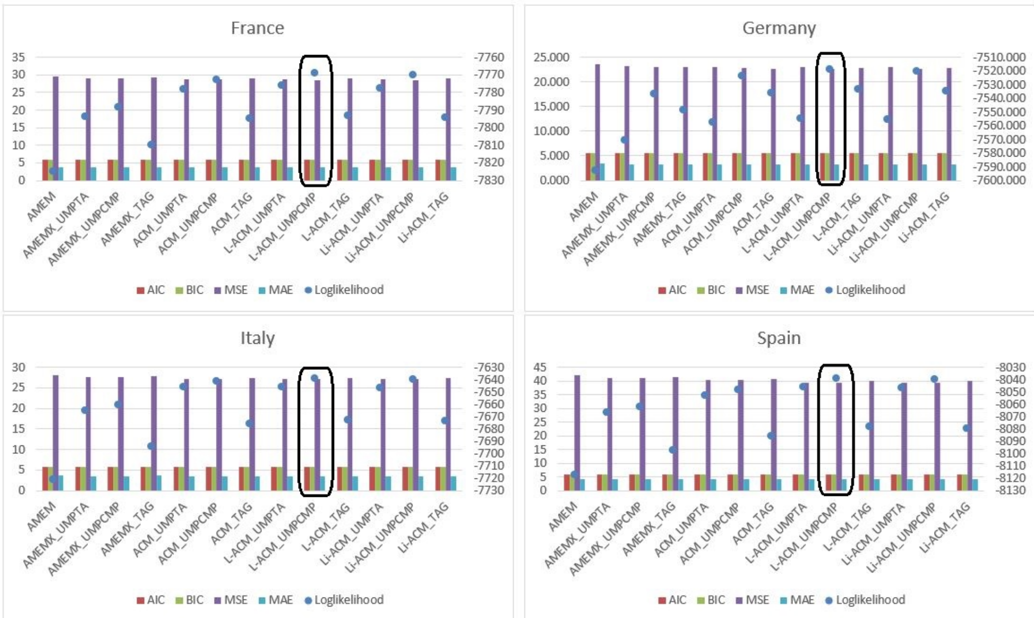

FIGURE 2.7: Model comparisons

Note: Model comparisons via Information Criteria (AIC and BIC) and forecasting capability (MSE and MAE) – Sample period: June 1, 2009 - December 31, 2019. Best model in the black box

2.4.2 The Model Confidence Set

In this Section, we compare the forecasting capability of the estimated models by implementing the Model Confidence Set (MCS) procedure. It is defined as the procedure needed to get the set of models that consists of the best models from a larger set of models (Hansen et al. (2011)) at a given significance level (10% in our case). MCS procedure is alternative to the procedure provided by Diebold and Mariano (1995) to compare the forecasting accuracy of two alternative models (the main advantages, here, is that we can consider a full set of models). MCS procedure is based on loss differentials between model i and j, $d_{ij,t}$, testing the null hypothesis $H_0 : E(d_{ij,t}) = 0$. If the null is rejected, the worst model is excluded from the set of the best models; the procedure ends when the null is no more rejected. The test underlying the procedure is a simple t-statistic concerning the average value of $d_{ij,t}$

$$t_{ij} = \frac{\bar{d}_{ij}}{\widehat{var}(\bar{d}_{ij})}$$

where $\widehat{var}(\bar{d}_{ij})$ is the estimator of $var(\bar{d}_{ij})$, obtained via the bootstrap procedure provided by Hansen et al. (2011).

We rely on this procedure to asses the one-step-ahead forecasting power of the model, basing on Quasi-Likelihood (QLike) as well as on mean square error (MSE) loss functions, which, according to Patton (2011), are consistent. In particular, QLike and MSE are two different measure of the variability of a dataset with respect to the value estimated through a statistical model: whereas MSE is the sum of the squared forecast errors ($\frac{1}{T}\sum_{t=1}^{T}(RV_t - R\hat{V}_t)^2$), QLike is measured as $\frac{1}{T}\sum_{t=1}^{T}(\frac{RV_t}{\mu_t} - ln(\frac{RV_t}{\mu_t}) - 1)$.

For this purpose we split the sample in correspondence of three different dates making the one-step-ahead out-of-sample forecast for the following year. The choice of the splitting dates is crucial for assessing the forecasting power of our proxies: in this respect, the first date we selected is "December 31, 2014" in order to exclude the most important unconventional policy programme, the EAPP. Therefore, we decided on "December 31, 2016", when the EAPP had already been launched for 2 years so that we have enough observations to evaluate it, and on "December 31, 2018" in order to consider the full period of the programme.

As shown in Figures 2.8-2.13, both the loss functions give us similar results, the most important of which is that the EAPP programme is crucial to improve the forecasting power of our proxies. This intuition could be better explained by considering the sub-samples individually. MCS results shown in Figures 2.8 and 2.9 refer to the forecasting period ending in December 2015 (before the EAPP announcement): the AMEMX model referring to the quantitative proxy is the best model (green bar, in Figure 2.9) for France and Italy, whereas the L-ACM is the best one in the case of Germany. In addition, for the France, Italy and Spain almost all the models belong to the best set. Crucially, the AMEM is included in the best set (as represented by the blue bars, whereas the black bars represent models excluded from the best set of models) in all the cases, meaning that - before the EAPP announcement - it has the same forecasting power as more complex models which include unconventional policies proxies.

Results change drastically if we refer to the estimation period ending in December 2016 (Figures 2.10 and 2.11), in which ECB purchased assets up to €80 billion per month, causing a remarkable change in the ECB balance sheet composition. Consequently, now the qualitative proxies have the higher forecasting power and the AMEM is always excluded from the best set of models. Importantly, our model specification (ACM) belongs to the best set in all the cases and it is the best model for Germany. The importance of this programme is also confirmed by the last MCS procedure, the one referring to the estimation period ending in December 2018 (Figures 2.12 and 2.13), which include the full implementation period of the EAPP. Here, in all the cases, the best model is the Li-ACM, with the UMP/CMP as a proxy (Figure 2.13). More precisely, in 2 out of 4 cases it is the only model belonging to the best set, which includes also the AMEMX and the L-ACM for France and Italy, respectively.

In sum, it emerges how unconventional policies played a crucial role in reducing stock market volatility.

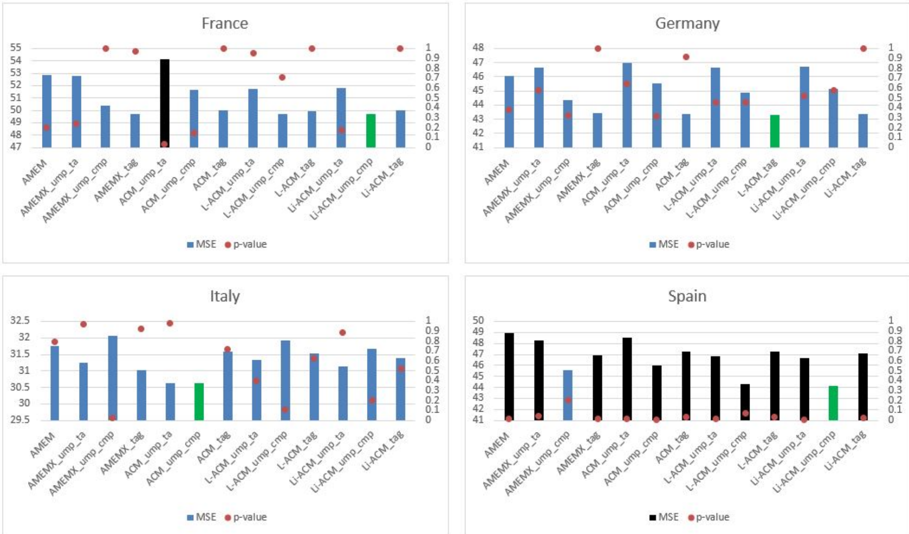

FIGURE 2.8: Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

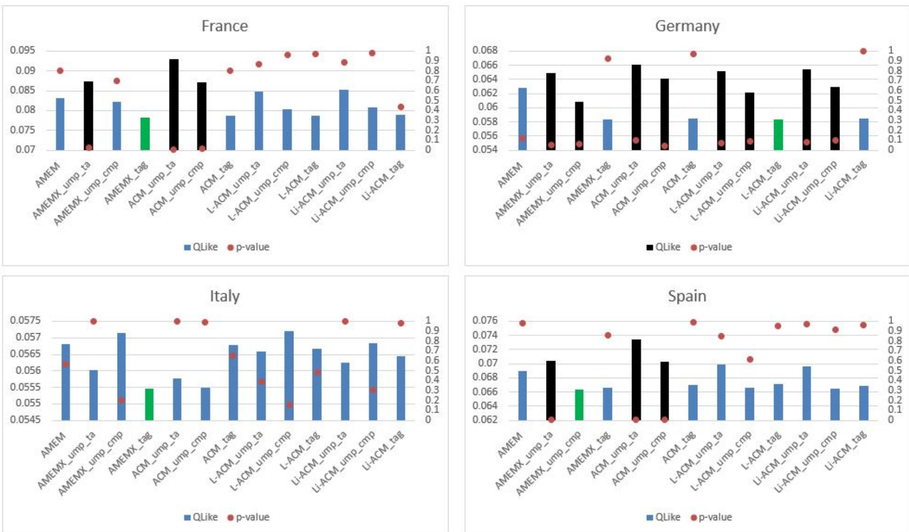

FIGURE 2.9: Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

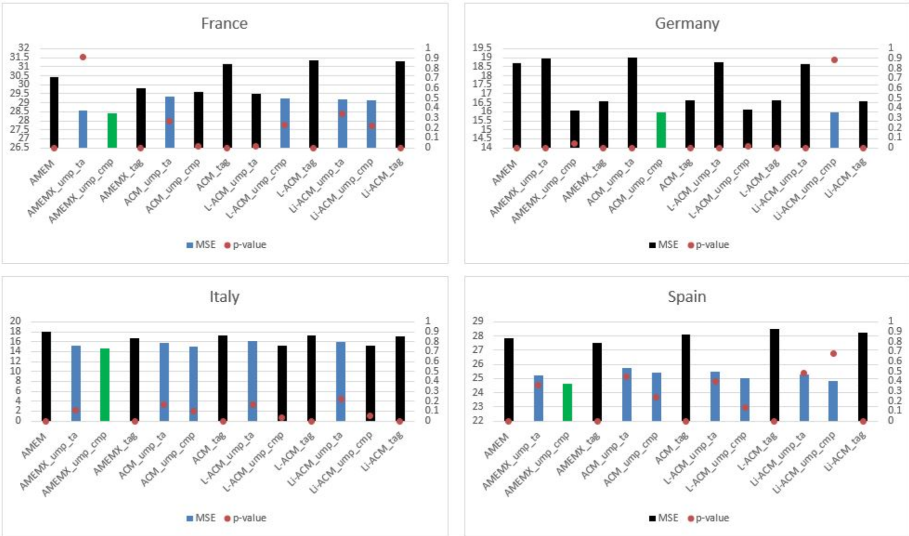

FIGURE 2.10: Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

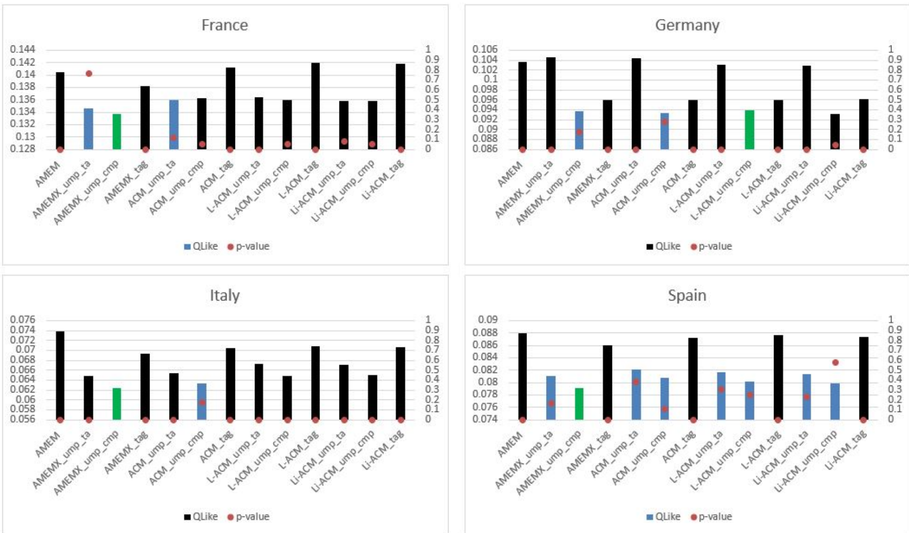

FIGURE 2.11: Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

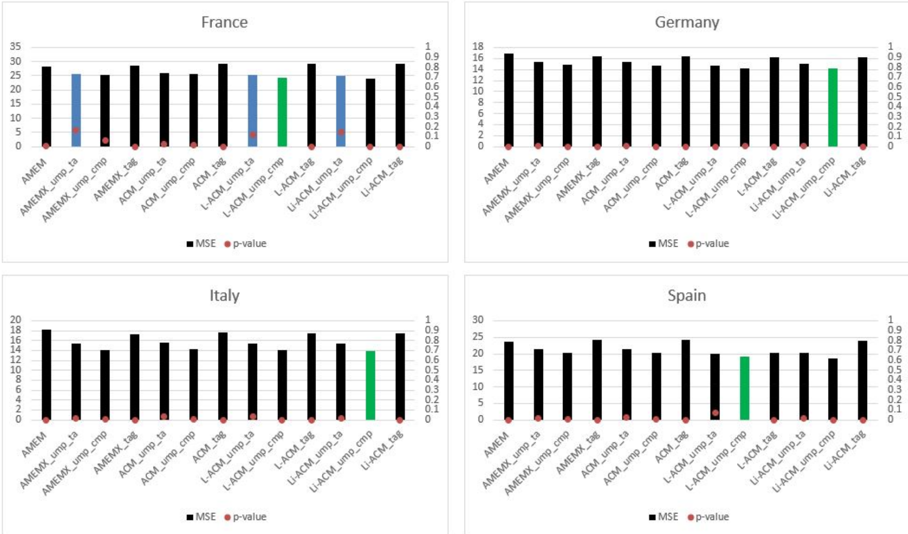

FIGURE 2.12: Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

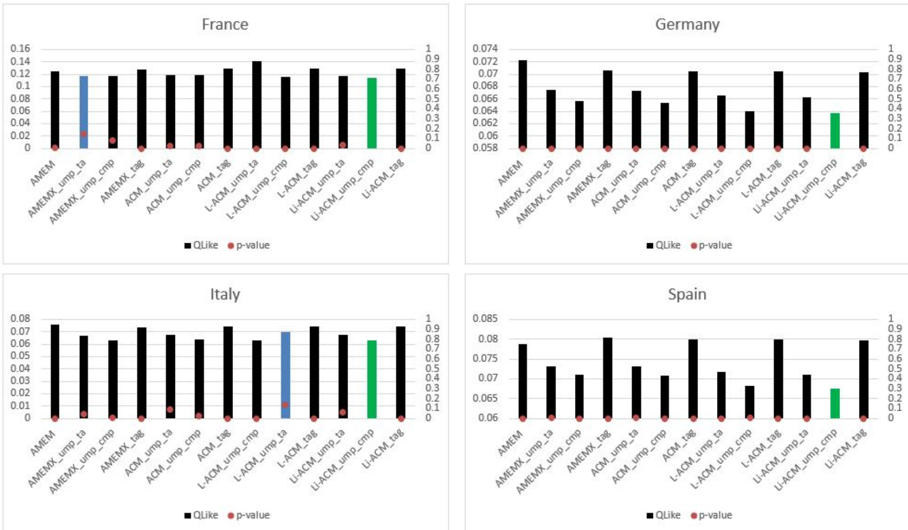

FIGURE 2.13: Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

2.4.3 How long does volatility take to converge to its long-run level?

In this section a multi-step forecasting procedure is employed to answer two crucial questions: i) how long do quantitative easing policies affect stock market volatility? ii) What would be the volatility response to a higher quantitative easing shock?

Once again, in carrying out this analysis we consider three sub-samples for the estimation period, which end in December 2014, 2016 and 2018, respectively, whereas the forecasting period refers to the following year. Results, basing on models defined in Appendix A.3 are shown in Figures 2.14 to 2.31. Of course the duration of the effects depends on the country as well as on the considered proxy. What is interesting to notice, in a first analysis, is how, before (the first sub-sample) and after (the last sub-sample) the EAPP, volatility has a downward trend: this is a further signal of the considerable effect that unconventional monetary polices had on stock market volatility.

Starting from the estimation period ending in August 2014 (Figures 2.14-2.16) the duration lasts between 30 (as estimated for the German index using the Li-ACM and the UMP/CMP proxy) and 78 days (as it results, for the same index, from the estimation of AMEMX with the UMP/TA proxy).

Results do not change significantly when we consider the estimation period ending in August 2016 (Figures 2.17-2.19). Here, the minimum (50 days) is estimated for Italy via AMEMX (UMP/TA as a proxy), whereas the maximum duration (103 days) is estimated via ACM for Germany, using the TAgrowth.

Finally, for the last estimation period (Figures 2.20-2.22), the minimum (41 days) is obtained via Li-ACM, with the UMP/TA as a proxy, for Italy, whereas the maximum (124 days) duration refers to France estimated via Li-ACM, using the TAgrowth as a proxy. As it could be noticed, independently on the duration, once the quantitative easing effect is completely absorbed by the market, the volatility converges to its long run level. Moreover, it seems that this long run level does not depend on the considered model.

Finally we analysis how volatility would have responded to a higher quantitative easing shock. For this purpose we increase the quantitative easing proxies by one standard deviation, which is 0.726 (In the case of the UMP/CMP proxy). As summarized in Figures 2.23-2.31, there is a significant reduction in the long run volatility level: focusing on the last sub-sample, for example, the average reduction is 1.52 for Germany, 1.83 for France and Italy, 2.36 for Spain. More important, in line with the no statically significance of the coefficient associated to the TAgrowth proxy, a positive shock in that variable does not lead to a significant change in the long run level of volatility.

In conclusion, once again, it appears clear the role played by unconventional policies in reducing stock market volatility as well as the fact that they are crucial tools available to Central Banks, especially in periods of interest rates close to the zero lower bound.

FIGURE 2.14: Multi-step forecast. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

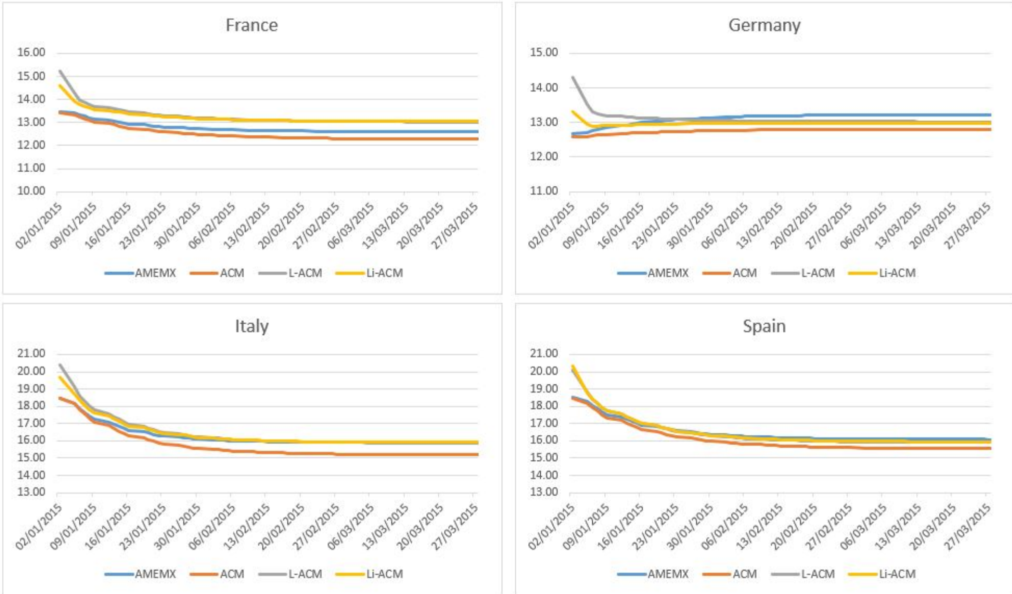

FIGURE 2.15: Multi-step forecast. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

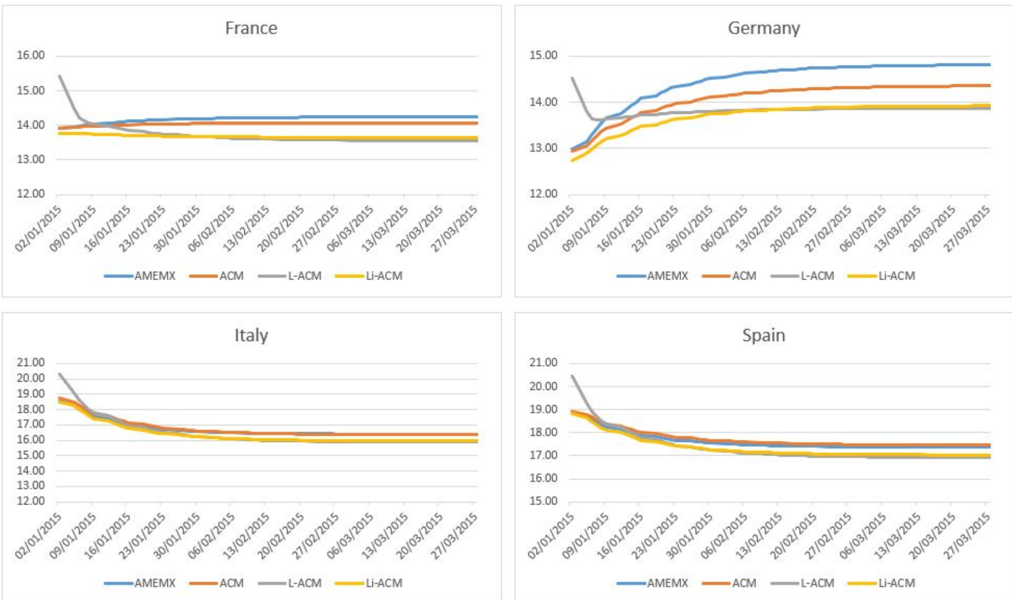

FIGURE 2.16: Multi-step forecast. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

FIGURE 2.17: Multi-step forecast. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

FIGURE 2.18: Multi-step forecast. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

FIGURE 2.19: Multi-step forecast. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

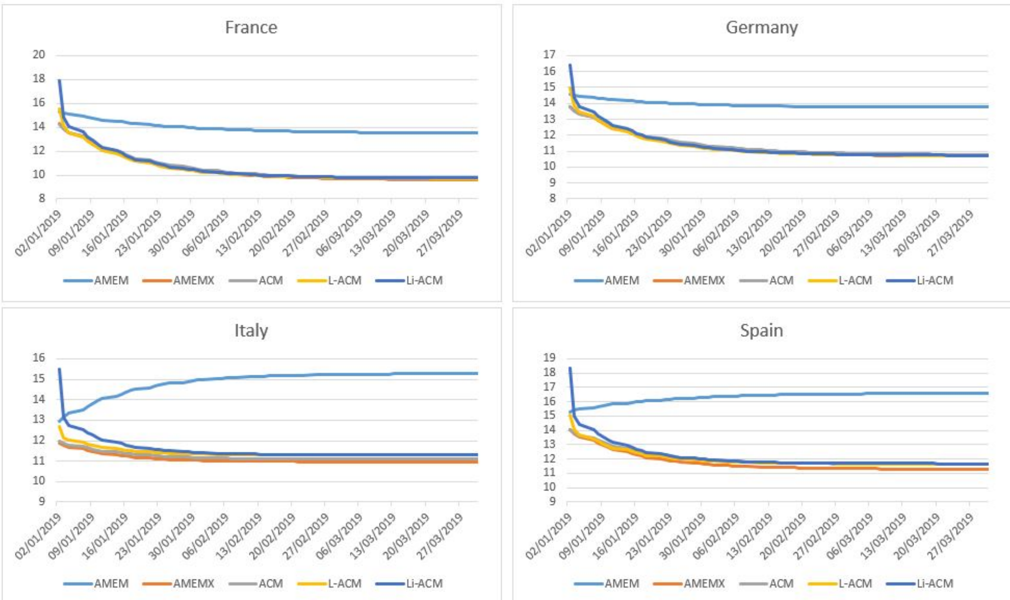

FIGURE 2.20: Multi-step forecast. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

FIGURE 2.21: Multi-step forecast. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

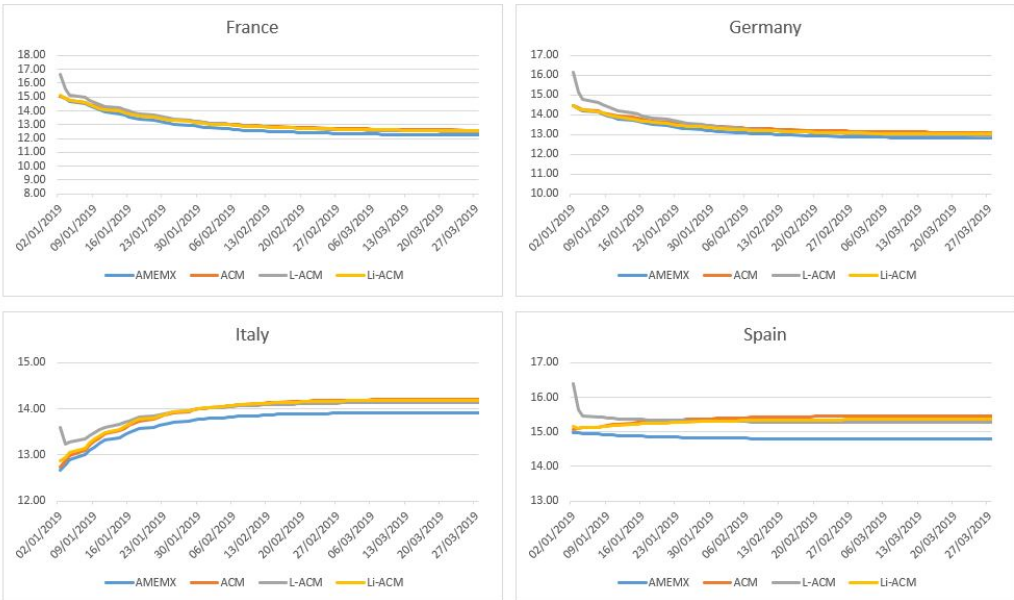

FIGURE 2.22: Multi-step forecast. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

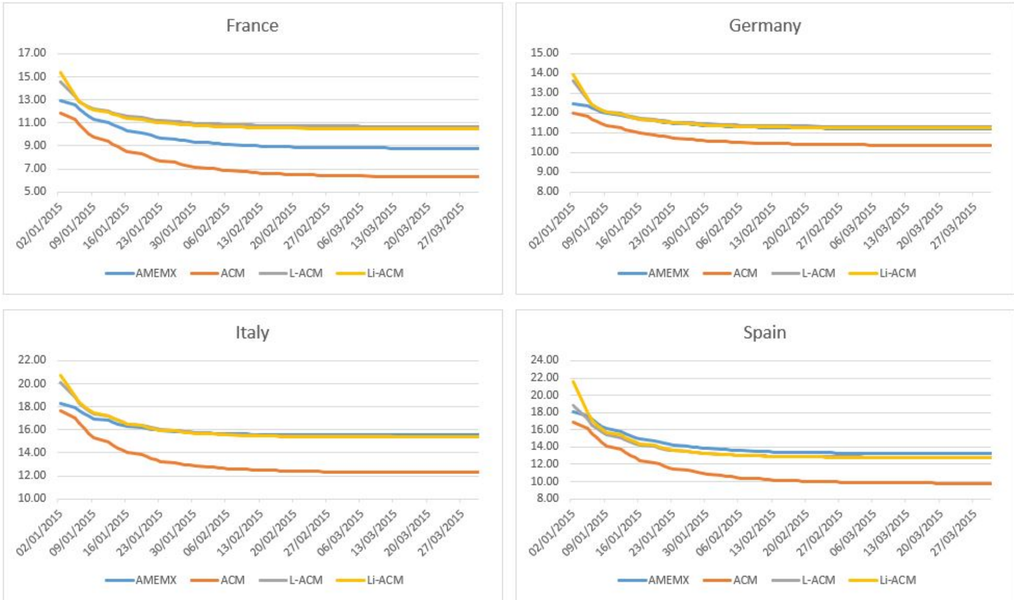

FIGURE 2.23: Multi-step forecast assuming 1sd shock. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

FIGURE 2.24: Multi-step forecast assuming 1sd shock. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

FIGURE 2.25: Multi-step forecast assuming 1sd shock. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2014. Forecast Period: Sample period: January 1, 2015 - December 31, 2015

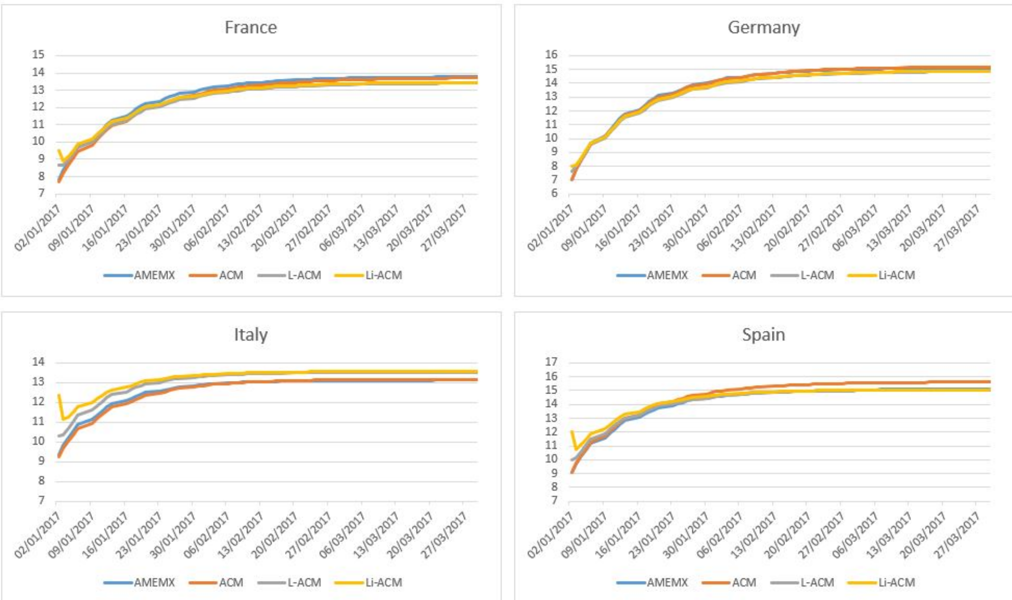

FIGURE 2.26: Multi-step forecast assuming 1sd shock. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

FIGURE 2.27: Multi-step forecast assuming 1sd shock. Proxy:UMP/CMP. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

FIGURE 2.28: Multi-step forecast assuming 1sd shock. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2016. Forecast Period: Sample period: January 1, 2017 - December 31, 2017

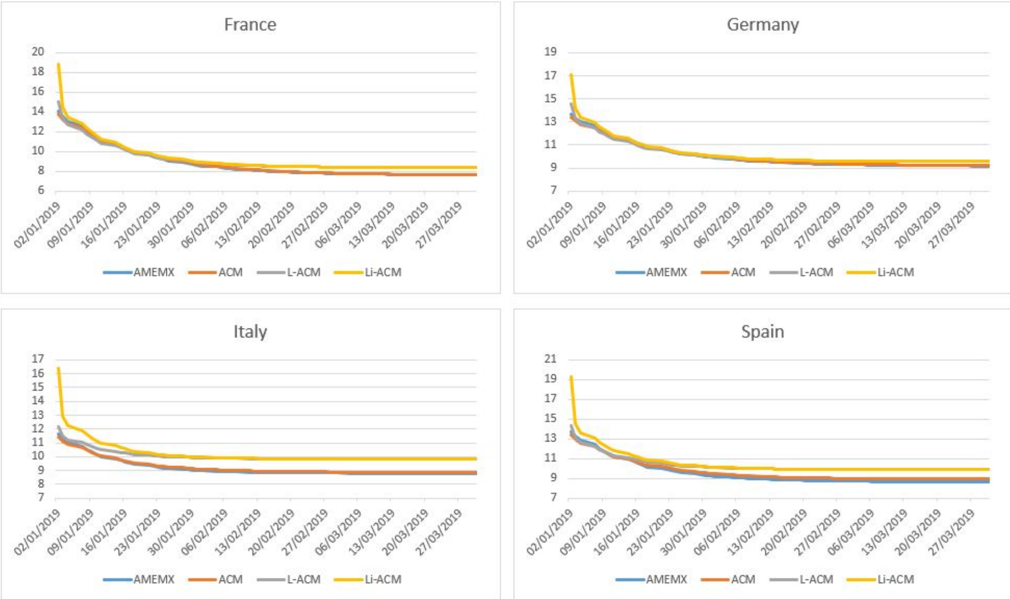

FIGURE 2.29: Multi-step forecast assuming 1sd shock. Proxy:UMP/TA. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

FIGURE 2.30: Multi-step forecast assuming 1sd shock. Proxy:UMP/CMP.Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

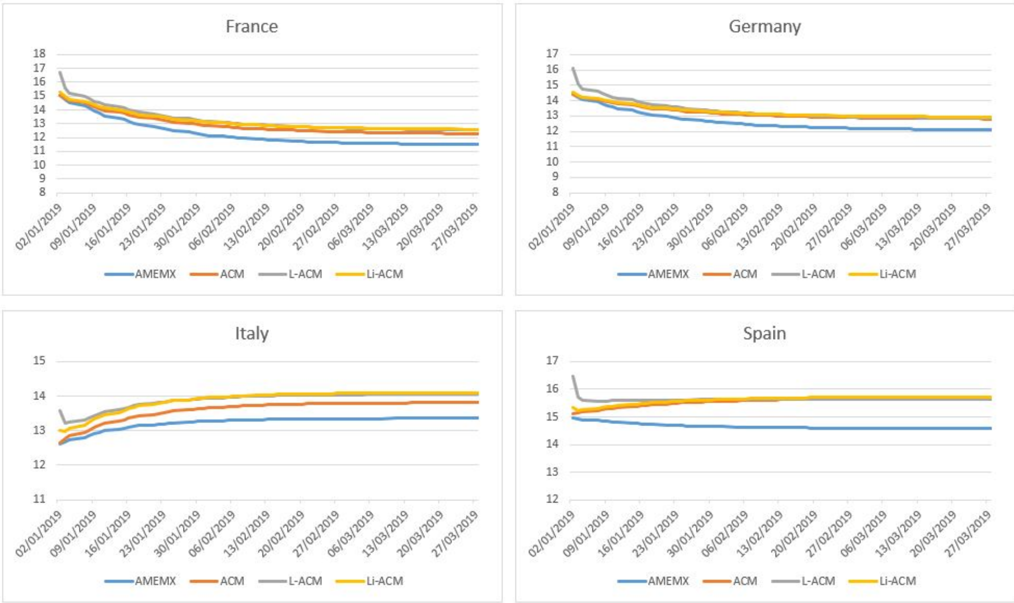

FIGURE 2.31: Multi-step forecast assuming 1sd shock. Proxy:TAgrowth. Estimation period: June 1, 2009 - December 31, 2018. Forecast Period: Sample period: January 1, 2019 - December 31, 2019

2.5 Conclusion

In this chapter we examined how unconventional monetary policies by ECB had affected realized volatility. The innovative feature of this study lies in the model we use, the Composite AMEM, which allows us to distinguish between a pure volatility mechanism and the part of volatility depending directly on quantitative easing policies. Therefore, after estimating the AMEM and the AMEM with exogenous variables, which serve as baseline models, we focus on the estimation of different specifications of the ACM, to allow unconventional policies to impact on volatility both in an additive and a multiplicative way. Results show how what matters for the effectiveness of these policies is the balance sheet composition rather than the balance sheet size. Indeed, it follows that an increase in securities held by ECB for monetary policy purposes relative to total asset (UMP/TA) reduces volatility in both core and peripheral countries, with disrupted countries generally beneficing more. A further proof derives from the estimation of the models through a different proxy, which basically tells us that an increase in securities purchased for QE programmes relative to securities held for conventional policies (UMP/CMP) also reduces market volatility. Model comparisons, based on information criteria and both MAE and MSE, show how the logistic specification of the multiplicative ACM is the best one, whereas UMP/CMP is the proxy working better. However, our proxies do not allow us to distinguish the specific effect of each policy, so that we cannot identify which of these extraordinary measures is more effective. This, of course, represents a first idea for future research, as well as the possibility to control also for spillovers among countries.

Moreover, we evaluated the out-of-sample forecasting performance of our models through the Model Confidence Set process from which it emerges the importance of the EAPP programme in making our proxies suitable for forecasting purposes. More in details, it derives how, before the EAPP announcement, the AMEM has the same forecasting performance of more complex models, whereas it is always excluded from the best set of models when we perform the MCS basing on sub-samples that include the EAPP programme. The importance of the EAPP is also evident from the multi-step forecast procedure, since we find that unconventional policies contribute in lowering volatility for a maximum of 124 business days, on the one hand, and that a further unconventional policies shock of one standard deviation, would have lowered stock market volatility up to 3 points, on the other.

In other words, our research is a further proof that unconventional monetary policy is a crucial instrument for Central Banks for restoring the proper functioning both of the economy and the financial markets and especially for achieving one of the main goals of central banks - i.e. to preserve financial stability - when interest rates are close to the zero lower bound.

Chapter 3

Unconventional Monetary Policies and Volatility Regime Shifts

Abstract

By augmenting the ACM models within the Markov switching (MS) framework, we aim to test whether the ECB was able to keep volatility within a low regime for an extended period of time. We estimate different Markov switching specifications basing on both the ACM and the multiplicative ACM, finding how the ECB was able to impact directly the part of volatility depending on unconventional policies, keeping it within a low regime for a minimum of 14 days (in the case of Italy) and a maximum of 53 days (Germany), as emerged from the MS(ξ)-LACM. Our proxy enters the model with the expected negative sign, but this effect is more than a short term effect, since the low regime is the prevailing regime in all the cases. MCS procedure gives evidence in favour of our intuition, with the MS(ξ)-LACM that is the best model in 3 out of 4 cases. Finally, the TVTP-AMEM shows how the ECB was able to affect the regime probabilities, since there exists a positive relationship between the low regime probability and the unconventional policy proxy, whereas the latter is negatively related to the high regime probability.

Keywords: Markov switching model, ECB unconventional monetary policies, stock market volatility, ACM, Model Confidence Set.

3.1 Introduction

If the GARCH model is the most widely used in volatility analysis, it is because of its capability to reproduce the volatility persistence in a parsimonious way (in most of the cases it is sufficient to consider a GARCH (1,1) model). As anticipated in Chapter 1, some authors, and in particular Lamoureux and Lastrapes (1990), argue that the volatility persistence is actually caused by different regimes which volatility is affected to.

Most of the recent literature concerning the analysis of volatility subject to several regimes is based on the presence of a latent variable which follows a Markov Chain: for this reason, this kind of model is known as Markov switching (MS) model. In this Chapter, we extend the ACM in order to allow our two volatility components to be subject to regime switching. Therefore, we firstly try to capture regime changes in the proper volatility dynamics (so we aim to analyse how volatility responded to unconventional policies within each regime), whereas, in a further analysis, we allow changes in the part of volatility depending directly on unconventional policies, with the main purpose to test whether the ECB was able to affect directly this volatility component, keeping it within a low regime. For this purpose, we replace the announcements dummy variable with a switching constant, with the main aim to capture possible jumps in the ξ_t process, even those previously caused by the dummy variable. Results seem to support our intuition, in the sense that the ξ_t component stays in a low regime for a long period of time, with probability of remaining in the low regime that is higher than the high volatility regime probability. Moreover, for what concerns Italy and Spain, unconventional policies have also affected the probability of remaining in each regime directly, with the low regime probability positively related to the unconventional policy proxy, whereas the probability of remaining in the high regime reduces when the proxy increases.

The chapter is structured as follow. A literature review concerning Markov switching AMEM is discussed in Section 3.2, whereas the estimated models are analysed in Section 3.3. Section 3.4 presents results - which are divided according to the estimated models - together with model comparisons based on the MCS procedure, discussed in Section 3.5. Finally, Section 3.6 concludes with some remarks.

3.2 Multiplicative Error Models and regime changes

The analysis we carry out in this Chapter represents the natural extension of the analysis we have presented in the previous one. Whereas in Chapter 2 we have analysed the impact of ECB's unconventional monetary policies on the stock market volatility, here we focus on the possibility that this kind of policy has driven volatility regime shifts.

As anticipated in Chapter 1, Markov Switching extensions are provided also within the Multiplicative Error Model framework. Gallo and Otranto (2015) propose two different models with parameters of the mean equation subject to changes in regime. They focus on a Smooth Transition AMEM (ST-AMEM), where changes in regime are driven by a function (so that they account for progressive changes in regimes), and on the three regimes Markov Switching AMEM (MS(3)-AMEM), which, differently, allows for abrupt changes. The latter, in particular, is specified as:

$$
\begin{aligned}
& x_t = \mu_{t,s_t}\epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(a_{s_t}, \tfrac{1}{a_{s_t}}) \\
& \mu_{t,s_t} = \omega + \sum_{i=1}^{n} k_i I_{s_t} + \alpha_{s_t}(x_{t-1} - \mu_{t-1,s_{t-1}}) + \beta_{s_t}\mu_{t-1,s_{t-1}} + \\
& \qquad\qquad\qquad\qquad + \gamma_{s_t} D_{t-1}(x_{t-1} - \mu_{t-1,s_{t-1}})
\end{aligned} \tag{3.1}
$$

where I_{s_t} is equal to 1 if $s_t \leq i$, $k_1 = 0$ and $k_i \geq 0$ for $i = 2 \ldots n$, so that the constant in regime j is equal to $\omega + \sum_{i=1}^{j} k_i$ and it increases passing from the lower to the higher regime. As usual, s_t is a dichotomic discrete latent variable representing the regime at time t and following a first order Markov chain.

$$
Pr(s_t = j|s_{t-1} = i, s_{t-2} \ldots) = Pr(s_t = j|s_{t-1} = i) = p_{ij}
$$

Positiveness and stationarity conditions do not change with respect to the simpler AMEM, i.e. $\omega > 0, \sum_{i=1}^{n} k_i, \alpha_{s_t}, \beta_{s_t}, \gamma_{s_t} \geq 0$ and $0 < \alpha_{s_t} + \beta_{s_t} + \frac{\gamma_{s_t}}{2} < 1$[1], respectively; whereas the unconditional mean in state j is given by

[1] Actually, this condition for stationarity could be considered too strong. Indeed, Gallo and Otranto (2018) show how - given the properties of stationarity and ergodicity of the MS GARCH model (Francq et al., 2001) - the necessary condition for the MS-AMEM to be stationary and ergodic is $\sum_{s_t=1}^{n} \pi_{s_t} E[log(\alpha_{s_t} + \gamma_{s_t} D_{t-1})\epsilon_t + \beta_{s_t}] < 0$, where $\pi_{s_t}, s_t = 1 \ldots n$ represents the ergodic probabilities of each regime.

$$\mu_j = \frac{\omega + \sum_{i=1}^{n} k_i}{\alpha_j + \beta_j + \frac{\gamma_j}{2}}$$

Basing on a large sample of the S&P500 RV measure, they apply these models and compare their forecasting performance with respect to the performance of a large set of models, including the HAR (Corsi, 2009), the Heterogeneous AMEM (HAMEM) in which the mean equation follows the HAR model, the HAR with jumps (Andersen et al., 2007) and the MS-HAMEM. Firstly, from the model in (3.1), it emerges, as one might largely expect, that the low volatility regime has a higher duration, 87 days; whereas the duration of the medium and the high regimes is remarkably lower, 28 and 13 days respectively. In addition, whereas the probability of remaining in the low volatility regime is 99%, they find a strong relationship between the intermediate and the high regime: when the model is in the intermediate (high) regime, there is a higher probability of passing in the high (intermediate) regime than the probability of moving in the low volatility regime. Finally, considering model comparisons, based on the MCS procedure, they find how the ST-AMEM is always in the best set of models, while the MS-AMEM shows good in-sample and one-step-ahead forecasting performance. In conclusion, this analysis represents evidence supporting the view that changes in the level of volatility are actually present in many stock indexes and that this slow-moving underlying trend can be adequately captured by a Markov switching model.

Following the intuition of Bollerslev et al. (2016), Cipollini et al. (2020) insert both an interaction and a curvature term in the MS-AMEM, to account for heteroskedasticity in Realized Volatility measurement errors, which arises because of the strong correlation between the square root of the Integrated Quarticity (IQ) and the Realized Volatility itself. Let the model be

$$\begin{aligned} RV_t &= \mu_{t,s_t}\epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(a_{s_t}, \tfrac{1}{a_{s_t}}) \\ \mu_{t,s_t} &= \omega + \sum_{i=1}^{n} k_i I_{s_t} + (\alpha_{s_t} + \alpha_E X_{t-1})RV_{t-1} + \beta_{s_t}\mu_{t-1,s_{t-1}} + \gamma_{s_t} D_{t-1} RV_{t-1} \end{aligned} \tag{3.2}$$

when $X_{t-1} = 0$ the model is the MS-AMEM; when $X_{t-1} = \sqrt{RQ_{t-1}}$, where RQ (Realized Quarticity) is a proxy for IQ, we have the MS-AMEMQ; finally when $X_{t-1} = RV_{t-1}$ the model, which they called MS-AMEM2, accounts for the curvature term. Basing on a sample of 28 big cap plus the S&P500, they find how the significance of α_E disappears when they consider changes in regime. In other words, while both the interaction and the curvature terms improve the in and out-of-sample performance of the HAR family models, the MS-AMEM is always the best model, that is the interaction/curvature term does not improve the forecasting ability of the model. This allow them to conclude that "the statistical significance of the interaction/curvature term provides evidence that accounting for heteroskedasticity and a slow-moving average level of volatility provides robustness to measurement errors" (Cipollini et al., 2020).

Basing on this considerations, and in particular on model (3.1), we propose an extension of the analysis carried out in the previous Chapter: whether in Chapter 2 we have proposed an analysis on the effect of unconventional policies on four Eurozone stock indexes, here we do something similar, trying to understand whether the ECB was able to keep volatility within a low volatility regime and how long this effect lasted.

3.3 The models

In this section the estimated models are discussed. To be sure that our models can measure the effort as well as the ability of the ECB in keeping volatility within a low regime as long as possible, we present different Markov switching Asymmetric Composite AMEM (MS-ACM) specifications, in all of which we consider only two regimes, low and high volatility regime, and allow for changes also in the shape parameter of the Gamma distribution, θ.

3.3.1 ACM with switching in ς process

In the simplest version of the $MS(\varsigma) - ACM$, we allow only the constant term to switch between a low and a high volatility regime. The model is specified as in (3.3):

$$\begin{aligned}
&RV_t = \mu_{t,s_t}\epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta_{s_t}, \tfrac{1}{\vartheta_{s_t}}) \\
&\mu_{t,s_t} = \varsigma_{t,s_t} + \xi_t \\
&\varsigma_{t,s_t} = \omega_0 + \omega_{1,s_t} + \alpha RV_{t-1} + \beta\varsigma_{t-1,s_{t-1}} + \gamma D_{t-1} RV_{t-1} \\
&\xi_t = \delta(E\left(x_t|\Psi_{t-1}\right) - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi\xi_{t-1}
\end{aligned} \tag{3.3}$$

Once again, changes in regime are driven by the latent variable s_t, which follows a first order Markov chain

$$Pr(s_t = j|s_{t-1} = i, s_{t-2}\ldots) = Pr(s_t = j|s_{t-1} = i) = p_{ij}$$

so that, our transition probabilities matrix, P, is given by

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

where each row of P sums to 1. Following Gallo and Otranto (2015) we ensure the volatility to increase passing from the low to the high regime, by computing the constant of the high volatility regime as $\omega + \omega_{s_t}$, which causes a specific unconditional mean equal to $\frac{\omega_0+\omega_{1,s_t}}{\alpha+\beta+\frac{\gamma}{2}}$, from which it derives that all parameter are required to be positive to ensure positiveness; differently, the stationarity condition is the same for both the regimes, i.e. $0 < \alpha + \beta + \frac{\gamma}{2} < 1$ and $\psi < 0$. A more general specification of the model is the one in which all parameters in ς_t are subject to changes in regime. However, relying on information criteria, the estimation of these models does not show signs of improvement, for this reason they are no longer considered in the following analysis.

In contrast to Chapter 2, where we measured how long the ECB unconventional policy measures lowered volatility by means of a multistep-ahead out-of-sample forecasting procedure, here, we have a different tool to achieve the same objective, that is the average duration of each volatility regime, i.e. $\frac{1}{1-p_{ii}}$ ($i = 0, 1$ for the low and high regime, respectively). In other words, we could interpret the average duration of the low volatility regime, as a measure of the ECB ability to decrease volatility for a certain period of time. Following the structure of the previous chapter, instead, also in this analysis we considered the two different multiplicative specifications of the ACM, which - within the Markov switching framework - are expressed as in (3.4) and (3.5), for the Logistic and the Linear specifications, respectively.

$$\begin{aligned} &RV_t = \mu_{t,s_t}\epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta_{s_t}, \tfrac{1}{\vartheta_{s_t}}) \\ &\mu_{t,s_t} = \varsigma_{t,s_t} 2\tfrac{exp(\xi_t)}{1+exp(\xi_t)} \\ &\varsigma_{t,s_t} = \omega_0 + \omega_{1,s_t} + \alpha RV_{t-1} + \beta\varsigma_{t-1,s_{t-1}} + \gamma D_{t-1}RV_{t-1} \\ &\xi_t = \delta(E\left(x_t|\Psi_{t-1}\right) - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi\xi_{t-1} \end{aligned} \tag{3.4}$$

$$\begin{aligned} &RV_t = \mu_{t,s_t}\epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta_{s_t}, \tfrac{1}{\vartheta_{s_t}}) \\ &\mu_{t,s_t} = \varsigma_{t,s_t}\xi_t \\ &\varsigma_{t,s_t} = \omega_0 + \omega_{1,s_t} + \alpha RV_{t-1} + \beta\varsigma_{t-1,s_{t-1}} + \gamma D_{t-1}RV_{t-1} \\ &\xi_t = (1-\psi) + \delta(E\left(x_t|\Psi_{t-1}\right) - \bar{x}) + \varphi(\Delta_t - \bar{\Delta}) + \psi\xi_{t-1} \end{aligned} \tag{3.5}$$

3.3.2 ACM with switching in ξ process

Assuming switching parameters in the proper volatility dynamics, ς_t, is the natural starting-point in our Markov switching analysis. However, it is possible that the switching process concerns the ξ_t component (we refers to this set of models through the prefix MS(ξ)). In order to consider also this scenario, we should modify our model by inserting a switching constant in the ξ_t component, which would replace our announcement variable. This is quite straightforward in the MS(ξ)-ACM and MS(ξ)-LACM, so that they are specified as in models (3.6) and (3.7).

$$\begin{aligned}
&RV_t = \mu_{t,st}\epsilon_t, \ \ \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta_{s_t}, \tfrac{1}{\vartheta_{s_t}}) \\
&\mu_{t,s_t} = \varsigma_t + \xi_{t,s_t} \\
&\varsigma_t = \omega + \alpha RV_{t-1} + \beta\varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\
&\xi_{t,s_t} = \varphi_0 + \varphi_{1,s_t} + \delta(E\left(x_t|\Psi_{t-1}\right) - \bar{x}) + \psi\xi_{t-1,s_{t-1}}
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
&RV_t = \mu_{t,st}\epsilon_t, \ \ \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta_{s_t}, \tfrac{1}{\vartheta_{s_t}}) \\
&\mu_{t,st} = \varsigma_t 2\tfrac{exp(\xi_{t,s_t})}{1+exp(\xi_{t,s_t})} \\
&\varsigma_t = \omega + \alpha RV_{t-1} + \beta\varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\
&\xi_{t,s_t} = \varphi_0 + \varphi_{1,s_t} + \delta(E\left(x_t|\Psi_{t-1}\right) - \bar{x}) + \psi\xi_{t-1,s_{t-1}}
\end{aligned} \tag{3.7}$$

where φ_0 represents the constant in the low volatility regime, increasing by φ_1 in the high volatility regime. Positiveness and stationarity conditions remain unchanged and in model (3.7) the logistic function ensures that the mean of the ξ_t process is still equal to 1, even though we are adding a constant term. This condition no longer holds when we consider the linear specification of the model. Therefore, within this specification we have to impose a further adjustment by means of the ergodic probabilities: in other words, we replace our constant term $\varphi_0 + \varphi_{1,s_t}$ with $[(1-\psi) + \varphi(s_t - E(s_t))]$. Let the model be

$$\begin{aligned}
&RV_t = \mu_{t,st}\epsilon_t, \ \ \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta_{s_t}, \tfrac{1}{\vartheta_{s_t}}) \\
&\mu_{t,st} = \varsigma_t \xi_{t,s_t} \\
&\varsigma_t = \omega + \alpha RV_{t-1} + \beta\varsigma_{t-1} + \gamma D_{t-1} RV_{t-1} \\
&\xi_{t,s_t} = (1-\psi) + \varphi(s_t - E(s_t)) + \delta(E\left(x_t|\Psi_{t-1}\right) - \bar{x}) + \psi\xi_{t-1,s_{t-1}}
\end{aligned} \tag{3.8}$$

where $E(s_t)$ is equal to the ergodic probability (let call it π) of regime 1, i.e. $\pi_1 = \frac{1-p_{00}}{2-p_{00}-p_{11}}$, as defined by Hamilton (1994).

To show that $E(\xi_t) = 1$, let ξ_t be expressed as

$$\xi_t = \varphi_0 + \varphi_1(s_t) + \delta(x_t - \bar{x}) + \psi\xi_{t-1}$$

by considering the unconditional expected value

$$E(\xi_t) = \varphi_0 + \varphi_1 E(s_t) + \delta(E(x_t) - \bar{x}) + \psi E(\xi_{t-1})$$

since $E(s_t) = 0 * \pi_0 + 1 * \pi_1 = \pi_1$, $E(x_t) = \bar{x}$ and $E(\xi_{t-1}) = E(\xi_t)$, then

$$\begin{aligned} E(\xi_t) - \psi E(\xi_{t-1}) &= \varphi_0 + \varphi_1\pi_1 + \delta(\bar{x} - \bar{x}) \\ E(\xi_t) &= \tfrac{\varphi_0 + \varphi_1\pi_1}{1-\psi} \\ E(\xi_t) = 1 \Rightarrow \tfrac{\varphi_0 + \varphi_1\pi_1}{1-\psi} &= 1 \Rightarrow \varphi_0 = (1-\psi) - \varphi_1\pi_1 \end{aligned}$$

Differently from models (3.6) and (3.7), in this model, the constant term in the low regime is given by $[1 - \psi - (\varphi\pi_1)]$, which increases to $[1 - \psi + \varphi - (\varphi\pi_1)]$ in the high regime.

Through these models we aim to understand whether the ECB was able to manage directly the part of volatility depending on unconventional policy measures: of course, what we expect is a higher probability to stay in the low volatility regime as well as a negative sign for the unconventional policy proxy.

3.3.3 Time Varying Transition Probability MS-AMEM

In models described so far, we have assumed that the ECB affects one of the two volatility components and that both the probabilities regimes do not vary over time. In a last analysis, we do something different by focusing on the particular scenario that ECB unconventional policies do not affect directly the stock market volatility, but it could rather affect the probability of remaining in the two volatility regimes. In other words, we estimate a simple MS-AMEM with time-varying transition probabilities (TVTP) depending on our unconventional policies proxy. The model, called TVTP-AMEM, is specified as in equation (3.9)

$$\begin{aligned} RV_t &= \mu_{t,s_t}\epsilon_t, \;\; \epsilon_t|\Psi_{t-1} \sim Gamma(\vartheta_{s_t}, \tfrac{1}{\vartheta_{s_t}}) \\ \mu_{t,s_t} &= \omega_0 + \omega_{1,s_t} + \alpha RV_{t-1} + \beta\varsigma_{t-1,s_{t-1}} + \gamma D_{t-1}RV_{t-1} \end{aligned} \tag{3.9}$$

As usual, regime at time t are represented by s_t, which follows a first order Markov chain with probability (collected in the time-varying transition probability matrix, P) changing over time, according to our unconventional policies proxy, x_t:

$$P = \begin{bmatrix} \frac{exp(\varphi_0+\delta_0 E(x_t|\Psi_{t-1}))}{1+exp(\varphi_0+\delta_0 E(x_t|\Psi_{t-1}))} & 1-\frac{exp(\varphi_0+\delta_0 E(x_t|\Psi_{t-1}))}{1+exp(\varphi_0+\delta_0 E(x_t|\Psi_{t-1}))} \\ 1-\frac{exp(\varphi_1+\delta_1 E(x_t|\Psi_{t-1}))}{1+exp(\varphi_1+\delta_1 E(x_t|\Psi_{t-1}))} & \frac{exp(\varphi_1+\delta_1 E(x_t|\Psi_{t-1}))}{1+exp(\varphi_1+\delta_1 E(x_t|\Psi_{t-1}))} \end{bmatrix}$$

What we expect is a higher impact on the low probability regime, which should be higher than the high regime probability, if the ECB unconventional policies are effective in reducing stock market volatility (as it emerges from the analysis in the previous Chapter). Therefore, this would result in a positive δ_0 coefficient.

As it is typical, we estimate all Markov switching models described above by means of the Hamilton filter and smoother, as introduced in Hamilton (1994), together with the solution proposed by Kim (1994) to solve the problems arising due to the dependence of μ_{t,s_t} on s_{t-1}, which causes the need to track all the possible paths of the regime between the first and the last observation. In particular, after each step of Hamilton filter, we collapse the 4 possible values of μ_t into 2 values, through a weighted average at time $t-1$,

$$\mu_{t,s_t} = \frac{\sum_{i=1}^{n} Pr[s_{t-1}=i, s_t=j|\Psi_t]\mu_{t,s_{t-1},s_t}}{Pr[s_t=j|\Psi_t]}$$

where μ_{t,s_{t-1},s_t} is the estimate of the unknown variable weighted with the filtering probabilities obtained by the Hamilton filter (see Appendix B.1 for more details on the filter).

3.4 Estimation results

Once again, we estimate the MS-AMEM as a baseline model. Coefficients (Table 3.1) are highly significant, with a low regime constant (which is no statistically significant only in the case of Spain) ranging between 0.261 and 1.115 for Spain and Germany, respectively. It is interesting to notice how peripheral countries have a lower average level of volatility in the low regime; however, as largely expected, in the high volatility regime we get a higher average value ($\omega_0 + \omega_1$) for Italy (3.73) and Spain (3.2). Even though Italy and Spain are the countries that suffered more the consequences of the financial crisis, it seems that their volatility measure responded quite differently, with volatility for Italy being more sensitive to volatility shocks ($\alpha + \frac{\gamma}{2}$ is about 0.3), whereas for Spain a crucial role is played by the conditional variance persistence (as expressed by the coefficient β). In these markets, the low volatility regime dominates the high regime, which has a probability (p_{11})

close to zero and not statistically significant: therefore, it is likely identified with volatility spikes; moreover, we find the gamma distribution has a higher variability in the high regime. Differently, both the high and low regimes are well defined for France and Germany, in which probabilities are close to 1 and statistically significant at 1% level.

When we augment the MS-AMEM with our exogenous variables, results (Table 3.2) remain almost unchanged. Generally, α and γ coefficients increase, whereas β decreases. This, of course, could be interpreted as a sign that volatility is even more sensitive to shocks and that the weight of the past conditional volatility is now less predominant (the reduction of beta is particularly significant in disrupted countries, where it reduces of about 7%, for Italy, and 4.6%, for Spain). Nonetheless, volatility persistence ($\alpha + \beta + \frac{\gamma}{2}$) is quite high, ranging between 86% (France) and 93% (Spain).

For what concerns the announcement variable, it is usually excluded from Markov switching analysis. The logic of this choice is fairly straightforward: in a Markov switching framework, where we capture abrupt changes in the level of volatility, a jump variable (like the announcement variable) could cause computational issues since it is a dummy variable and then it captures abrupt changes as well. Despite, the validity of this consideration, in our analysis the announcement variable still has a strong explanatory power, considering the fact that it is significant at the 99% confidence level.

The unconventional policy proxy[2] is significant and it enters the model with the expected negative sign. More precisely, it is higher in peripheral countries (where it is significant at 1% level) than in Germany, where it has a lower level of significance (10%).
Besides the significance of the proxy, a further indication of the efficiency of the unconventional policy in reducing volatility derives from probabilities coefficients. Whereas probabilities remain unchanged for France and Germany, the probability of remaining in the low regime increases in the case of Italy and Spain.

Finally, from the Ljung-Box statistics, the models are still affected from residual autocorrelation at a 1% level. This is the case of Spain (both in the MS-AMEM and MS-AMEMX) and Germany at the highest lag of the MS-AMEMX.

[2] It should be specified that, given the results obtained in the previous analysis, here, we focus just on the UMP/CMP proxy, which is turned out to be the best in terms of AIC and BIC.

TABLE 3.1: MS-AMEM

a)	**France**	**Germany**	**Italy**	**Spain**
ω_0	0.981 ***	1.115 ***	0.522 ***	0.261
	(0.009)	(0.076)	(0.122)	(0.188)
ω_1	0.627 ***	0.865 ***	3.211 ***	2.941 ***
	(0.075)	(0.139)	(0.59)	(0.438)
α	0.137 ***	0.155 ***	0.23 ***	0.156 ***
	(0.017)	(0.018)	(0.027)	(0.027)
β	0.699 ***	0.691 ***	0.681 ***	0.766 ***
	(0.018)	(0.02)	(0.03)	(0.032)
γ	0.125 ***	0.097 ***	0.073 ***	0.074 ***
	(0.011)	(0.012)	(0.009)	(0.008)
p_{00}	0.999 ***	0.997 ***	0.925 ***	0.851 ***
	(0.001)	(0.002)	(0.049)	(0.068)
p_{11}	0.999 ***	0.996 ***	0.000	0.000
	(0.001)	(0.004)	(0.000)	(0.000)
θ_0	7.288 ***	9.714 ***	13.773 ***	12.364 ***
	(0.34)	(0.4)	(0.834)	(0.758)
θ_1	8.366 ***	10.892 ***	3.947 ***	4.4 ***
	(0.359)	(0.677)	(1.43)	(0.889)
b)				
Ljung-Box 1 lag	0.089 *	0.307	0.037 **	0.000 ***
Ljung-Box 5 lag	0.057 *	0.071 *	0.07 *	0.000 ***
Ljung-Box 10 lag	0.026 **	0.034 **	0.092 *	0.000 ***

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 3.2: MS-AMEMX. Proxy: UMP/CMP

a)	**France**	**Germany**	**Italy**	**Spain**
ω_0	1.498 ***	1.145 ***	1.026 ***	0.886 *
	(0.199)	(0.087)	(0.043)	(0.489)
ω_1	0.919 ***	0.766 ***	2.803 ***	2.604 ***
	(0.064)	(0.133)	(0.634)	(0.575)
α	0.127 **	0.172 ***	0.242 ***	0.151 ***
	(0.052)	(0.017)	(0.021)	(0.024)
β	0.672 ***	0.675 ***	0.633 ***	0.731 ***
	(0.061)	(0.021)	(0.019)	(0.05)
γ	0.127 ***	0.097 ***	0.078 ***	0.086 ***
	(0.017)	(0.008)	(0.008)	(0.024)
δ	-0.29 **	-0.054 *	-0.247 ***	-0.277 ***
	(0.137)	(0.032)	(0.039)	(0.092)
φ	1.358 ***	0.98 ***	1.494 ***	1.279 ***
	(0.376)	(0.321)	(0.332)	(0.385)
p_{00}	0.989 ***	0.998 ***	0.922 ***	0.878 ***
	(0.013)	(0.002)	(0.043)	(0.098)
p_{11}	0.983 ***	0.996 ***	0.066	0.046
	(0.016)	(0.004)	(0.058)	(0.069)
θ_0	8.306 ***	9.677 ***	14.143 ***	12.356 ***
	(0.637)	(0.413)	(0.786)	(1.213)
θ_1	7.898 ***	11.3 ***	4.144 ***	4.226 ***
	(0.617)	(0.719)	(1.252)	(1.385)
b)				
Ljung-Box 1 lag	0.829	0.348	0.421	0.000 ***
Ljung-Box 5 lag	0.119	0.076 *	0.424	0.000 ***
Ljung-Box 10 lag	0.283	0.005 ***	0.595	0.007 ***

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

3.4.1 MS(ς)-ACM results

The estimation of the MS(ς)-ACM, improves the significance of our parameters. In the MS(ς)-ACM (Table 3.3), the impact of the announcement as well as that of the proxy (which is significant at 1% level also for Germany) is pretty high, with the announcement effect that is higher in disrupted countries (3.18 and 2.9 for Italy and Spain, respectively), whereas the long run effect for Germany and French is higher than that observed for Italy.
Once again, it emerges a higher probability of remaining in the low volatility regime (regime 0) for Italy (0.918) and Spain (0.892); Moreover, in these countries the probability of moving from high to low volatility regime is higher than the probability of moving from low to high volatility regime (computed as $1 - p_{11}$ and $1 - p_{00}$, respectively). This leads to an average duration of low volatility regime (given by $\frac{1}{1-p_{00}}$) of about 9 days for Spain and 12 days for Italy. The average duration is perhaps lower than one might expect but it is plausible if compared with the average duration of high volatility regime, which is about 1 day for these countries, meaning that the high volatility regime is basically represented by volatilities spikes. A different case is that of core countries where there is also a high probability to stay in the high volatility regime (0.979 and 0.997 for France and Germany, respectively). Here, we estimate an average duration of 83 days and 48 days for low and high volatility regimes for France; whereas for Germany we observe an average duration of 250 days for the low regime and 333 days for the high regime. Clearly, this result is in line with the higher long run effect (-0.75 for France and -0.998 for Germany) estimated for these markets, if compared with the value obtained for the other markets. Finally, the part of volatility depending directly on unconventional policies does not show an autoregressive dynamics, given that the AR coefficient is never significant. This dynamics is, instead, detected for France and Spain when we consider the MS(ς)-LACM (Table 3.4) and the MS(ς)-LiACM (Table 3.5). In both the multiplicative specifications the proxies are highly significant and they have the expected sign but, as noted in the previous chapter, the coefficients are remarkably lower than those estimated through the MS(ς)-ACM.

Even in the multiplicative specifications the residual autocorrelation is detected for Spain, whereas there is an improvement for what concerns the case of Germany (with respect to the ACM, in particular), where we never reject the null of autocorrelation at 1% significance level.

TABLE 3.3: MS(ς)-ACM. Proxy: UMP/CMP

a)	**France**	**Germany**	**Italy**	**Spain**
ω_0	1.305 ***	1.07 ***	0.876 ***	0.866 ***
	(0.215)	(0.079)	(0.111)	(0.193)
ω_1	0.846 ***	0.624 ***	2.856 **	2.54 ***
	(0.263)	(0.074)	(1.202)	(0.615)
α	0.113 ***	0.157 ***	0.215 ***	0.148 ***
	(0.014)	(0.014)	(0.022)	(0.019)
β	0.705 ***	0.688 ***	0.671 ***	0.739 ***
	(0.034)	(0.018)	(0.023)	(0.027)
γ	0.123 ***	0.099 ***	0.077 ***	0.084 ***
	(0.014)	(0.009)	(0.006)	(0.009)
δ	-0.75 **	-0.998 ***	-0.638 ***	-0.932 ***
	(0.323)	(0.176)	(0.116)	(0.22)
φ	2.757 ***	2.148 ***	3.177 ***	2.899 ***
	(0.502)	(0.42)	(0.482)	(0.57)
ψ	0.131	0.084	0.054	0.099
	(0.091)	(0.07)	(0.056)	(0.092)
p_{00}	0.988 ***	0.996 ***	0.918 ***	0.892 ***
	(0.014)	(0.002)	(0.061)	(0.064)
p_{11}	0.979 ***	0.997 ***	0.067	0.065
	(0.014)	(0.002)	(0.176)	(0.059)
θ_0	8.436 ***	9.527 ***	14.471 ***	12.236 ***
	(0.625)	(0.479)	(1.279)	(0.817)
θ_1	7.905 ***	11.039 ***	4.369 ***	4.212 ***
	(0.661)	(0.54)	(1.638)	(1.056)
b)				
Ljung-Box 1 lag	0.509	0.203	0.137	0.000 ***
Ljung-Box 5 lag	0.189	0.057 *	0.412	0.001 ***
Ljung-Box 10 lag	0.441	0.003 ***	0.57	0.015 **

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 3.4: MS(ς)-LACM. Proxy: UMP/CMP

a)	**France**	**Germany**	**Italy**	**Spain**
ω_0	1.414 ***	1.119 ***	0.835 ***	0.827 **
	(0.269)	(0.113)	(0.153)	(0.331)
ω_1	0.897 **	0.718 ***	3.003 ***	2.728 ***
	(0.385)	(0.094)	(1.054)	(0.531)
α	0.103 ***	0.149 ***	0.215 ***	0.136 ***
	(0.017)	(0.019)	(0.024)	(0.03)
β	0.697 ***	0.687 ***	0.67 ***	0.746 ***
	(0.04)	(0.022)	(0.027)	(0.045)
γ	0.128 ***	0.101 ***	0.078 ***	0.086 ***
	(0.017)	(0.011)	(0.007)	(0.017)
δ	-0.21 ***	-0.182 ***	-0.107 ***	-0.156 ***
	(0.041)	(0.039)	(0.02)	(0.028)
φ	0.464 ***	0.357 ***	0.491 ***	0.429 ***
	(0.089)	(0.079)	(0.071)	(0.08)
ψ	0.171 *	0.17	0.065	0.136 **
	(0.098)	(0.111)	(0.066)	(0.068)
p_{00}	0.988 ***	0.995 ***	0.915 ***	0.874 ***
	(0.007)	(0.002)	(0.066)	(0.064)
p_{11}	0.984 ***	0.997 ***	0.054	0.073
	(0.011)	(0.002)	(0.14)	(0.093)
θ_0	8.622 ***	9.709 ***	14.478 ***	12.555 ***
	(0.409)	(0.487)	(1.227)	(0.798)
θ_1	7.987 ***	11.151 ***	4.54 ***	4.582 ***
	(0.561)	(0.587)	(1.73)	(1.071)
b)				
Ljung-Box 1 lag	0.523	0.263	0.141	0.000 ***
Ljung-Box 5 lag	0.243	0.097 *	0.37	0.001 ***
Ljung-Box 10 lag	0.513	0.014 **	0.496	0.009 ***

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 3.5: MS(ς)-LiACM. Proxy: UMP/CMP

a)	**France**	**Germany**	**Italy**	**Spain**
ω_0	1.422 ***	1.129 ***	0.839 ***	0.834 ***
	(0.173)	(0.169)	(0.077)	(0.203)
ω_1	0.903 ***	0.729 ***	3.02 ***	2.781 ***
	(0.094)	(0.124)	(0.965)	(0.509)
α	0.103 ***	0.15 ***	0.216 ***	0.136 ***
	(0.021)	(0.03)	(0.023)	(0.019)
β	0.696 ***	0.685 ***	0.669 ***	0.745 ***
	(0.032)	(0.04)	(0.024)	(0.026)
γ	0.128 ***	0.101 ***	0.078 ***	0.086 ***
	(0.012)	(0.014)	(0.006)	(0.009)
δ	-0.107 ***	-0.097 ***	-0.055 ***	-0.08 ***
	(0.018)	(0.018)	(0.01)	(0.014)
φ	0.228 ***	0.177 ***	0.241 ***	0.212 ***
	(0.042)	(0.035)	(0.034)	(0.037)
ψ	0.153 **	0.127	0.05	0.116 *
	(0.68)	(0.092)	(0.053)	(0.067)
p_{00}	0.988 ***	0.995 ***	0.916 ***	0.877 ***
	(0.006)	(0.002)	(0.054)	(0.06)
p_{11}	0.984 ***	0.997 ***	0.056	0.073
	(0.008)	(0.002)	(0.134)	(0.057)
θ_0	8.63 ***	9.722 ***	14.465 ***	12.514 ***
	(0.4)	(0.485)	(1.05)	(0.764)
θ_1	7.974 ***	11.133 ***	4.526 ***	4.568 ***
	(0.507)	(0.582)	(1.551)	(0.97)
b)				
Ljung-Box 1 lag	0.553	0.293	0.154	0.000 ***
Ljung-Box 5 lag	0.245	0.106	0.387	0.001 ***
Ljung-Box 10 lag	0.513	0.015 **	0.513	0.009 ***

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

The multiplicative specifications, and in particular the MS(ς)-LACM, represent the best model (values in bold in Table 3.6) both in terms of AIC and BIC, in all the cases. There are only few exceptions represented by the MS(ς)-LiACM, which is the best model for Italy, in terms of MSE and MAE, and for Spain, also in terms of AIC and BIC. Therefore, in general, we could conclude that splitting the volatility, in order to account for the part depending on unconventional policies, improves the fit property and the forecasting power of the model (i.e. the AMEMX is never the best model not even in terms of MSE and MAE).
Figure 3.1 compares RV with the high volatility regime probability, estimated through MS(ς)-LACM[3]. By considering France and Germany, it is possible to notice how both the regimes are well defined: here, it clearly emerges the volatility clustering, coherently with the common view that high volatility periods last less than periods of low volatility. Indeed, the model is able to capture the long period of low volatility starting in July 2012 with the OMT announcement, as well as the high volatility period in 2015 (for Germany, in particular), in the first year of the EAPP programme, until the BCE decided on the increment of the amount of monthly purchases, in March 2016.

Something different happens in peripheral countries, which are generally more sensitive to news. As a consequence, for Italy and Spain the high volatility regime has a lower persistence than for France and Germany: here, it seems to coincide with unconventional monetary policy announcements, in line with the 1 day-average-duration computed before. This is shown in Figure 3.2, which plots the unconventional monetary policy announcements and the probability of being in the high volatility regime. It could be noticed how often they occur in the same day. Here, we highlight two important cases (red points in the Figure): the announcement on May 2nd, 2013 - which refers to details about the LTRO programme - and the announcement on March 10th, 2016 - by means of which the EAPP monthly purchases were increased from €60 to €80 billion. Therefore, in these countries, the high regime occurs often, interrupting the low volatility regime, but lasts remarkably less.

[3]We have selected this model based on the information criteria, as shown in Table 3.6.

TABLE 3.6: MS(ς) models comparison

France

	MS(ς)AMEM	MS(ς)AMEMX	MS(ς)ACM	MS(ς)L-ACM	MS(ς)Li-ACM
Log-lik	-7775.042	-7761.769	-7746.709	**-7736.75**	-7737.177
AIC	5.796	5.79	5.779	**5.772**	**5.772**
BIC	5.816	5.814	5.806	**5.798**	**5.798**
MSE	28.983	27.716	27.392	**27.071**	27.077
MAE	3.793	3.691	3.666	**3.65**	**3.65**

Germany

	MS(ς)AMEM	MS(ς)AMEMX	MS(ς)ACM	MS(ς)L-ACM	MS(ς)Li-ACM
Log-lik	-7517.393	-7492.079	-7476.945	**-7466.601**	-7467.234
AIC	5.604	5.589	5.578	**5.571**	**5.571**
BIC	5.624	5.613	5.605	**5.597**	**5.597**
MSE	22.571	22.309	22.257	**21.893**	21.9
MAE	3.301	3.291	3.279	**3.26**	**3.26**

Italy

	MS(ς)AMEM	MS(ς)AMEMX	MS(ς)ACM	MS(ς)L-ACM	MS(ς)Li-ACM
Log-lik	-7612.782	-7580.457	-7557.976	**-7554.533**	-7554.667
AIC	5.675	5.655	5.639	**5.636**	**5.636**
BIC	5.695	5.679	5.665	**5.662**	5.663
MSE	25.924	25.725	25.09	24.87	**24.867**
MAE	3.483	3.465	3.414	**3.404**	**3.404**

Spain

	MS(ς)AMEM	MS(ς)AMEMX	MS(ς)ACM	MS(ς)L-ACM	MS(ς)Li-ACM
Log-lik	-8039.317	-8004.488	-7991.708	**-7980.829**	-7981.034
AIC	5.993	5.971	5.962	**5.954**	**5.954**
BIC	6.012	5.995	5.988	5.98	**5.856**
MSE	39.349	38.975	38.414	36.921	**36.861**
MAE	4.083	4.082	4.071	4.022	**4.02**

Note: MS(ς) models comparison via Information Criteria (AIC and BIC) and forecast capability (MSE and MAE). Sample period: June 1, 2009 - December 31, 2019

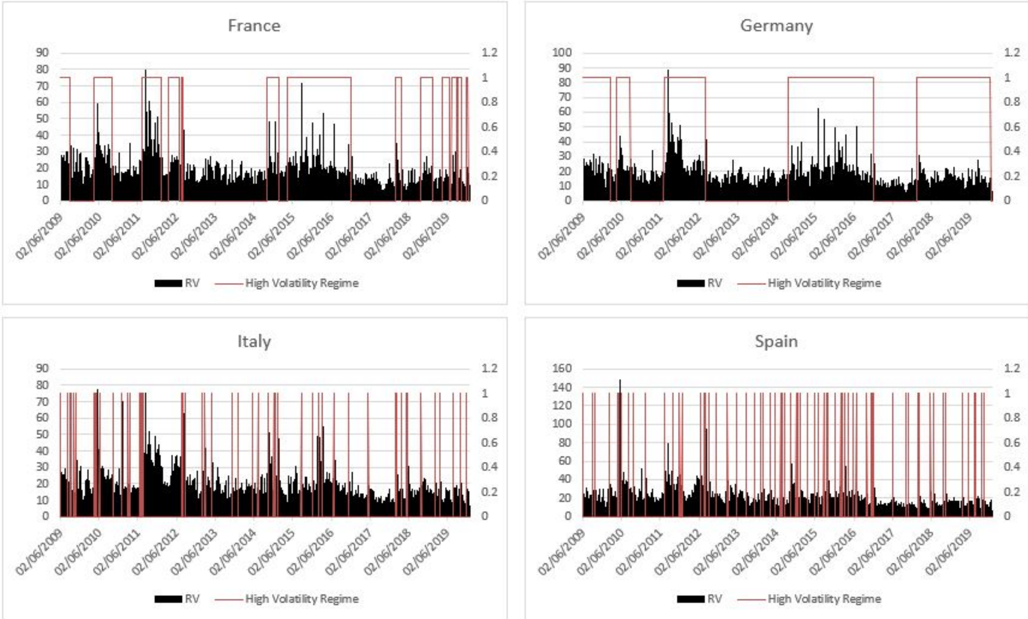

FIGURE 3.1: Realized Volatility and high volatility regime probability estimated from MS(ς)-LACM. Sample period: June 1, 2009 - December 31, 2019

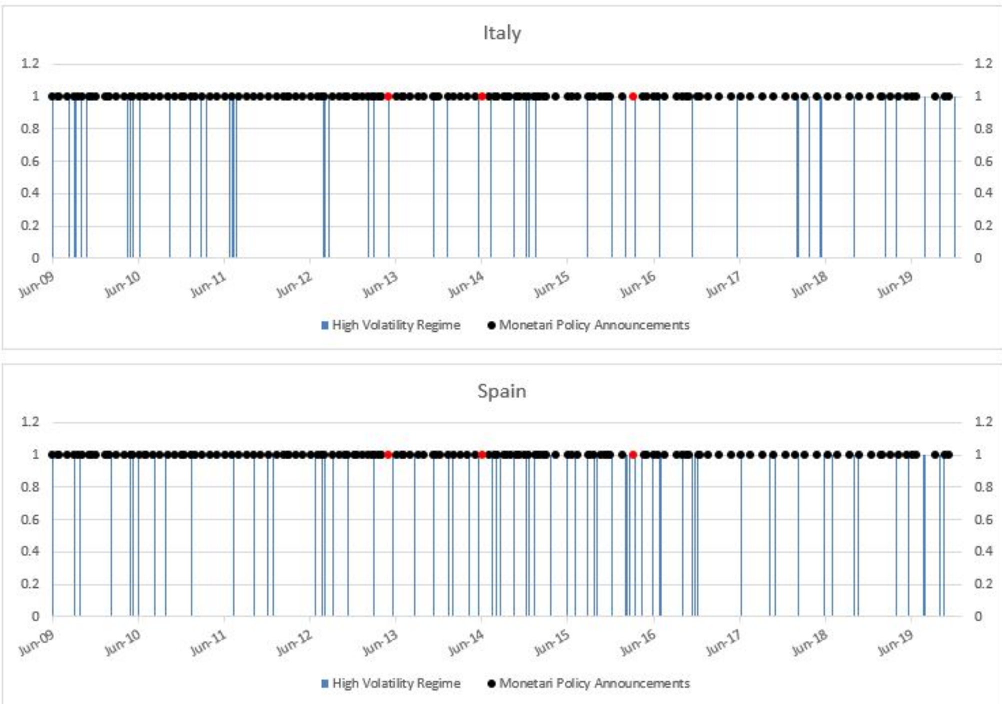

FIGURE 3.2: Unconventional policy announcements and high regime volatility probability estimated from MS(ς)-LACM. Sample period: June 1, 2009 - December 31, 2019

3.4.2 MS(ξ)-ACM results

As largely anticipated, in a further analysis, we have investigated the possibility that the switching process involves the ξ_t component. We analyse this scenario by replacing the announcements dummy variable with a switching constant: the idea behind this model is to capture possible jumps in this process, even those previously caused by the dummy variable. Results (Tables 3.7 - 3.9), seem to support this intuition. Focusing on the MS(ξ)-ACM and MS(ξ)-LACM, the constant in the low volatility regime is always equal to zero, whereas it increases remarkably in the high regime. As usual, that increment is larger in the MS(ξ)-ACM, where it ranges (Tables 3.7) between 4.56 (Italy) ad 6.42 (Germany). Also in this case, the unconventional policy proxy enters the model with the expected negative sign, with the strongest impact observed for Spain (-1.09) and the weakest recognized for Germany (-0.44). Interestingly, we do not identify significant differences in the ς parameters, if compared with those estimated from the MS(ς) models. Even considering the MS(ξ) specifications, the probability of remaining in the low regime is higher than that of the high volatility regime; however, differently from the previous set of models, both in the MS(ξ)-ACM and in the MS(ξ)-LACM, now the high regime probability coefficient is significant also in peripheral countries. Nonetheless, this does not change the duration of the high regime, which is 1 day for all the considered countries, while the duration of the low regime ranges between 14 days (Italy) and 53 days (Germany).

Results remain almost unchanged when we estimate the MS(ξ)-LiACM (Table 3.9). In analysing this model, it is necessary to remember that the constant in the low regime is no longer estimated, it is rather constructed empirically. In the low volatility regime, there are no differences in the level of the constant across countries (ranging between 0.86 and 0.96 for Germany and Italy, respectively), whereas, in the high regime, it increases of about 0.4 for Italy and Spain; for France and Germany, instead, the constant increases by about 0.1 in the high volatility regime. Finally, none of the models show a significant autoregressive coefficient (ψ), but in all the cases they successfully remove residual autocorrelation.

Interestingly, in all the cases these Markov switching specifications clearly improve the statistical property of our model: the null hypothesis of the Ljung-Box test is never rejected at 1% level in all the considered countries. Differently from the other specifications, also for Spain our model is able to capture the autoregressive structure of volatility.

Once again, model comparisons in Table 3.10 shows how the MS(ξ)-LACM works better in core countries, whereas the linear specification is the best for what concerns Italy and Spain. For this reason, in what follows, we refers to these specifications in carrying out a probability inference analysis.

Since our constant is estimated in place of the dummy variable, what is important to check is whether the constant is able to capture the volatility jump caused by the monetary policy announcements. For this reason, in Figure 3.3, we compare the weighted average of the constant with the announcement variable, for the case of Italy. In brief, it seems that the weighted average jumps in correspondence of monetary policy announcements and it is the case, in particular, of the unconventional policy announcements. Here, we highlight the two dates analysed in Figure 3.3 (May 2nd, 2013 and March 10th, 2016) plus the announcement on June 5th, 2014, which refers to the establishment of a new TLTRO programme. It is interesting to notice how returns responded to these announcements. Whereas the announcement on May 2nd, 2013 did not affect stock price, with all the indexes closing around the parity with respect to the previous day, the other two announcements had a stronger impact: in details, investors perceived favourably the launch of the TLTRO in June 2014, with an increment of 1.1% for France and Spain and of about 1.5% for Italy; differently, the increase of monthly purchases within the EAPP depressed stock market returns, with a reduction between 0.5% for Italy and 2.3% for Germany. More important, in all the cases the probability of the high regime is near to 1, confirming the intuition that our constant could successfully replace the announcement variable in order to allow the switching process in the ξ_t component. Moreover, we observe a reduction in the weighted average of φ after the announcements, consistently with the average duration of the high volatility regime that is equal to 1 in this country.

This scenario is also confirmed in Table 3.11 in which: the first column refers to the number of cases in which the process is in the high volatility regime (as represented by the smoothed probabilities, i.e. $P_T(s_t = 1)$ in Table 3.11); the second column, instead, counts the number of cases in which the high regime corresponds to a monetary policy announcement; finally, in the last column a sort of "rate of success" is computed, i.e. the number of cases in which the high volatility regime corresponds to the announcement, over the number of cases in which the process is in the high volatility regime as well. The rate of success is about 22%, meaning that, during the sample period, the process was in the high volatility regime because of a monetary policy announcement in 1 out of 4 cases. The rate of success is perhaps less than one might expect; however, in analysing this result, it should be kept in mind that only in 74 out of 144 cases announcements concerns the unconventional policy measures and that in most of the cases the announcement was largely expected from market agents.

TABLE 3.7: MS(ξ)-ACM. Proxy: UMP/CMP

a)	**France**	**Germany**	**Italy**	**Spain**
ω	0.853 ***	0.661 ***	1.063 ***	1.059 ***
	(0.083)	(0.095)	(0.18)	(0.029)
α	0.142 ***	0.185 ***	0.228 ***	0.151 ***
	(0.016)	(0.016)	(0.02)	(0.015)
β	0.732 ***	0.722 ***	0.649 ***	0.732 ***
	(0.02)	(0.021)	(0.032)	(0.017)
γ	0.112 ***	0.082 ***	0.08 ***	0.086 ***
	(0.01)	(0.009)	(0.012)	(0.008)
δ	-0.775 ***	-0.44 ***	-0.741 ***	-1.09 ***
	(0.125)	(0.105)	(0.135)	(0.125)
φ_0	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
φ_1	6.275 ***	6.422 ***	4.556 ***	6.161 ***
	(1.743)	(1.941)	(1.324)	(2.379)
ψ	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
p_{00}	0.964 ***	0.981 ***	0.928 ***	0.943 ***
	(0.024)	(0.014)	(0.028)	(0.032)
p_{11}	0.222 **	0.304 **	0.338 ***	0.314 ***
	(0.1)	(0.119)	(0.118)	(0.082)
θ_0	8.852 ***	11.249 ***	15.033 ***	11.99 ***
	(0.375)	(0.489)	(0.979)	(0.704)
θ_1	3.271 ***	2.598 ***	4.112 ***	3.777 ***
	(0.71)	(0.735)	(0.836)	(0.682)
b)				
Ljung-Box 1 lag	0.711	0.161	0.403	0.22
Ljung-Box 5 lag	0.263	0.121	0.449	0.087 *
Ljung-Box 10 lag	0.364	0.276	0.639	0.34

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 3.8: MS(ζ)-LACM. Proxy: UMP/CMP

a)	France	Germany	Italy	Spain
ω	0.862 ***	0.684 ***	1.084 ***	1.135 ***
	(0.081)	(0.077)	(0.142)	(0.09)
α	0.137 ***	0.184 ***	0.223 ***	0.144 ***
	(0.016)	(0.015)	(0.023)	(0.015)
β	0.733 ***	0.721 ***	0.65 ***	0.731 ***
	(0.021)	(0.02)	(0.028)	(0.016)
γ	0.114 ***	0.082 ***	0.079 ***	0.086 ***
	(0.01)	(0.009)	(0.007)	(0.006)
δ	-0.159 ***	-0.09 ***	-0.125 ***	-0.183 ***
	(0.022)	(0.019)	(0.019)	(0.022)
φ_0	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
φ_1	1.227 ***	1.32 ***	0.749 ***	1.001 ***
	(0.126)	(0.206)	(0.165)	(0.009)
ψ	0.000	0.000	0.000	0.035
	(0.000)	(0.000)	(0.000)	(0.074)
p_{00}	0.961 ***	0.981 ***	0.923 ***	0.94 ***
	(0.016)	(0.013)	(0.025)	(0.017)
p_{11}	0.193 *	0.299 *	0.293 ***	0.26 ***
	(0.104)	(0.174)	(0.072)	(0.098)
θ_0	8.905 ***	11.274 ***	15.034 ***	12.029 ***
	(0.325)	(0.453)	(0.839)	(0.483)
θ_1	3.431 ***	2.617 ***	4.217 ***	4.092 ***
	(0.628)	(0.69)	(0.754)	(0.798)
b)				
Ljung-Box 1 lag	0.957	0.044 **	0.189	0.369
Ljung-Box 5 lag	0.238	0.045 **	0.204	0.098 *
Ljung-Box 10 lag	0.42	0.157	0.334	0.326

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 3.9: MS(ξ)-LiACM. Proxy: UMP/CMP

a)	**France**	**Germany**	**Italy**	**Spain**
ω	1.545 ***	1.617 ***	1.12 ***	1.172 ***
	(0.22)	(0.14)	(0.108)	(0.215)
α	0.108 ***	0.156 ***	0.23 ***	0.149 ***
	(0.029)	(0.018)	(0.031)	(0.026)
β	0.709 ***	0.677 ***	0.651 ***	0.731 ***
	(0.041)	(0.02)	(0.032)	(0.04)
γ	0.132 ***	0.101 ***	0.082 ***	0.089 ***
	(0.015)	(0.01)	(0.02)	(0.013)
δ	-0.139 ***	-0.108 ***	-0.06 ***	-0.088 ***
	(0.037)	(0.014)	(0.009)	(0.021)
φ_0	0.886	0.864	0.964	0.932
φ_1	0.18 ***	0.189 ***	0.347 ***	0.447 ***
	(0.034)	(0.019)	(0.103)	(0.144)
ψ	0.000	0.000	0.000	0.034
	(0.001)	(0.001)	(0.000)	(0.2)
p_{00}	0.992 ***	0.995 ***	0.92 ***	0.939 ***
	(0.005)	(0.002)	(0.035)	(0.03)
p_{11}	0.996 ***	0.998 ***	0.296	0.264 **
	(0.006)	(0.001)	(0.229)	(0.12)
θ_0	8.69 ***	9.576 ***	15.108 ***	12.05 ***
	(0.515)	(0.469)	(1.214)	(0.635)
θ_1	7.616 ***	10.855 ***	4.328 ***	4.159 ***
	(0.46)	(0.531)	(1.16)	(0.828)
b)				
Ljung-Box 1 lag	0.498	0.302	0.18	0.394
Ljung-Box 5 lag	0.115	0.081 *	0.199	0.098 *
Ljung-Box 10 lag	0.297	0.018 **	0.328	0.326

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 3.10: MS(ξ) models comparison

France

	MS(ξ) ACM	MS(ξ) L-ACM	MS(ξ) Li-ACM
Log-lik	-7757.171	**-7752.837**	-7763.977
AIC	5.787	**5.784**	5.791
BIC	5.813	**5.81**	5.816
MSE	26.135	**24.78**	27.946
MAE	3.674	**3.64**	3.733

Germany

	MS(ξ) ACM	MS(ξ) L-ACM	MS(ξ) Li-ACM
Log-lik	-7484.367	**-7481.553**	-7491.33
AIC	5.584	**5.582**	5.588
BIC	5.61	**5.608**	5.612
MSE	20.958	**19.898**	22.223
MAE	3.264	**3.246**	3.287

Italy

	MS(ξ) ACM	MS(ξ) L-ACM	MS(ξ) Li-ACM
Log-lik	-7587.671	-7587.282	**-7586.442**
AIC	5.661	5.66	**5.659**
BIC	5.687	5.687	**5.683**
MSE	24.314	22.856	**22.658**
MAE	3.396	3.356	**3.344**

Spain

	MS(ξ) ACM	MS(ξ) L-ACM	MS(ξ) Li-ACM
Log-lik	-8008.474	-8000.679	**-8000.329**
AIC	5.974	5.968	**5.967**
BIC	6.001	5.995	**5.992**
MSE	36.292	32.84	**32.552**
MAE	4.007	3.93	**3.918**

Note: MS(ξ) models comparison via Information Criteria (AIC and BIC) and forecast capability (MSE and MAE). Sample period: June 1, 2009 - December 31, 2019

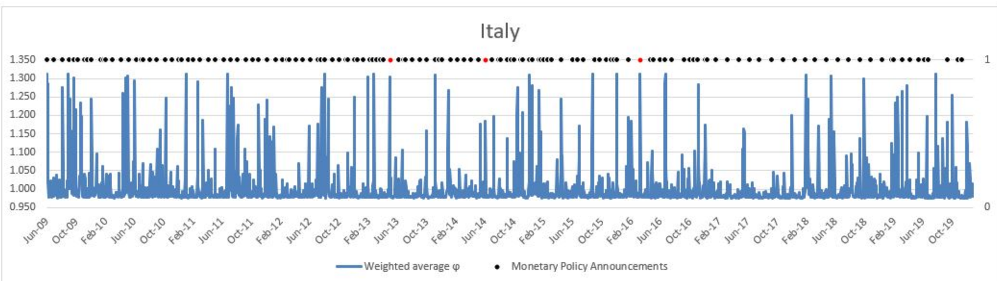

FIGURE 3.3: Weighted average of φ resulting from the MS(ξ) $-$ $LiACM$ and announcement variable. Sample period: June 1, 2009 - December 31, 2019

TABLE 3.11: Announcement variable and high volatility regime

	$P_T(s_t = 1) > 0.5$	$P_T(s_t = 1) > 0.5$ and $\Delta_t = 1$	Rate of Success
France	29	6	21%
Germany	23	5	22%
Italy	96	22	23%
Spain	62	15	24%

Note: Comparing high volatility regime ($P_T(s_t = 1) > 0.5$) and the announcement variable. Models: MS(ξ) L-ACM (France and Germany) and MS(ξ)-LiACM (Italy and Spain). Sample period: June 1, 2009 - December 31, 2019

3.4.3 TVTP-AMEM results

The last analysis we carry out represents a further proof that ECB gave a crucial contribution in maintaining volatility as low as possible. This conclusion derives form the TVTP model we have estimated on the index of the two peripheral countries in our sample[4]. Results are shown in Table 3.12. The first two columns refer to the model estimated without considering any constraint: all the coefficients of the mean equation are highly significant and they are close to coefficients estimated via the MS-AMEMX. More interesting is the analysis concerning probability coefficients. In particular, looking at delta coefficients, it turns out a no significant coefficient in the high volatility regime for Spain. Generally, it could depend from the fact that, as we have seen before, we have few observations for this particular regime and, for this reason, we prefer to discuss the model estimated by setting $\delta_1 = 0$. Results, which are reported in the last two columns of Table 3.12, are similar to those deriving from the unrestricted model. φ coefficient is higher in the low volatility regime, meaning that, even in absence of unconventional policies (i.e. a fixed probability Markov switching model) the probability of remaining in the low regime is higher than probability of remaining in the high regime. More important, in both cases δ_0 is positive and significant at 1% level: this means that, once the process is in the low regime, an increase of our proxy (for example because of the implementation of a new programme as well as the strengthening of existing programmes) rises the probability of staying in that regime, rather than to move in the other.

Finally, both the models seem to be particularly weak in terms of Ljung-Box statistics: in details, residual autocorrelation is present not only in the

[4]Unfortunately, we have had some computational problems in estimating this model on the core countries so that results for these indexes are not available.

case of Spain - where it emerges quite clearly, i.e. the p-values always approach zero - but also in the case of Italy, at the first lag.

Similarly to the analysis made through the MS(ς)-LACM, Figure 3.4 compares the high volatility regime with the RV measure (This analysis is based on the restricted model, which turns out to be the best model in terms of AIC and BIC, as shown in Table 3.13). Also in this case the high volatility regime seems to coincide with volatility spikes. In general, we can say that the low volatility regime is the one prevailing during the sample period, and only few times the process moves to the high volatility regime. Indeed, coherently with the no-significance of δ_1, there are just few cases in which it emerges an extended period of high volatility. Two examples are represented by the Greek crisis period (around May 2010) and the period around May 2012, when the crisis reached also Italy and Spain.

Therefore, we could conclude that with our analysis we find evidence supporting the view that the ECB was able to exert a calming effect in financial markets, keeping volatility as low as possible and contributing to mitigate losses caused by the worst financial crisis the Europe experienced since the Great Depression in 1929.

TABLE 3.12: TVTP-AMEM. Proxy: UMP/CMP

a)	**Unrestricted**		**Restricted**	
	Italy	**Spain**	**Italy**	**Spain**
ω_0	0.663 ***	0.586 ***	0.665 ***	0.592 ***
	(0.1)	(0.113)	(0.1)	(0.154)
ω_1	3.389 ***	3.43 ***	3.337 ***	3.466 ***
	(0.505)	(0.49)	(0.489)	(0.448)
α	0.214 ***	0.136 ***	0.214 ***	0.137 ***
	(0.024)	(0.019)	(0.025)	(0.02)
β	0.678 ***	0.756 ***	0.678 ***	0.754 ***
	(0.026)	(0.024)	(0.026)	(0.028)
γ	0.075 ***	0.079 ***	0.075 ***	0.079 ***
	(0.007)	(0.008)	(0.008)	(0.008)
φ_0	1.507 ***	1.13 ***	1.492 ***	1.14 ***
	(0.433)	(0.352)	(0.433)	(0.325)
φ_1	-8.728 *	-2.778 ***	-4.262 *	-3.191 **
	(4.558)	(0.761)	(2.316)	(1.304)
δ_0	1.026 ***	0.915 ***	1.029 ***	0.938 ***
	(0.315)	(0.19)	(0.296)	(0.196)
δ_1	-6.982 ***	-0.399	-	-
	(1.512)	(0.259)		
θ_0	14.404 ***	12.594 ***	14.446 ***	12.545 ***
	(0.734)	(0.585)	(0.716)	(0.541)
θ_1	4.896 ***	5.051 ***	4.93 ***	5.017 ***
	(1.174)	(0.687)	(1.149)	(0.672)
b)				
Ljung-Box 1 lag	0.003 ***	0.000 ***	0.004 ***	0.000 ***
Ljung-Box 5 lag	0.038 *	0.000 ***	0.049 *	0.000 ***
Ljung-Box 10 lag	0.126	0.001 ***	0.153	0.001 ***

Note: Dependent variable: Annualized realized kernel volatility. **a)** Coefficients (s.e. in parenthesis) and **b)** Ljung-Box statistics (p-values). P–values: *(<10%); **(< 5%); ***(< 1%). Sample period: June 1, 2009 - December 31, 2019. Observations: 2685

TABLE 3.13: TVTP-AMEM models comparison.

	Italy		Spain	
	Unrestricted	**Restricted**	**Unrestricted**	**Restricted**
Log-lik	-7593.703	**-7593.684**	-8014.585	**-8014.638**
AIC	5.665	**5.664**	5.978	**5.977**
BIC	5.689	**5.686**	6.002	**5.999**
MSE	25.169	**25.147**	**37.733**	37.776
MAE	3.411	**3.41**	**3.987**	3.99

Note: Comparing Restricted and Unrestricted TVTP-AMEM via Information Criteria (AIC and BIC) and forecast capability (MSE and MAE). Sample period: June 1, 2009 - December 31, 2019

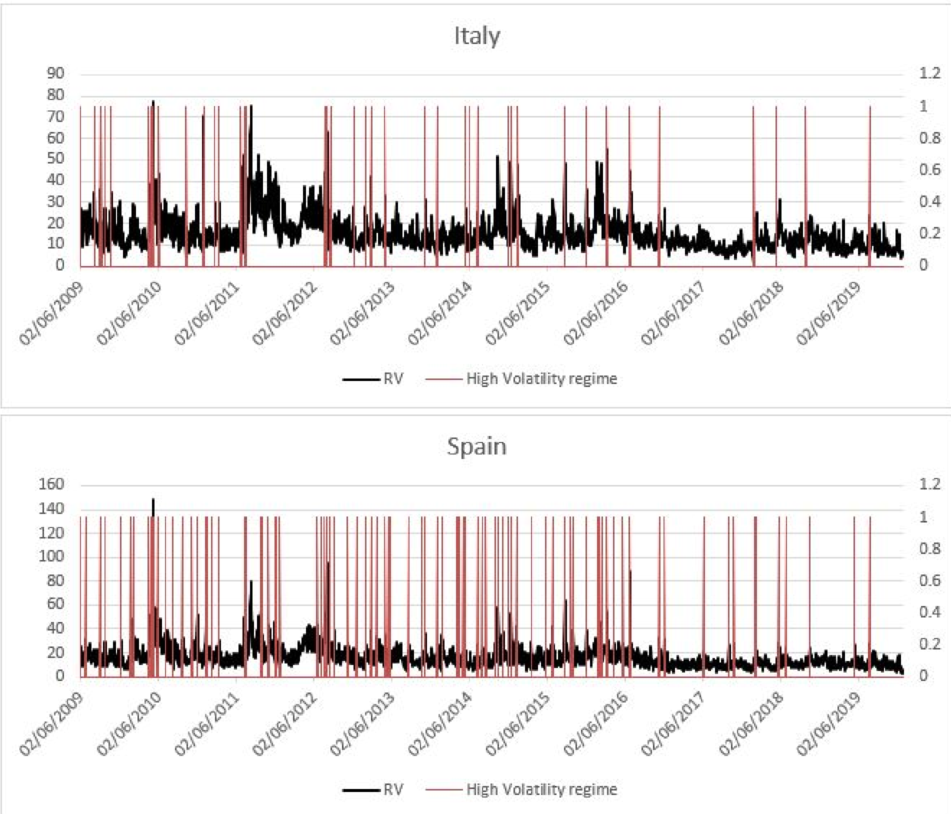

FIGURE 3.4: TVTP-AMEM: RV and High volatility regime. Sample period: June 1, 2009 - December 31, 2019

3.5 Comparing forecasting performance

In this section an analysis on the forecasting performance of the estimated models is presented. Similarly to the analysis we have carried out in chapter 2, we base model comparisons on the MCS procedure, with the 90% confidence level, basing on the one-step-ahead forecast for the period going from January 1st, 2019 to December 31st, 2019. In addition, for each of the stock indexes, we consider the best model resulting from the analysis in chapter 2 as a benchmark model, that is the Li-ACM with UMP/CMP as a proxy for unconventional policies. As shown in Figure 3.5, in terms of MSE, the MS-(ξ)-LACM is confirmed to be the best model (the green bar) in 3 out of 4 cases, i.e. France, Germany and Spain, where the MS-(ξ)-ACM[5] is the only model belonging to the best set (as represented by the blue bars). In addition, the latter is the model with the best one-step-ahead forecast performance for Italy. What is important to notice is how none of the MS(ς) models belong to the best set of models, giving evidence in favour of our intuition that ECB successfully impacted the part of volatility depending directly on unconventional policies; moreover, neither the benchmark model nor the MS-(ς)-AMEM enter the best set of models (models excluded from the best set are represented by the black bars), highlighting the best forecasting performance of more complex models.

Basing on this consideration, the MS-(ξ)-LACM is selected to carry out the multi-step-ahead forecast analysis. Differently from the multi-step-ahead forecasting analysis in Chapter 2, the presence of regime switching makes it hard for volatility to converge to a long run level. Indeed, as shown in Figure 3.7, in most of the models also the forecasted volatility is subject to the two different regimes and, more important, it emerges how the forecasted series is able to reproduce (Figure 3.8) the non-linear features which characterize the RV series. This is also confirmed by Figure 3.9, which compares the realized volatility during the forecasting period and the estimated probability of being in the high regime. Basically, in all the indexes, it is evident how the model is able to reproduce the volatility spikes observed in May - probably because of the European election - or in August, when there was a period of instability due to the fall of the government in Italy.

[5]This model is the best one in 3 out of 4 cases if we consider the QLike, as shown in Figure 3.6.

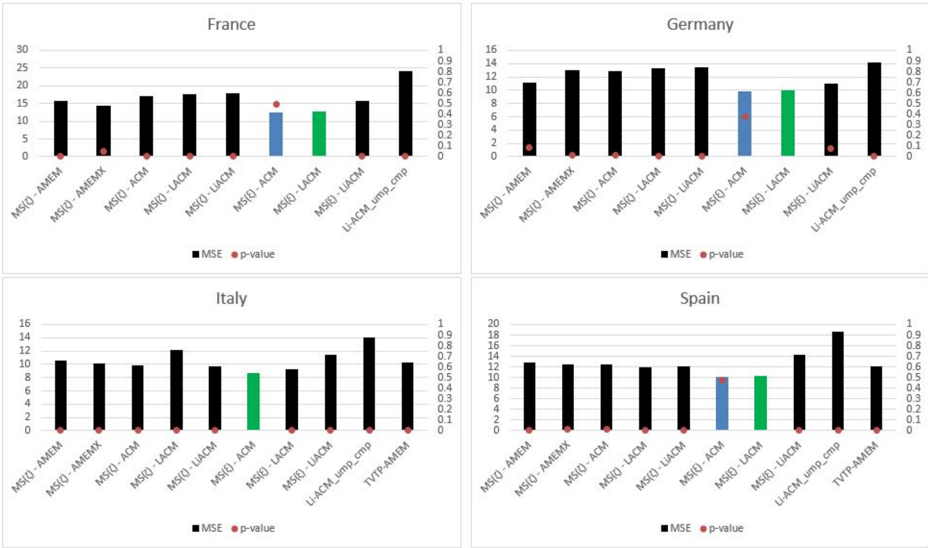

FIGURE 3.5: Model Confidence Set. Loss function: MSE. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

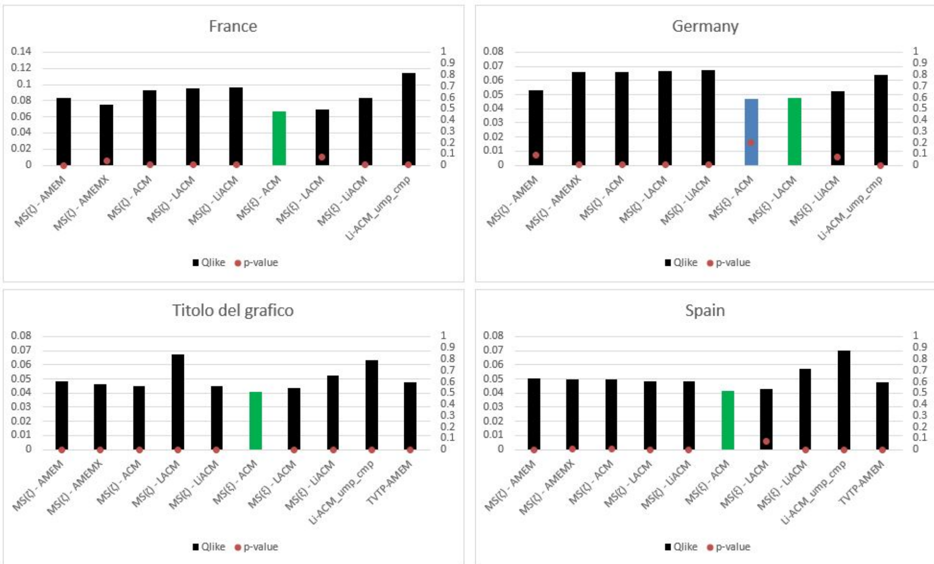

FIGURE 3.6: Model Confidence Set. Loss function: Qlike. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019

Note. Bars: green (the best model); blue (models belonging to the best set); black (excluded models).

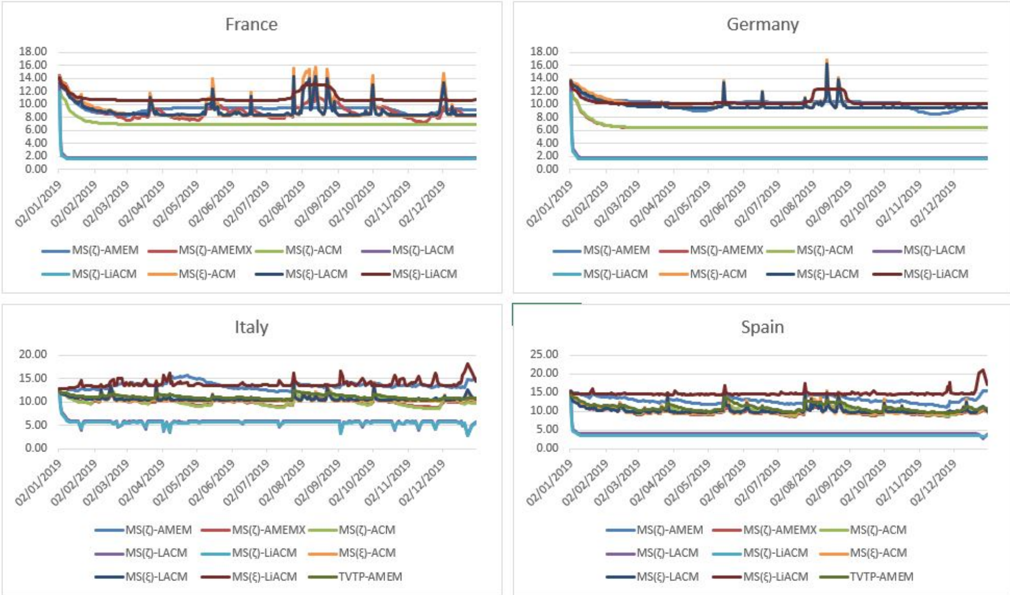

FIGURE 3.7: Multi-step forecast. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019

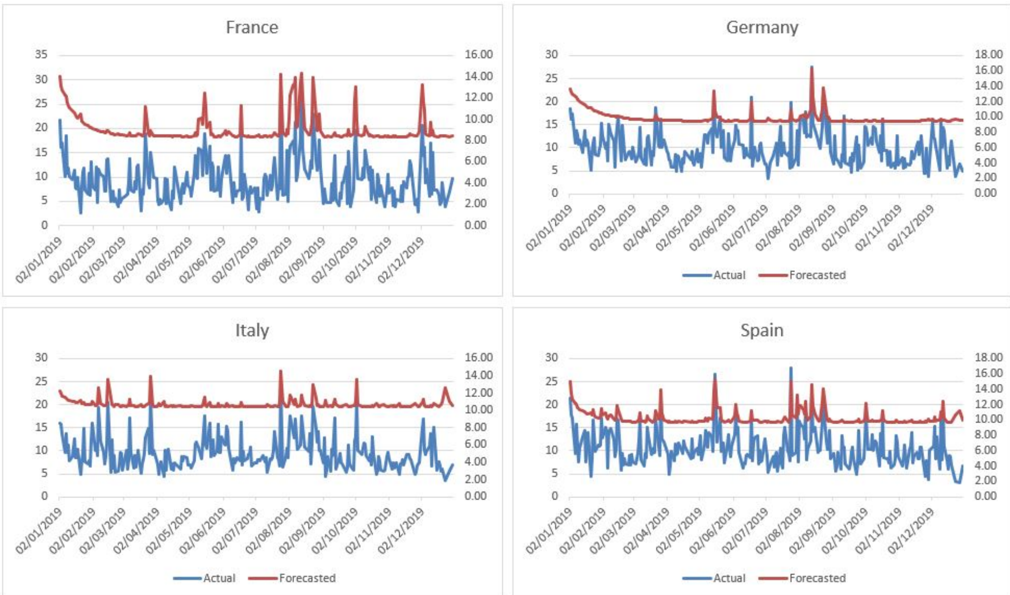

FIGURE 3.8: Actual vs forecasted volatility. Model: MS(ξ)-LACM. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019

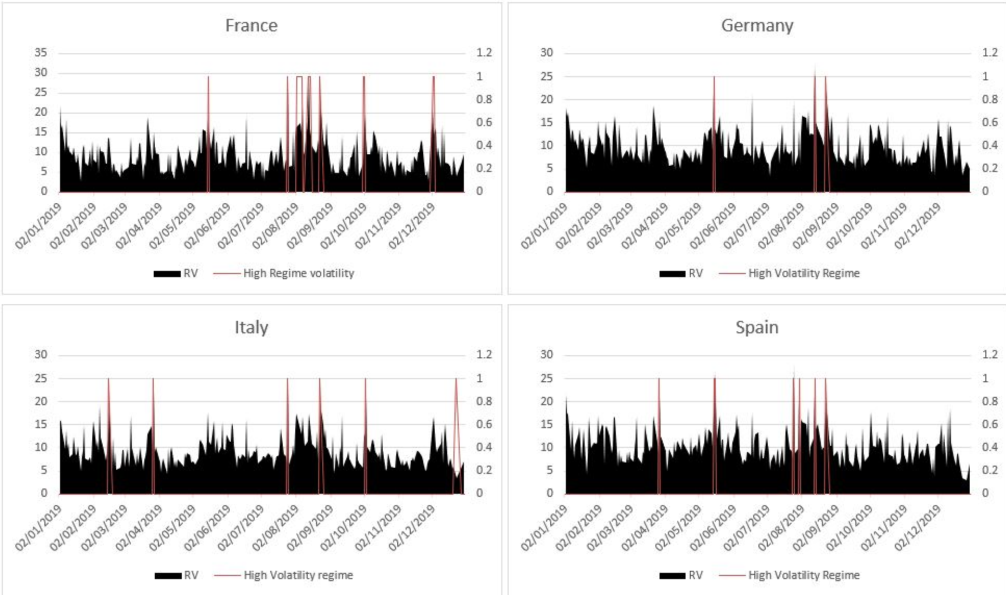

FIGURE 3.9: RV and forecasted high probability regime. Model: MS(ξ)-LACM. Estimation period: June 1, 2009 - December 31, 2018. Forecast period: January 1, 2019 - December 31, 2019

3.6 Conclusion

In this chapter, the models we have presented in Chapter 2 are extended within the Markov switching framework. More in details, we consider only two regimes (corresponding to the low and the high volatility regime, respectively) allowing the switching in the constant parameter and aiming to test the ECB ability to keep volatility within the low regime. In this respect we consider different specifications of the MS-ACM. In a first analysis, we allow changes in regimes in the constant of the ς process; in a second set of models, we analyse whether the ECB was able to affect directly the part of volatility depending on unconventional policies, as represented by the ξ process. In this regard, we have replaced the announcement variable with a switching constant, which requires the use of the ergodic probability in the MS(ξ)-LiACM specification. Finally, by means of a Time Varying Transition Probabilities Markov switching model, we have analysed whether the ECB has also affected the regimes probabilities. In line with what we find in Chapter 2, results give a clear evidence in favour of the calming effect that unconventional policies have on market volatility.

The difference between core and peripheral countries emerges quite clearly in the MS(ς)-ACM models: the low volatility regime is the one prevailing in peripheral countries, with an average duration of 12 and 9 days for Italy and

Spain, respectively. In core countries, instead, both the regimes are well defined, with an average duration of 83 (low regime) and 48 (high regime) days for France, while the average duration is even higher for Germany (250 and 333 days). The difference disappears in the second analysis, which refers to the estimation of the MS(ξ)-ACM models. Focusing on the additive and on the logistic specification, which are the best models according to the MCS procedure, it emerges how the low volatility regime is the prevailing regime in all the cases, with an average duration between 14 (Italy) and 53 days (Germany). More important, the constant in the low regime is equal to zero, whereas there is a remarkable increase in the high regime (up to 1.32 points, if we focus on the MS(ξ)-LACM): in other words, it seems that our switching constant was able to reproduce the announcement variable, with jumps in correspondence of announcements days (which in most of the cases correspond to the high volatility regime). In addition - differently to the MS(ς)-ACMs as well as to the TVTP-AMEM - the MS(ξ)-ACMs are the only ones that are able to capture the autoregressive structure of volatility for all the indexes - i.e. the null hypothesis of the Ljung-Box test, which states the presence of residual autocorrelation, is never rejected at a 1% significance level.

Interesting results also derive from the TVTP-AMEM, from which it emerges a positive coefficient associated with the proxy in the low regime, meaning that an increase in the level of the proxy leads to a higher probability of remaining in this regime; differently, the proxy enters the high regime probability with a negative sign, so that as the proxy increases, the probability of remaining in this regime decreases. However, even though this model has a MSE lower than that of some MS-ACM specifications, it never belongs to the best set of model, as it turns out from the MCS procedure. The latter shows as the MS(ξ)-ACM is the best model for Italy, whereas it belongs to the best set for the other countries, for which the MS(ξ)-LACM is always the best model.

Therefore, in conclusion, it seems that the calming effect of the ECB unconventional monetary policies it is more than a short-term effect, since the ECB was able to impact directly the part of volatility depending on this kind of policy, keeping it within a low regime for an extended period of time.

Appendix Chapter 2

A.1 Preliminary analysis

In this appendix we present a preliminary analysis on the proxies employed in this research. Table A.1 shows the descriptive statistics. As expected, the qualitative proxies (UMP/TA and UMP/CMP) have similar characteristics. It derives from the fact that they differ only in the definition of their denominator. Whereas in the UMP/CMP the denominator is defined as the sum of conventional monetary policy tools in the ECB balance sheet (i.e. Open Market Operations Marginal Lending Facility, Deposit Facility, Autonomous Liquidity Factor and Current Account) in the UMP/TA the denominator is constructed by adding also the unconventional policies themselves. For this reason the UMP/TA has values between 0 and 1, while the other proxy could have values grater than 1. These similar characteristics are also shown by Figure A.1 which plots the evolution of the variables over time. While the TA growth is a stationary series as long as it represents a growth rate, the other proxies are not stationary and it emerges clearly the upward trend they have since 2015, when the ECB decided on the EAPP. As said, the aim of this appendix is to perform a preliminary analysis concerning the proxy in order to understand what is the Data Generating Process (DGP) under the variables themselves. Looking at Figure A.2, which plots the ACF and PACF of our series, it seems they follow a White noise process. Actually, whereas it is the case of the UMP/TA, both UMP/CMP ad TAgrowth need a further analysis. In the case of the UMP/CMP - which requires to consider the first difference of the variable in order to make it stationary - even though the correlogram (Figure A.3) suggests an ARIMA (4,1,3), it emerges how the GDP process driving this proxy is an ARIMA(4,1,1) with the second specific lag for the MA component. The white noise residuals (A.4) obtained from the estimation of this model (estimation results are briefly reported in Table A.2) supports the choice of this ARIMA specification so that we can use the current fitted values as a proxy for our unconventional policy measures. Finally, for what concerns the TAgrowth proxy, the GDP seems to follow an ARIMA (4,0,1) as shown in Figure A.2. This is also confirmed by Figure A.5, which shows the correlogram of model residuals, whereas results are shown in Table A.3.

TABLE A.1: Unconventional policies proxies' descriptive statistics. Sample period: June 1, 2009 to December 31, 2019. Observation: 2686.

	UMP/TA	UMP/CMP	TAgrowth
Mean	0.227	0.645	0.000
Median	0.069	0.074	0.000
Min	0.000	0.000	-0.822
Max	0.681	2.132	0.82
St.Dev.	0.268	0.725	0.65
Skewness	0.358	0.617	0.32
Kurtosis	-1.705	-1.352	65.591

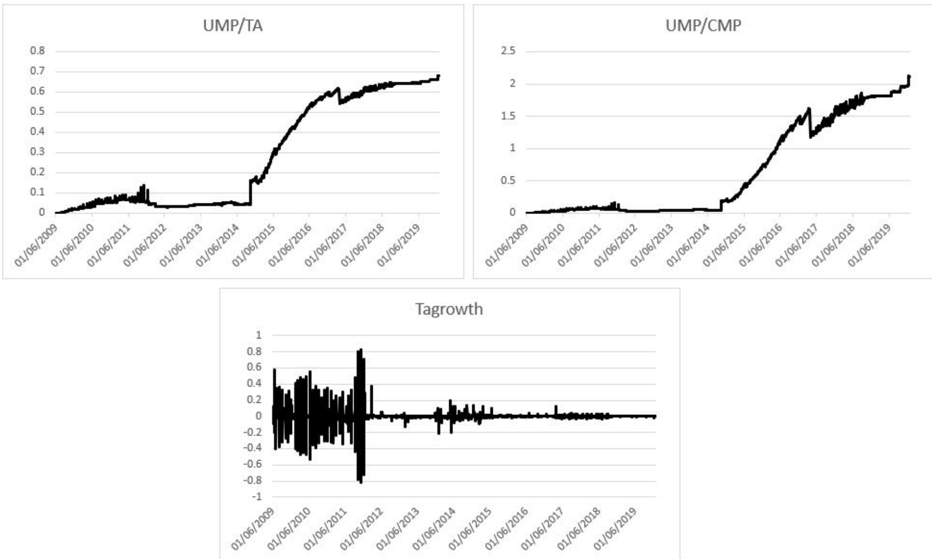

FIGURE A.1: UMP/TA, UMP/CMP and TAgrowth. Sample Period: 01/01/2009 – 30/12/2019.

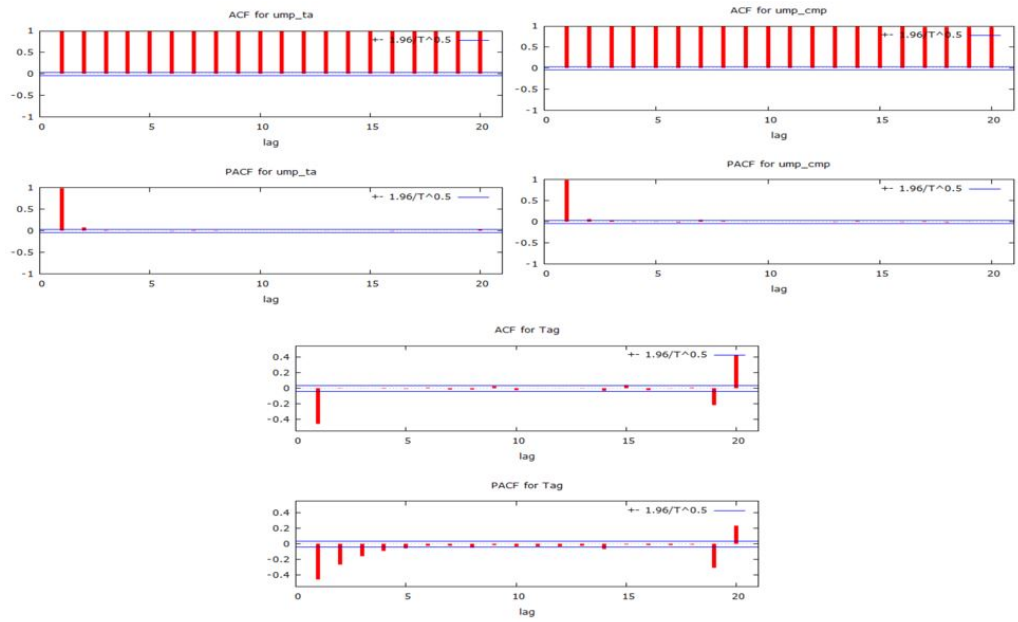

FIGURE A.2: ACF and PACF

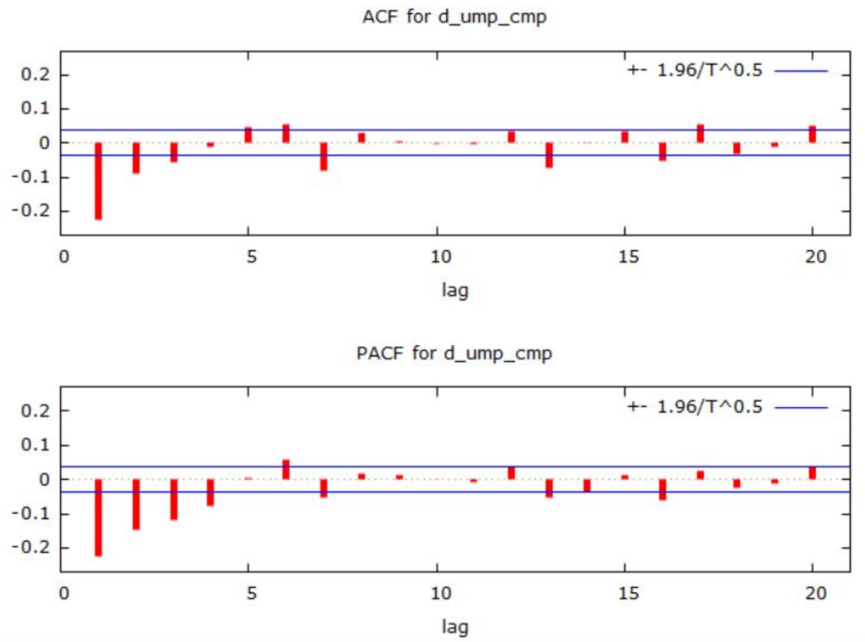

FIGURE A.3: ACF and PACF of the UMP/CMP first differences

TABLE A.2: ARIMA (4,1,2). Dependent variable: UMP/CMP. Observations: 2685

	coeff	s.e.	z	p-value
constant	0.001	0.000	3.659	0.000
AR_1	-0.284	0.019	-14.795	1.67E-49
AR_2	-0.505	0.111	-4.541	5.6e-06
AR_3	-0.219	0.03	-7.23	4.82e-013
AR_4	-0.129	0.024	-5.457	4.83e-08
MA_2	0.321	0.114	2.81	0.005

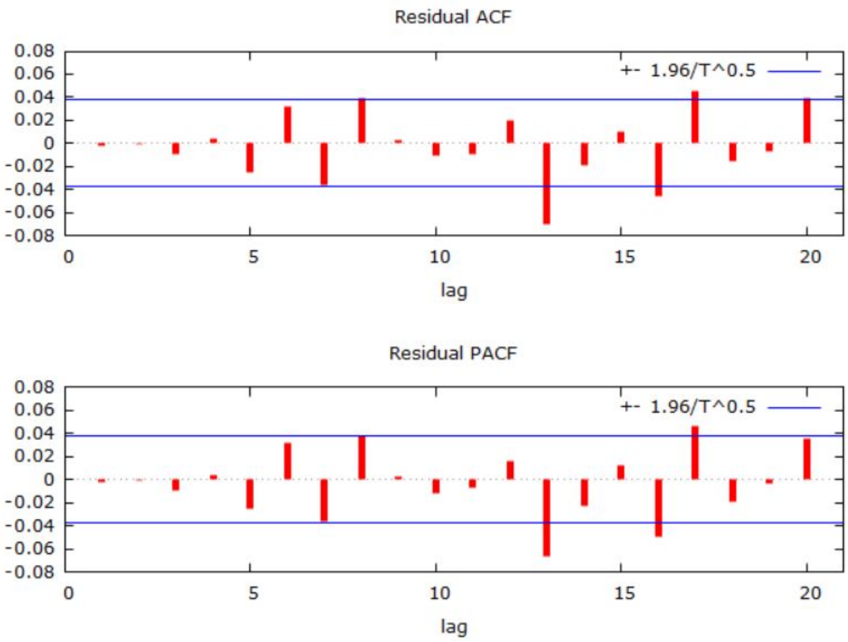

FIGURE A.4: ACF and PACF from ARIMA(4,1,1). Proxy: UMP/CMP

TABLE A.3: ARIMA (4,0,1). Dependent variable: TAgrowth. Observations: 2685

	coeff	s.e.	z	p-value
constant	0.000	0.000	1.353	0.176
AR_1	0.204	0.037	5.488	4.07e-08
AR_2	0.141	0.029	4.812	1.49e-06
AR_3	0.093	0.025	3.722	0.000
AR_4	0.06	0.023	2.652	0.008
MA_1	-0.856	0.031	-27.19	8.17e-163

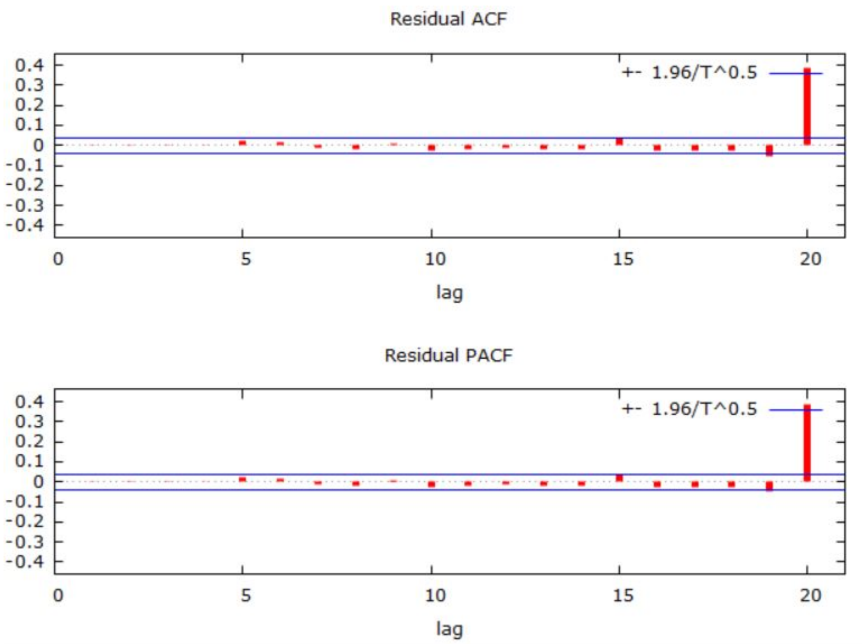

FIGURE A.5: ACF and PACF from ARMA(4,0,1). Proxy: TAgrowth

A.2 Marginal effects

In this appendix we derive, for each model, the marginal effects of the policy variables on the volatility in terms of first partial derivatives of $\mu_{t+\tau}$ with respect to x_{t-1} and Δ_t, as illustrated in Section 2.3.2 (Table 2.2).

AMEMX

$$
\begin{aligned}
RV_t &= \mu_t \epsilon_t \\
\mu_t &= \omega + (\alpha + \gamma D_{t-1}) RV_{t-1} + \beta \mu_{t-1} + \delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) \\
\mu_{t+1} &= \omega + (\alpha + \gamma D_t) RV_t + \beta \mu_t + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) \\
&= \omega + (\alpha + \gamma D_t) RV_t + \beta[\omega + (\alpha + \gamma D_{t-1}) RV_{t-1} + \beta \mu_{t-1} + \\
&\quad + \delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star)] + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) \\
&= \omega(1+\beta) + \alpha(RV_t + \beta RV_{t-1}) + \gamma(D_t RV_t + \beta D_{t-1} RVt - 1) + \beta^2 \mu_{t-1} + \\
&\quad + \delta[(x_t - x^\star) + \beta(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+1} - \Delta^\star) + \beta(\Delta_t - \Delta^\star)] \\
\mu_{t+2} &= \omega + (\alpha + \gamma D_{t+1}) RV_{t+1} + \beta \mu_{t+1} + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) \\
&= \omega + (\alpha + \gamma D_{t+1}) RV_{t+1} + \beta[\omega(1+\beta) + \alpha(RV_t + \beta RV_{t-1}) + \\
&\quad + \gamma(D_t RV_t + \beta D_{t-1} RVt - 1) + \beta^2 \mu_{t-1} + \delta[(x_t - x^\star) + \beta(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+1} - \Delta^\star) + \beta(\Delta_t - \Delta^\star)] + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) \\
&= \omega(1+\beta+\beta^2) + \alpha(RV_{t+1} + \beta RV_t + \beta^2 RV_{t-1}) + \\
&\quad + \gamma(D_{t+1} RV_{t+1} + \beta D_t RV_t + \beta^2 D_{t-1} RVt - 1) + \beta^3 \mu_{t-1} + \\
&\quad + \delta[(x_{t+1} - x^\star) + \beta(x_t - x^\star) + \beta^2(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+2} - \Delta^\star) + \beta(\Delta_{t+1} - \Delta^\star) + \beta^2(\Delta_t - \Delta^\star)] \\
&\vdots \\
\mu_{t+\tau} &= \omega(1 + \beta + \cdots + \beta^t au) + \alpha(RV_{t+\tau-1} + \cdots + \beta^\tau RV_{t-1}) + \\
&\quad + \gamma(D_{t+\tau-1} RV_{t+\tau-1} + \cdots + \beta^\tau D_{t-1} RVt - 1) + \beta^{\tau+1} \mu_{t-1} + \\
&\quad + \delta[(x_{t+\tau-1} - x^\star) + \cdots + \beta^\tau(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+\tau} - \Delta^\star) + \cdots + \beta^\tau(\Delta_t - \Delta^\star)].
\end{aligned}
$$

By taking the first partial derivatives, we get the marginal effect of x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
δ	$\beta\delta$	$\beta^2\delta$	$\cdots$	$\beta^\tau\delta$

and the marginal effect of Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
φ	$\beta\varphi$	$\beta^2\varphi$	$\cdots$	$\beta^\tau\varphi$

ACM

$$RV_t = \mu_t \epsilon_t$$

$$\begin{aligned}
\mu_t &= \varsigma_t + \xi_t \\
\mu_t &= \omega + (\alpha + \gamma D_{t-1})RV_{t-1} + \beta\varsigma_{t-1} + \delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1}
\end{aligned}$$

$$\begin{aligned}
\mu_{t+1} &= \omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \psi\xi_t \\
&= \omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t + \delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \\
&\quad + \psi[\delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1}] \\
&= \omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t + \delta[(x_t - x^\star) + \psi(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+1} - \Delta^\star) + \psi(\Delta_t - \Delta^\star)] + \psi^2\xi_{t-1}
\end{aligned}$$

$$\begin{aligned}
\mu_{t+2} &= \omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1} + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \psi\xi_{t+1} \\
&= \omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1} + \delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \\
&\quad + \psi\{\delta[(x_t - x^\star) + \psi(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+1} - \Delta^\star)\psi(\Delta_t - \Delta^\star) + \psi^2\xi_{t-1}\} \\
&= \omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1} + \delta[(x_{t+1} - x^\star) + \psi(x_t - x^\star) + \\
&\quad + \psi^2(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+2} - \Delta^\star) + \psi(\Delta_{t+1} - \Delta^\star) + \psi^2(\Delta_t - \Delta^\star)] + \psi^3\xi_{t-1}
\end{aligned}$$

$$\vdots$$

$$\begin{aligned}
\mu_{t+\tau} &= \omega + (\alpha + \gamma D_{t+\tau-1})RV_{t+\tau-1} + \beta\varsigma_{t+\tau-1} + \delta[(x_{t+\tau-1} - x^\star) + \cdots + \psi^\tau(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+\tau} - \Delta^\star) + \cdots + \psi^\tau(\Delta_t - \Delta^\star)] + \psi^{\tau+1}\xi_{t-1}.
\end{aligned}$$

By taking the first partial derivatives, we get the marginal effect of x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
δ	$\psi\delta$	$\psi^2\delta$	$\cdots$	$\psi^\tau\delta$

and the marginal effect of Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
φ	$\psi\varphi$	$\psi^2\varphi$	$\cdots$	$\psi^\tau\varphi$

L-ACM

$$RV_t = \mu_t \epsilon_t$$

$$\begin{aligned}
\mu_t &= 2\varsigma_t \frac{exp(\xi_t)}{1+exp(\xi_t)} \\
&= 2\varsigma_t \frac{exp[\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_t]}{1+exp[\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_t]} \\
\mu_{t+1} &= 2\varsigma_{t+1} \frac{exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi\xi_t]}{1+exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi\xi_t]} \\
&= 2\varsigma_{t+1}[\frac{exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi(\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_{t-1})]}{1+exp[\delta(x_t-x^\star)+\varphi(\Delta_{t+1}-\Delta^\star)+\psi(\delta(x_{t-1}-x^\star)+\varphi(\Delta_t-\Delta^\star)+\psi\xi_{t-1})]}] \\
&= 2\varsigma_{t+1}[\frac{exp[\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}]}{1+exp[\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}]}] \\
\mu_{t+2} &= 2\varsigma_{t+2} \frac{exp[\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\xi_{t+1}]}{1+exp[\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\xi_{t+1}]} \\
&= 2\varsigma_{t+2} \\
&\quad [\frac{exp(\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\{\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}\})}{1+exp(\delta(x_{t+1}-x^\star)+\varphi(\Delta_{t+2}-\Delta^\star)+\psi\{\delta[(x_t-x^\star)+\psi(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+1}-\Delta^\star)+\psi(\Delta_t-\Delta^\star)]+\psi^2\xi_{t-1}\})}] \\
&= 2\varsigma_{t+2} \\
&\quad [\frac{exp(\delta[(x_{t+1}-x^\star)+\psi(x_t-x^\star)+\psi^2(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+2}-\Delta^\star)+\psi(\Delta_{t+1}-\Delta^\star)+\psi^2(\Delta_t-\Delta^\star)]+\psi^3\xi_{t-1})}{1+exp(\delta[(x_{t+1}-x^\star)+\psi(x_t-x^\star)+\psi^2(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+2}-\Delta^\star)+\psi(\Delta_{t+1}-\Delta^\star)+\psi^2(\Delta_t-\Delta^\star)]+\psi^3\xi_{t-1})}] \\
&\vdots \\
&= 2\varsigma_{t+\tau}[\frac{exp(\delta[(x_{t+\tau-1}-x^\star)+\cdots+\psi^\tau(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+\tau}-\Delta^\star)+\cdots+\psi^\tau(\Delta_t-\Delta^\star)]+\psi^{\tau+1}\xi_{t-1})}{1+exp(\delta[(x_{t+\tau-1}-x^\star)+\cdots+\psi^\tau(x_{t-1}-x^\star)]+\varphi[(\Delta_{t+\tau}-\Delta^\star)+\cdots+\psi^\tau(\Delta_t-\Delta^\star)]+\psi^{\tau+1}\xi_{t-1})}]
\end{aligned}$$

By taking the first partial derivatives, we get the marginal effect of x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$2\varsigma_t\delta\frac{exp(\xi_t)}{[1+exp(\xi_t)]^2}$	$2\varsigma_{t+1}\psi\delta\frac{exp(\xi_{t+1})}{[1+exp(\xi_{t+1})]^2}$	$2\varsigma_{t+2}\psi^2\delta\frac{exp(\xi_{t+2})}{[1+exp(\xi_{t+2})]^2}$	$\cdots$	$2\varsigma_{t+\tau}\psi^\tau\delta\frac{exp(\xi_{t+\tau})}{[1+exp(\xi_{t+\tau})]^2}$

and the marginal effect of Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$2\varsigma_t\varphi\frac{exp(\xi_t)}{[1+exp(\xi_t)]^2}$	$2\varsigma_{t+1}\psi\varphi\frac{exp(\xi_{t+1})}{[1+exp(\xi_{t+1})]^2}$	$2\varsigma_{t+2}\psi^2\varphi\frac{exp(\xi_{t+2})}{[1+exp(\xi_{t+2})]^2}$	$\cdots$	$2\varsigma_{t+\tau}\psi^\tau\varphi\frac{exp(\xi_{t+\tau})}{[1+exp(\xi_{t+\tau})]^2}$

Li-ACM

$$
\begin{aligned}
RV_t &= \mu_t \epsilon_t \\
\mu_t &= \varsigma_t \xi_t \\
&= [\omega + (\alpha + \gamma D_{t-1})RV_{t-1} + \beta\varsigma_{t-1}][\delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1}] \\
\mu_{t+1} &= [\omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t][\delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \psi\xi_t] \\
&= [\omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t]\{\delta(x_t - x^\star) + \varphi(\Delta_{t+1} - \Delta^\star) + \\
&\quad + \psi[\delta(x_{t-1} - x^\star) + \varphi(\Delta_t - \Delta^\star) + \psi\xi_{t-1}]\} \\
&= [\omega + (\alpha + \gamma D_t)RV_t + \beta\varsigma_t]\{\delta[(x_t - x^\star) + \psi(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+1} - \Delta^\star) + \psi(\Delta_t - \Delta^\star)] + \psi^2\xi_{t-1}]\} \\
\mu_{t+2} &= [\omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1}][\delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \psi\xi_{t+1}] \\
&= [\omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1}]\{\delta(x_{t+1} - x^\star) + \varphi(\Delta_{t+2} - \Delta^\star) + \\
&\quad + \psi[\delta[(x_t - x^\star) - \psi(x_{t-1} - x^\star)] + \varphi[(\Delta_{t+1} - \Delta^\star) + \psi(\Delta_t - \Delta^\star)] + \psi^2\xi_{t-1}]\} \\
&= [\omega + (\alpha + \gamma D_{t+1})RV_{t+1} + \beta\varsigma_{t+1}]\{\delta[(x_{t+1} - x^\star) + \psi(x_t - x^\star) + \psi^2(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+2} - \Delta^\star) + \psi(\Delta_{t+1} - \Delta^\star) + \psi^2(\Delta_t - \Delta^\star)] + \psi^3\xi_{t-1}]\} \\
&\vdots \\
\mu_{t+\tau} &= [\omega + (\alpha + \gamma D_{t+\tau-1})RV_{t+\tau-1} + \beta\varsigma_{t+\tau-1}]\{\delta[(x_{t+\tau-1} - x^\star) + \psi^\tau(x_{t-1} - x^\star)] + \\
&\quad + \varphi[(\Delta_{t+\tau} - \Delta^\star) + \cdots + \psi^\tau(\Delta_t - \Delta^\star)] + \psi^{\tau+1}\xi_{t-1}]\}.
\end{aligned}
$$

By taking the first partial derivatives, we get the marginal effect of x_{t-1}

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$\delta\varsigma_t$	$\psi\delta\varsigma_{t+1}$	$\psi^2\delta\varsigma_{t+2}$	$\cdots$	$\psi^\tau\delta\varsigma_{t+\tau}$

and the marginal effect of Δ_t

on μ_t	on μ_{t+1}	on μ_{t+2}	$\cdots$	on $\mu_{t+\tau}$
$\varphi\varsigma_t$	$\psi\varphi\varsigma_{t+1}$	$\psi^2\varphi\varsigma_{t+2}$	$\cdots$	$\psi^\tau\varphi\varsigma_{t+\tau}$

A.3 Multi-step forecasting

In this appendix we present the formula we used to perform the Multi–step forecasting procedure.

AMEMX

$$
\begin{aligned}
RV_T &= \mu_T \epsilon_T \\
\mu_T &= \omega + \alpha RV_{T-1} + \beta \mu_{T-1} + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) \\
\mu_{T+1} &= \omega + \alpha RV_T + \beta \mu_T + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) \\
&= \omega + \alpha RV_T + \beta[\omega + \alpha RV_{T-1} + \beta \mu_{T-1} + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star)] + \\
&\quad + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) \\
&= \omega(1+\beta) + \beta^2 \mu_{T-1} + \alpha(\beta RV_{T-1} + RV_T) + \delta[\beta(x_{T-1} - x^\star) + (x_T - x^\star)] + \\
&\quad + \varphi[\beta(\Delta_T - \Delta^\star) + (\Delta_{T+1} - \Delta^\star)] \\
&\vdots \\
\mu_{T+\tau} &= \omega(1 + \beta + \beta^2 + .. + \beta^\tau) + \beta^{\tau+1} \mu_{T-1} + \alpha \sum_{i=-1}^{\tau-1} \beta^{\tau-1+i} RV_{T-i} + \\
&\quad + \delta \sum_{i=-1}^{\tau-1} \beta^{\tau-1+i} (x_{T-i} - x^\star) + \varphi \sum_{i=0}^{\tau} \beta^{\tau-i} (\Delta_{T+i} - \Delta^\star)
\end{aligned}
$$

For $\tau \to \infty$ the process converges to the unconditional mean:

$$
\mu = \frac{\omega + \delta(x_{T-1} - \bar{x}) + \varphi(\Delta_T - \bar{\Delta})}{1 - \alpha - \beta - \frac{\gamma}{2}}
$$

ACM

$$
\begin{aligned}
RV_T &= \mu_T \epsilon_T \\
\mu_T &= \varsigma_t + \xi_t \\
&= \omega + \alpha RV_{T-1} + \beta \varsigma_{T-1} + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) + \psi \xi_{T-1} \\
\mu_{T+1} &= \omega + \alpha RV_T + \beta \varsigma_T + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) + \psi \xi_T \\
&= \omega + \alpha RV_T + \beta(\omega + \alpha RV_{T-1} + \beta \varsigma_{T-1}) + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) + \\
&\quad + \psi[\delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \bar{\Delta}^\star) + \psi \xi_{T-1})] \\
&= \omega(1+\beta) + \beta^2 \varsigma_{T-1} + \alpha(\beta RV_{T-1} + RV_T) + \delta[\psi(x_{T-1} - x^\star) + (x_T - x^\star)] + \\
&\quad + \varphi[\psi(\Delta_T - \Delta^\star) + (\Delta_{T+1} - \Delta^\star)] + \psi^2 \xi_{T-1} \\
&\vdots \\
\mu_{T+\tau} &= \omega(1 + \beta + \beta^2 + .. + \beta^\tau) + \beta^{\tau+1} \varsigma_{T-1} + \alpha \sum_{i=-1}^{\tau-1} \beta^{\tau-1+i} RV_{T-i} + \\
&\quad + \delta \sum_{i=-1}^{\tau-1} \psi^{\tau-1+i}(x_{T-i} - \bar{x}) + \varphi \sum_{i=0}^{\tau} \psi^{\tau-i}(\Delta_{T+i} - \bar{\Delta}) + \psi^\tau \xi_{T-1}.
\end{aligned}
$$

Similarly to the previous case, for $\tau \to \infty$ the unconditional mean is given by:

$\mu = [\frac{1}{1-(\alpha+\frac{\gamma}{2})(\frac{1}{1-\beta})}][\frac{\omega}{1-\beta} + \frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]$

L-ACM

$$RV_T = \mu_T \epsilon_T$$

$$\begin{aligned}
\mu_T &= 2\varsigma_T[\frac{exp(\xi_T)}{(1+exp(\xi_T))}] \\
&= 2[\omega + \alpha RV_{T-1} + \beta\varsigma_{T-1}]\{\frac{exp[\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1}]}{1+exp[\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1}]}\}
\end{aligned}$$

$$\begin{aligned}
\mu_{T+1} &= 2[\omega + \alpha RV_T + \beta\varsigma_T]\{\frac{exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi\xi_T]}{1+exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi\xi_T]}\} \\
&= 2[\omega + \alpha RV_T + \beta(\omega + \alpha RV_{T-1} + \beta\varsigma_{T-1})] \\
&\quad \{\frac{exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi(\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1})]}{1+exp[\delta(x_T-x^\star)+\varphi(\Delta_{T+1}-\Delta^\star)+\psi(\delta(x_{T-1}-x^\star)+\varphi(\Delta_T-\Delta^\star)+\psi\xi_{T-1})]}\} \\
&= 2[\omega(1+\beta) + \beta^2\varsigma_{T-1} + \alpha(\beta RV_{T-1} + RV_T)] \\
&\quad \{\frac{exp[\delta(\psi(x_{T-1}-x^\star)+(x_T-x^\star))+\varphi(\psi(\Delta_T-\Delta^\star)+(\Delta_{T+1}-\Delta^\star))+\psi^2\xi_{T-1}]}{1+exp[\delta(\psi(x_{T-1}-x^\star)+(x_T-x^\star))+\varphi(\psi(\Delta_T-\Delta^\star)+(\Delta_{T+1}-\Delta^\star))+\psi^2\xi_{T-1}]}\}
\end{aligned}$$

$$\begin{aligned}
\mu_{T+\tau} &= 2[\omega(1+\beta+\beta^2+..+\beta^\tau) + \beta^{\tau+1}\varsigma_{T-1} + \alpha\sum_{i=-1}^{\tau-1}\beta^{\tau-1+i}RV_{T-i}] \\
&\quad \{\frac{exp[\delta\sum_{i=-1}^{\tau-1}\psi^{\tau-1+i}(x_{T-i}-x^\star)+\varphi\sum_{i=0}^{\tau}\psi^{\tau-i}(\Delta_{T+i}-\Delta^\star)+\psi^\tau\xi_{T-1}]}{1+exp[\delta\sum_{i=-1}^{\tau-1}\psi^{\tau-1+i}(x_{T-i}-x^\star)+\varphi\sum_{i=0}^{\tau}\psi^{\tau-i}(\Delta_{T+i}-\Delta^\star)+\psi^\tau\xi_{T-1}]}\}
\end{aligned}$$

for $\tau \to \infty$

$$\mu = \frac{\{2(\frac{\omega}{1-\beta})[\frac{exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}{1+exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}]\}}{1-2(\frac{\alpha+\frac{\gamma}{2}}{1-\beta})[\frac{exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}{1+exp[\frac{\delta(x_{T-1}-\bar{x})+\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}]}$$

Li-ACM

$$
\begin{aligned}
RV_T &= \mu_T \epsilon_T \\
\mu_T &= \varsigma_T \xi_T \\
&= [\omega + \alpha RV_{T-1} + \beta \varsigma_{T-1}][(1-\psi) + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) + \psi \xi_{T-1}] \\
\mu_{T+1} &= [\omega + \alpha RV_T + \beta \varsigma_T][(1-\psi) + \delta(x_T - x^s tar) + \varphi(\Delta_{T+1} + \Delta^\star) + \psi \xi_T)] \\
&= [\omega + \alpha RV_T + \beta(\omega + \alpha RV_{T-1} + \beta \varsigma_{T-1})] \\
&\quad \{(1-\psi)) + \delta(x_T - x^\star) + \varphi(\Delta_{T+1} - \Delta^\star) + \psi[(1-\psi) + \delta(x_{T-1} - x^\star) + \varphi(\Delta_T - \Delta^\star) + \\
&\quad + \psi \xi_{T-1})\} \\
&= [\omega(1+\beta) + \beta^2 \varsigma_{T-1} + \alpha(\beta RV_{T-1} + RV_T)] \\
&\quad [\psi(1-\psi) + \delta(x_t - x^\star) + \psi(x_{T-1} - x^\star)) + \varphi((\Delta_{T+1} - \Delta^\star) - \psi(\Delta_T - \Delta^\star) + \psi^2 \xi_{T-1})]
\end{aligned}
$$

$$
\begin{aligned}
\mu_{T+\tau} &= [\omega(1 + \beta + \beta^2 + .. + \beta^\tau) + \beta^{\tau+1} \varsigma_{T-1} + \alpha \sum_{i=-1}^{\tau-1} \beta^{\tau-1+i} RV_{T-i}] \\
&\quad [(1-\psi)(1 + \psi + \psi^2 + .. + \psi^\tau) + \delta \sum_{i=-1}^{\tau-1} \psi^{\tau-1+i}(x_{T-i} - x^\star) + \\
&\quad + \varphi \sum_{i=0}^{\tau} \psi^{\tau-i}(\Delta_{T+i} - \Delta^\star) + \psi^\tau \xi_{T-1}]
\end{aligned}
$$

for $\tau \to \infty$

$$
\mu = [\tfrac{\omega}{1-\beta} + \tfrac{1-\psi-\delta(x_{T-1}-\bar{x})-\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]\{\tfrac{1}{1-(\frac{\alpha+\frac{\gamma}{2}}{1-\beta})[\frac{1-\psi-\delta(x_{T-1}-\bar{x})-\varphi(\Delta_T-\bar{\Delta})}{1-\psi}]}\}
$$

Appendix Chapter 3

B.1 Hamilton filter and smoother

The Hamilton filter and smoother represents the main tool used to obtain the Maximum-likelihood estimator in the Markov Switching framework. Since it is a non-linear recursive algorithm, it needs a numerical optimization procedure from which the likelihood function is obtained as a by-product. The filter was proposed by Hamilton (1994) and it is based on the evaluation of the optimal forecast of the regime, by means of three different information sets:

1. the set based on information prior to time t (Ψ_{t-1}), used to compute the prediction probabilities: $P(s_t = i|\Psi_{t-1}, \Theta)$. Where Ψ_{t-1} represents the information set available at time $t-1$, whereas Θ is the set of estimated parameters;

2. the set based on past and current information (Ψ_t), which is used to derive the filtering probabilities: $P(s_t = i|\Psi_t, \Theta)$;

3. the full sample information set (Ψ_T), necessary to obtain the smoothed probabilities $P(s_t = i|\Psi_T, \Theta)$.

Therefore, let y_t be the variable of interest, the purpose is to estimate the Quasi-log-maximum-likelihood, which is expressed by means of the density functions conditional on the past information

$$L_t(\Theta) = \frac{1}{T}\sum_{t=1}^{T} lnf(y_t|\Psi_{t-1}, \Theta) \tag{10}$$

$f(y_t|\Psi_{t-1}, \Theta)$ is the weighted average of the density of y_t conditional on the past information and on the state, where the weights are represented by the prediction probabilities. Therefore, by assuming a two state Markov switching process

$$f(y_t|\Psi_{t-1}, \Theta) = \sum_{i=0}^{1} P(s_t = i|\Psi_{t-1}, \Theta) f(y_t|s_t = i, \Psi_{t-1}, \Theta)] \tag{11}$$

The conditional density function is crucial for the computation of the filtering probability, which are defined as the probability, conditional on the current information, of being in a given regime at each time point.

$$P(s_t = i|\Psi_t, \Theta) = \frac{P(s_t=i|\Psi_{t-1},\Theta) f(y_t|s_t=i,\Psi_{t-1},\Theta)}{f(\Psi_t|y_{t-1},\Theta)} \tag{12}$$

Basically, the filtered probabilities are given by the ratio between the probability of moving to a regime i, from each other regime, and the total probability of y_t, conditional on past information.
Finally, starting from the filtering probabilities we could easily predict the next period prediction probabilities, that is

$$P(s_{t+1} = i|\Psi_t, \Theta) = p_{0i} P(s_t = 0|\Psi_t, \Theta) + p_{1i} P(s_t = 1|\Psi_t, \Theta) \tag{13}$$

where p_{0i} and p_{1i} are the transition probabilities, expressed as $P(s_{t+1} = i|s_t = 0)$ and $P(s_{t+1} = i|s_t = 1)$, respectively.

As said, it is an iterative procedure: by choosing the initial value of the prediction probabilities, we can compute the density function of y_t conditional both on the past information, $f(y_t|\Psi_{t-1}, \Theta)$, and on the past information and the current regime, $f(y_t|s_t = i, \Psi_{t-1}, \Theta)$. Once the density function in equation (10) is estimated, the set of parameters $\hat{\Theta}$ are used to re-estimate the probabilities, which, in turn, will be used to re-estimate the density functions again, and so on until the convergence is achieved.

According to Hamilton (1994), given the stationarity and ergodic assumptions of the Markov chain, a possible initial value for the prediction probabilities is represented by the unconditional probabilities. To obtain the unconditional probabilities it is necessary to consider the eigenvalues of the transition probability matrix, which, in turn, derives from the solution of the characteristic polynomial $(P - \lambda I)$, where P is the transition probability matrix. In the two state Markov switching model, the eigenvalues should satisfy

$$0 = \begin{bmatrix} p_{00} - \lambda & 1 - p_{11} \\ 1 - p_{00} & p_{11} - \lambda \end{bmatrix}$$

$$(p_{00} - \lambda)(p_{11} - \lambda) - [(1 - p_{00})(1 - p_{11})] = 0$$

$$p_{00}p_{11} - (p_{00} + p_{11})\lambda + \lambda^2 - 1 + p_{00} + p_{11} - p_{00}p_{11} = 0$$

$$\lambda^2 - (p_{00} + p_{11})\lambda - 1 + p_{00} + p_{11} = 0$$

$$(\lambda - 1) - (\lambda + 1 - p_{00} - p_{11}) = 0$$

Therefore, the eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 1 - p_{00} - p_{11}$. According to Hamilton (1994) λ_2 is within the unit circle if and only if $0 < p_{00} + p_{11} < 2$; whereas the process is ergodic and irreducible if $p_{00} + p_{11} > 0$ and $p_{00}, p_{11} < 0$.

Finally, the eigenvector (π) - and, hence, the unconditional probabilities, $P(s_t = i)$ - associated to λ_1 is

$$\pi = \begin{bmatrix} \frac{1-p_{11}}{2-p_{00}-p_{11}} \\ \\ \frac{1-p_{00}}{2-p_{00}-p_{11}} \end{bmatrix}$$

Smoothing probabilities. Once the model is estimated, we need an accurate measure to assess our estimated process in each period of time within our sample. In other words, we need a measure that, basing on the full information set Ψ_T, allows us to understand the probability of each regime at time $t < T$. Of course it is something different from filtering probabilities, which, since they refers to the current information set, are based on less information and hence could be less accurate. Therefore, the difference between filtering and smoothing probabilities is that whereas the former represents a real-time assessment of the current state (so, they are computed recursively), the latter are an ex-post measure of the state at time t.

Generally, the smoothing probabilities are computed through the algorithm proposed by Kim (1994), starting from

$$P(s_t = i | s_{t+1} = j, \Psi_T, \Theta) = \frac{p_{ij} P(s_t = i | \Psi_t, \Theta)}{P(s_{t+1} = j | \Psi_t, \Theta)} \tag{14}$$

in a two state Markov switching process, the smoothing probabilities are expressed as

$$P(s_t = i|\Psi_T, \Theta) = P(s_{t+1} = 0|\Psi_T, \Theta)P(s_t = i|s_{t+1} = 0, \Psi_T, \Theta)+$$
$$+P(s_{t+1} = 1|\Psi_T, \Theta)P(s_t = i|s_{t+1} = 1, \Psi_T, \Theta) \tag{15}$$
$$= P(s_t = i|\Psi_t, \Theta)[\tfrac{p_{i0}P(s_{t+1}=0|\Psi_T,\Theta)}{P(s_{t+1}=0|\Psi_t,\Theta)} + \tfrac{p_{i1}P(s_{t+1}=1|\Psi_T,\Theta)}{P(s_{t+1}=1|\Psi_t,\Theta)}]$$

Usually, the filtering probability are used as initial value to iterate this set of equation backward, in order to get the estimated smoothing probabilities (they are estimated since QMLE $\hat{\Theta}$ is used in place of Θ).

Bibliography

C. Altavilla, D. Giannone, and M. Lenza. The financial and macroeconomic effects of OMT announcements. ECB working paper, 2014.

C. Altavilla, G. Carboni, and R. Motto. Asset purchase programmes and financial markets: lessons from the euro area. ECB working paper, 2015.

T. G. Andersen and T. Bollerslev. Answering the skeptics: yes, standard volatility models do provide accurate forecasts. International Economic Review, 39:885–905, 1998.

T. G. Andersen, T. Bollerslev, and F. X. Diebold. Roughing it up: including jump components in the measurement, modeling and forecasting of return volatility. Review of Economics and Statistics, 89:701–720, 2007.

P. Andrade, J. H. Breckenfelder, F. De Fiore, P. Karadi, and O. Tristani. The ECB's asset purchase programme: an early assessment. 2016.

A. Apostolou and J. Beirne. Volatility spillovers of Federal Reserve and ECB balance sheet expansions to emerging market economies. 2017.

L. Baele. Volatility spillover effects in european equity markets. Journal of Financial and Quantitative Analysis, 40(2):373–401, 2005.

M. Balatti, C. Brooks, M. P. Clements, and K. Kappou. Did quantitative easing only inflate stock prices? Macroeconomic evidence from the US and UK. Macroeconomic Evidence from the US and UK (September 13, 2016), 2016.

O. E. Barndorff-Nielsen, P. R. Hansen, A. Lunde, and N. Shephard. Designing realised kernels to measure the ex-post variation of equity prices in the presence of noise. Econometrica, 76:1481–1536, 2008.

R. M. Beetsma, F. De Jong, M. Giuliodori, and D. Widijanto. The impact of news and the SMP on realized (co) variances in the eurozone sovereign debt market. CEPR Discussion Paper No. DP9803, 2014.

F. Black. Interest rates as options. the Journal of Finance, 50(5):1371–1376, 1995.

J. Boeckx, M. Dossche, and G. Peersman. Effectiveness and transmission of the ECB's balance sheet policies. CESifo Working Paper Series, 2014.

T. Bollerslev. Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31:307–327, 1986.

T. Bollerslev, A. J. Patton, and R. Quaedvlieg. Exploiting the errors: a simple approach for improved volatility forecasting. Journal of Econometrics, 192: 1–18, 2016. ISSN 0304-4076.

A. N. Bomfim. Pre-announcement effects, news effects, and volatility: monetary policy and the stock market. Journal of Banking & Finance, 27(1): 133–151, 2003.

F. Breedon, J. S. Chadha, and A. Waters. The financial market impact of UK quantitative easing. Oxford Review of Economic Policy, 28(4):702–728, 2012.

C. T. Brownlees, F. Cipollini, and G. M. Gallo. Intra-daily volume modeling and prediction for algorithmic trading. Journal of Financial Econometrics, 9:489–518, 2011.

C. T. Brownlees, F. Cipollini, and G. M. Gallo. Multiplicative Error Models. In L. Bauwens, C. Hafner, and S. Laurent, editors, Volatility Models and Their Applications, pages 223–247. Wiley, 2012.

J. Cai. A Markov model of switching-regime ARCH. Journal of Business & Economic Statistics, 12(3):309–316, 1994.

M. Casiraghi, E. Gaiotti, M. L. Rodano, and A. Secchi. The impact of unconventional monetary policy on the italian economy during the sovereign debt crisis. Bank of Italy Occasional Paper, (203), 2013.

K. F. Chan and P. Gray. Volatility jumps and macroeconomic news announcements. Journal of Futures Markets, 38(8):881–897, 2018.

H. Chen, V. Cúrdia, and A. Ferrero. The macroeconomic effects of large-scale asset purchase programmes. The economic journal, 122(564):F289–F315, 2012.

A. Ciarlone and A. Colabella. Spillovers of the ECB's non-standard monetary policy into CESEE economies. Ensayos sobre política económica, 34(81): 175–190, 2016.

A. Ciarlone and A. Colabella. Asset price volatility in EU-6 economies: how large is the role played by the ECB? Bank of Italy Temi di Discussione (Working Paper) No, 1175, 2018.

M. Ciccarelli, J. A. Garcia, and C. Montes-Galdon. Unconventional monetary policy and the anchoring of inflation expectations. ECB working paper, 2017.

F. Cipollini, G. M. Gallo, and E. Otranto. Realized volatility forecasting: robustness to measurement errors. International Journal of Forecasting, 2020.

N. Converse. The impact of unconventional monetary policy on financial uncertainty in emerging markets. Technical report, mimeo, 2015.

F. Corsi. A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics, 7:174–196, 2009.

F. Corsi and R. Reno. Volatility determinants: heterogeneity, leverage, and jumps. Technical report, Citeseer, 2009.

V. Curdia and M. Woodford. The central-bank balance sheet as an instrument of monetary policy. Journal of Monetary Economics, 58(1):54–79, 2011.

S. d'Amico, W. English, D. López-Salido, and E. Nelson. The Federal Reserve's large-scale asset purchase programmes: rationale and effects. The Economic Journal, 122(564):F415–F446, 2012.

R. A. De Santis. Impact of the asset purchase programme on euro area government bond yields using market news. Economic Modelling, 86:192–209, 2020.

F. X. Diebold and R. S. Mariano. Comparing predictive accuracy. Journal of Business & Economic Statistics, 13(3):253–263, 1995.

Z. Ding, C. W. J. Granger, and R. F. Engle. A long memory property of stock market returns and a new model. Journal of Empirical Finance, 1(1):83–106, 1993.

M. J. Dueker. Markov switching in GARCH processes and mean-reverting stock-market volatility. Journal of Business & Economic Statistics, 15(1): 26–34, 1997.

R. F. Engle. Autoregressive Conditional Heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50:987–1007, 1982.

R. F. Engle. New frontiers for ARCH models. Journal of Applied Econometrics, 17:425–446, 2002.

R. F. Engle and G. M. Gallo. A multiple indicators model for volatility using intra-daily data. Journal of Econometrics, 131:3–27, 2006.

R. F. Engle and G. J. Lee. A permanent and transitory component model of stock return volatility. In R. F. Engle and H. White, editors, Cointegration, Causality, and Forecasting: A Festschrift in Honor of Clive W. J. Granger, pages 475–497. Oxford University Press, Oxford, 1999.

F. Eser and B. Schwaab. Evaluating the impact of unconventional monetary policy measures: empirical evidence from the ECB's securities markets programme. Journal of Financial Economics, 119(1):147–167, 2016.

C. Francq, M. Roussignol, and J.-M. Zakoïan. Conditional heteroskedasticity driven by hidden Markov chains. Journal of Time Series Analysis, 22:197–220, 2001.

M. Fratzscher, M. L. Duca, and R. Straub. ECB unconventional monetary policy: market impact and international spillovers. IMF Economic Review, 64(1):36–74, 2016.

G. M. Gallo and E. Otranto. Forecasting realized volatility with changing average levels. International Journal of Forecasting, 31:620–634, 2015. ISSN 0165-1765.

G. M. Gallo and E. Otranto. Combining sharp and smooth transitions in volatility dynamics: a fuzzy regime approach. Journal of the Royal Statistical Society: Series C (Applied Statistics), 67(3):549–573, 2018.

L. Gambacorta, B. Hofmann, and G. Peersman. The effectiveness of unconventional monetary policy at the zero lower bound: a cross-country analysis. Journal of Money, Credit and Banking, 46(4):615–642, 2014.

L. Gambetti and A. Musso. The macroeconomic impact of the ECB's expanded asset purchase programme (APP). 2017.

G. Georgiadis and J. Gräb. Global financial market impact of the announcement of the ECB's asset purchase programme. Journal of Financial Stability, 26:257–265, 2016.

E. Ghysels, J. Idier, S. Manganelli, and O. Vergote. A high-frequency assessment of the ECB Securities Markets Programme. Journal of the European Economic Association, 15(1):218–243, 2017.

L. R. Glosten, R. Jagannanthan, and D. E. Runkle. On the relation between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance, 48(5):1779–1801, 1993.

S. M. Goldfeld. A Markov model for switching regression. Journal of econometrics, 1:3–16, 1973.

S. F. Gray. Modeling the conditional distribution of interest rates as a regime-switching process. Journal of Financial Economics, 42(1):27–62, 1996.

M. Haas, S. Mittnik, and M. Paolella. A new approach to Markov-switching GARCH models. Journal of Financial Econometrics, 2:493 – 530, 2004.

R. Haitsma, D. Unalmis, and J. de Haan. The impact of the ECB's conventional and unconventional monetary policies on stock markets. Journal of Macroeconomics, 48:101–116, 2016.

J. Hamilton and R. Susmel. Autoregressive conditional heteroskedasticity and changes in regime. Journal of Econometrics, 64:307–333, 1994.

J. D. Hamilton. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica: Journal of the Econometric Society, pages 357–384, 1989.

J. D. Hamilton. Time Series Analysis. Princeton University Press, 1994.

J. D. Hamilton and G. Lin. Stock market volatility and the business cycle. Journal of applied econometrics, 11(5):573–593, 1996.

B. E. Hansen. The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP. Journal of applied Econometrics, 7 (S1):S61–S82, 1992.

P. R. Hansen, A. Lunde, and J. M. Nason. The Model Confidence Set. Econometrica, 79(2):453–497, 2011.

M. Hattori, A. Schrimpf, and V. Sushko. The response of tail risk perceptions to unconventional monetary policy. American Economic Journal: Macroeconomics, 8(2):111–36, 2016.

M. Joyce, A. Lasaosa, I. Stevens, M. Tong, et al. The financial market impact of quantitative easing in the UK. International Journal of Central Banking, 7(3):113–161, 2011.

G. Kapetanios, H. Mumtaz, I. Stevens, and K. Theodoridis. Assessing the economy-wide effects of quantitative easing. The Economic Journal, 122 (564):F316–F347, 2012.

D. Kenourgios, S. Papadamou, and D. Dimitriou. Intraday exchange rate volatility transmissions across QE announcements. Finance Research Letters, 14:128–134, 2015.

C. Kim. Dynamic linear models with Markov switching. Journal of Econometrics, 60:1–22, 1994.

A. Krishnamurthy, S. Nagel, and A. Vissing-Jorgensen. ECB policies involving government bond purchases: impact and channels. Review of Finance, 22(1):1–44, 2018.

C. G. Lamoureux and W. D. Lastrapes. Heteroskedasticity in stock return data: volume versus GARCH effects. The Journal of Finance, 45:221–229, 1990.

F. Ma, M. Wahab, D. Huang, and W. Xu. Forecasting the realized volatility of the oil futures market: a regime switching approach. Energy Economics, 67:136–145, 2017.

D. B. Nelson. Conditional heteroskedasticity in asset returns: a new approach. Econometrica, 59:347–370, 1991.

E. Otranto. Capturing the spillover effect with multiplicative error models. Communications in Statistics-Theory and Methods, 44(15):3173–3191, 2015.

A. J. Patton. Volatility forecast comparison using imperfect volatility proxies. Journal of Econometrics, 160(1):246 – 256, 2011. ISSN 0304-4076. Realized Volatility.

G. Peersman. Macroeconomic effects of unconventional monetary policy in the euro area. ECB Working Paper, 2011.

M. H. Pesaran and R. P. Smith. Counterfactual analysis in macroeconometrics: an empirical investigation into the effects of quantitative easing. Research in Economics, 70(2):262–280, 2016.

J. H. Rogers, C. Scotti, and J. H. Wright. Evaluating asset-market effects of unconventional monetary policy: a multi-country review. Economic Policy, 29(80):749–799, 2014.

L. Rompolis. The effectiveness of unconventional monetary policy on risk aversion and uncertainty. Bank of Greece, Working Paper Series, 2017.

C. Rosa. How'unconventional'are large-scale asset purchases? the impact of monetary policy on asset prices. The Impact of Monetary Policy on Asset Prices (May 1, 2012). FRB of New York Staff Report, (560), 2012.

A. Shogbuyi and J. M. Steeley. The effect of quantitative easing on the variance and covariance of the UK and US equity markets. International Review of Financial Analysis, 52:281–291, 2017.

J. M. Steeley and A. Matyushkin. The effects of quantitative easing on the volatility of the gilt-edged market. International review of financial analysis, 37:113–128, 2015.

U. Szczerbowicz et al. The ECB unconventional monetary policies: have they lowered market borrowing costs for banks and governments? International Journal of Central Banking, 11(4):91–127, 2015.

J. Tan and V. Kohli. The effect of FED's quantitative easing on stock volatility. Available at SSRN 2215423, 2011.

F. Urbschat and S. Watzka. Quantitative easing in the euro area–an event study approach. The Quarterly Review of Economics and Finance, 2019.

K. Voutsinas and R. A. Werner. New evidence on the effectiveness of "Quantitative Easing" in japan. Technical report, CFS working paper, 2011.

J. H. Wright. What does monetary policy do to long-term interest rates at the zero lower bound? The Economic Journal, 122(564):F447–F466, 2012.

J. C. Wu and F. D. Xia. Measuring the macroeconomic impact of monetary policy at the zero lower bound. Journal of Money, Credit and Banking, 48 (2-3):253–291, 2016.