



Reply to comment

# A critical analysis towards research perspectives Reply to comments on “Modeling human behavior in economics and social science”

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## Abstract

We take advantage of the challenging comments to the modeling approach we proposed in [35] to look ahead at a number of applications of the methods to the alternative questions these comments raise. In turn, our effort results in a number of interesting and valuable research perspectives. The presentation goes along three main lines. In the first line, we summarize briefly the aims and results in [35]. In the second section we give a technical the issues raised and, finally, the focus moves to the above mentioned research perspectives.

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## 1. Introduction

We take advantage of the various comments to our paper [35] to elaborate further upon technicalities, and to present an additional critical analysis, and finally to look ahead of potential interesting research perspectives coming from somewhat straightforward applications of the methodology we have proposed.

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As mentioned, [35] is devoted to the analysis of the complex interactions between human behaviors and social-economics phenomena. This topic has been critically analyzed in [35] towards possible applications. In that paper, we discuss the idea that mathematical tools can deeply contribute to somewhat higher understanding of the dynamics of social and economic systems. In more detail, the mathematical approach mainly refers to [3], where methods of statistical mechanics and evolutionary game theory have been adopted for modeling of social systems taken as living and hence complex systems. Additional important references have addressed the mathematical theory of collective learning processes, among which [22,23] and of evolutionary game theory, among which [41,42]. Theoretical aspects have been specifically addressed by a recent literature, where a variety of applications have been developed based upon the aforementioned mathematical approach see [2,3,14,24,33,34,36].

Our paper [35] has motivated a number of sharp and interesting comments, that deserve attention and motivate further discussion, that can potentially lead to new research perspectives. These comments have all contributed to stimulate our attention towards the contents of our paper, have forced us to think to the approach we propose in new perspective, and hence have promoted reasoning on potential new and valuable research.

The rationale supporting the approach proposed in [35] stems from the idea that the development of the methods coming from the kinetic theory allows the researcher to include detailed features of living system, can provide a systemic approach, and hence a field theory, for the modeling of socio-economical systems. Such a systemic approach is not only capable to reduce the large number of variables provided for the description of a complex living system, but it is also able to capture the main complexities of living, hence behavioral systems (on the point see [24]).

We begin our discussion by briefly summarize the approach we propose, so that the reply to the various comments can be properly framed. The approach is developed along the following sequential steps:

- 1) The entities that comprise the system, called *active particles*, are aggregated into different groups of interest called *functional subsystems* (FS). Active particles within the same FS share a common strategy called *activity* which define their *microscopic state*. The overall state of the system is delivered by a probability distribution over the said activity variable in each FS.
- 2) Active particles interact within the same functional subsystem as well as with particles of other subsystems, while generally they interact also with FSs viewed as a whole being represented by their mean value.
- 3) The evolution of the probability distribution is obtained by a balance of particles within elementary volumes of the space of microscopic states, where the inflow and outflow of particles is related to the interactions. The dynamics of interactions at the microscopic scale are modeled by theoretical tools of *stochastic behavioral games*.
- 4) Mathematical models are obtained by implementing the aforementioned modeling of interactions in the general mathematical structure.
- 5) Validation of models follows by comparisons with empirical data and by their ability to reproduce qualitatively emerging behaviors.

The mathematical structure presented in [35] and derived according to the aforementioned rationale is as follows:

$$\begin{aligned}
 \partial_t f_i(t, u) = & \sum_{h,k=1}^n \int_{D_u \times D_u} \eta_{h,k}[\mathbf{f}](u_*, u^*) \mathcal{A}_{h,k}^i[\mathbf{f}](u_* \rightarrow u|u_*, u^*) f_h(t, u_*) f_k(t, u^*) du_* du^* \\
 & - f_i(t, u) \sum_{k=1}^n \int_{D_u} \eta_{i,k}[\mathbf{f}](u, u^*) f_k(t, u^*) du^*, \\
 & + \sum_{h,k=1}^n \int_{D_u} v_{h,k}[\mathbf{f}](u_*, \mathbb{E}_k^1) \mathcal{B}_{h,k}^i[\mathbf{f}](u_* \rightarrow u|u_*, \mathbb{E}_k^1[f_k]) f_h(t, u_*) du_* \\
 & - f_i(t, u) \sum_{k=1}^n v_{i,k}[\mathbf{f}](u, \mathbb{E}_k^1),
 \end{aligned} \tag{1}$$

where  $\mathbf{f}$  denotes the set of all distribution functions and where square brackets have been used to denote the dependence on the distribution functions which highlight the nonlinear nature of interactions. In addition, for completeness, the terms that appear in Eq. (1) are:

- The *interaction rates*  $\eta_{h,k}[\mathbf{f}](u_*, u^*)$  and  $\nu_{h,k}[\mathbf{f}](u_*, \mathbb{E}_k^1)$ , which model the frequency of interactions between a candidate  $h$ -particle with state  $u_*$  and a field  $k$ -particle with state  $u^*$  or a  $k$ -th FS viewed as a whole being represented by their mean activity  $\mathbb{E}_k^1$ ;
- The *transition probability density*  $\mathcal{A}_{h,k}^i[\mathbf{f}](u_* \rightarrow u|u_*, u^*)$ , which denotes the probability density that a candidate  $h$ -particle shifts, after an interaction with a field  $k$ -particle, to the state of the test  $i$ -particle, such that the microscopic states of the candidate  $h$ -particle and the field  $k$ -particle are  $u_*$  and  $u^*$  respectively;
- The *transition probability density*  $\mathcal{B}_{h,k}^i[\mathbf{f}](u_* \rightarrow u|u_*, \mathbb{E}_k^1)$ , which denotes the probability density that a candidate  $h$ -particle shifts, after an interaction with the  $k$ -th FS, to the state of the test  $i$ -particle, such that the microscopic state of the candidate  $h$ -particle is  $u_*$  and that the mean activity of the  $k$ -th FS is  $\mathbb{E}_k^1$ .

## 2. Discussion of the comments

The authors of the comments have addressed a number of interesting problems concerning mathematical topics, analytic and computational, interactions between socio-economic sciences and applied sciences, and the interactions between social sciences and economics. Some authors have treated, in their comments, more than one topic. We find convenient to refer our reply to the specific topics we have selected out of the comments we have received. Then, for each topic, we have analyzed the contributions of the comments. Subsequently we propose our interpretation of the comment. According to our bias, we have selected four topics, along which we structure our reply. The first two of these topics mainly focus on mathematical problems; the following two topics deal with potential applications. The comments of some authors cover different topics, hence different references may address to the same author.

**Analytic and computational problems** The application of mathematical models to socio-economic questions tends to generate analytic and computational problems. As for the former, the classical problem consists in investigating the asymptotic behaviors of the solutions, which has been posed in [45]. This comment addresses a problem which has not yet achieved in general contexts, but only in special cases. In fact, the proof of existence of solutions can be obtained by the fixed-point analysis, which rapidly yields local existence, while geometrical prolongation for arbitrarily large time is obtained thanks to conservation of numbers of particles. On the other hand, the application of the Schrauder fixed point theorem leads to existence, however not to the uniqueness of solutions. This open problem is discussed in the last section of our paper.

Focusing now on computational problems, we agree with the comment [39], where Monte Carlo methods are available for studying the empirical behavior of the class of equations treated in [35]. Obviously, if new approaches towards the selection of the most appropriate mathematical tools become available, then this issue should be properly examined again under a new light.

The derivation of macro equations from the underlying description of the micro scale is a classical problem which has been mentioned in comments [6,25] that refer to biological applications [8,50], which developed after the survey paper [11]. Comments [25] clearly suggests to investigate how far the micro–macro derivation is of interest also in social and economics sciences. We feel comfortable to reply positively. In fact, it would be interesting studying models with space structure as social and economic systems are heterogeneously distributed in the economy. This challenging objective has been already touched, however lightly, in our paper referring to the methodological approach presented in [19]. Advanced mathematical tools are needed to tackle this interesting problem which requires transferring the knowledge developed in the field of active particles [8,11,50] to the case of economic agents. Applied mathematicians are attracted by the challenging analytic problems such as global solvability [15] and blow up of solution [16], posed by new type of nonlinearity included in macro models derived from the underlying description of the micro scale.

**General modeling issues** Modeling perspectives have attracted most of the authors of comments. Various topics have been presented. Each of them is examined in the following

1. The comments [1,6,31,39,45] have discussed the problem of the continuous approximation of the dependent variable, namely the probability distribution in Eq. (1). This is an important question because the overall system is finite, while letting to infinity the number of particles is an approximation of physical reality. The comment [31]

also refers to symmetric and non symmetric interactions [5], short and long range interactions [13], and the collective expression of a swarm intelligence [28]. Moreover, it poses the interesting problem of including Darwinist type mutation and selection dynamics in the modeling approach. Indeed the mathematical structure of Eq. (1) includes this dynamics which is an important feature of the class of systems discussed in our paper.

Indeed, the suggestion of developing swarm models towards modeling of socio-economic systems is interesting. Therefore, we will return to this matter in the last section, as it appears to have a very interesting research perspective, while at present we simply remark additional work on the present state of the art on modeling swarms is needed to tackle the challenging problem.

2. The problem of looking for a mathematical structure appropriate to provide the conceptual framework towards the derivation of models for the class of systems under consideration has been discussed by various comments [1, 6,45] referring either to the use of the mathematical theory of swarms or hybrid system mixing deterministic and stochastic dynamics [40].
3. Comment [40] has further stressed that, with respect to [35], there is also the need of studying systems where different dynamics simultaneously occur. Of course, we agree with this comment. The author has also mentioned the possibility of modeling systems where social dynamics is conditioning a mechanical dynamics. The author has referred, due to their research activity, to modeling social dynamics in crowd dynamics [7,10]. We do agree that the interaction of “soft” and “hard” sciences is one of the research fields which, in this century, wait for answers to highly challenging problems.
4. The design of databases that allow the research to test models against real dynamics, which have been actually experienced, has been mentioned in [1], where the personal professional experience of the author has definitely motivated her comment. This topic was treated in [3], by some pioneer ideas that deserve attention towards potential research perspectives. The scientific community has not yet reached a commonly shared scientific opinion on this matter, which will be further discussed in the next section.
5. The whole contents of [35] includes speculations on the complex interaction between the so called “soft” and “hard” sciences. Some of the comments touch this topic. However, two comments [32,46] are specifically focused on the former. We do believe that this topic deserves attention as stated in [37], who shows that the contribution of hard sciences to soft sciences is becoming increasingly important over time. Significant examples are mentioned in [32] focusing on [18,33]. Comment [46] indicates that developing appropriate approaches to reduce the gap between hard and soft sciences is a key issue in the study of socio-economic systems, and subsequently focus on the role of ethical behaviors in economics. The author provides specific references [14,30,37] that support the statements of the comment. We do agree with [46]. Additional reasonings on this topic are proposed in the next section.
6. A concise introduction on the complex interactions between politics of governments, democracy, and growth of nations is presented in [49] that basically refers to the literature reviewed in [38]. Subsequently, the following questions/challenges are given: (i) Is it possible, within the proposed modeling approach, to study the interaction between the dynamics of institutional changes and the changes in the distribution of political power related to the economic structure of the society? (ii) Is the kinetic framework suitable to study dynamic political economics phenomena in the presence of stochastic shocks, thus allowing dynamic in which the properties sharply change at a threshold value? (iii) Is there a possibility of quantifying “turbulence” within a kinetic model thus allowing empirical tests? Our reply here will be technical, while a deeper analysis will be given in the next section. These questions appear as challenging research perspectives. Remaining at a technical level, we stress that the modeling approach is in two steps, namely first a general mathematical structure is built to be suitable to include, at least at a quantitative level, the complexity features of the class of systems under consideration; subsequently models are derived by modeling interactions involving all entities which characterize the system. The structure presented in [35], referring to the general framework proposed in [3], can include the specific dynamics mentioned in [49]. Providing a positive reply also to quantitative aspects also means developing a proper research program. We do not naively claim that the result can be rapidly achieved, but we simply state that we are optimistic as we shall justify in the next section.

## Potential applications

1. Two interesting points are raised in [43]. The first of these, discussed also in [25,20], regards a multiscale approach of the modelization of social systems; the second one suggests an interesting application of the kinetic theory to Corporate Finance. With respect to the first point, which has already been treated above, we only add that it is clear that a reasonably general mathematical background is needed in particular when modeling social system, where the collective behavior of large systems evolving in space and time are described at different spatial and temporal scales, and are treated in a consistent way based on the information on the individual behavior. Examples showing that such transition usually lead to dramatic changes of the mathematical structure of the model may be found in [7]–[17]; these examples are mainly in self-propelled particles and biology. Regarding to the role of the managers' empire preservation problem, it is indeed interesting to explore patterns emerging from interactions between self-interested managers and firm value maximizing shareholders. When including, for instance, such a dynamic of interaction, would a model be able to highlight early warning signals of a distortive tendency of managers to preserve their empire or reputations? We believe that models in the kinetic theory framework would shed light into the complex interaction between market wages of managers and growth of the firm value. Moreover, it would be interesting to consider feedback effects as those pointed out in [43], in which a stagnant economy boosts financing constraints and may reduce firm-level investment.
2. An application to the phenomenon of aggregation/secession of countries is suggested by the interesting comment [48]. A first step toward a kinetic modelization of this actual and important phenomenon may be found in [2], where it has been proposed a model where the system is a nation in which interest groups that express their attitude towards a process of secession have been individuated and modeled as interacting functional subsystems. In the authors' opinion, suggestions coming from comment [48] can be very useful in improving the proposed model of [2].
3. Comment [44] suggests to analyze behavioral reactions of banks as individual entities interacting in a regulatory landscape. The adaptive features of the kinetic approach may provide, in the authors' opinion, a well suited theoretical background to model such phenomena, such as regulatory capture issue in the banking sector. This is due to the fact that the proposed approach would allow, as also suggested by the authors of the comment, to introduce rules which are endogenously arising from the system itself and possibly leading to instabilities in the financial systems.

## 3. Looking ahead to research perspectives

The analysis of the various comments suggests a large extent of agreement between the readers opinion and our viewpoints on the challenging research field covered by our paper [35]. Moreover, most of the comments have opened a window on possible research perspectives, as already observed in the preceding section. Before dealing specifically with the main topic of this section on research perspectives, we mention the three key questions we have raised in [46], namely: are researchers from economics and social sciences, and researchers from mathematics or physics willing to work in a sincere interdisciplinary way? Do individuals in general, and governments and elites in particular, really want to move forward the frontier of our knowledge, and too improve the wealth of their nations? Are governments really interested in the advice of researchers?

These questions focus on the effective utility of the interaction between mathematics and economics. The author of [46] correctly claims that this interaction can be practically useful if a positive answer can be given to the questions above. Indeed, research activity should be addressed to show how the aforementioned interaction is successful. Bearing all the above in mind, three research hints are given in the following, where the selection is far from being exhaustive as it is based upon the authors' bias. We cover three topics: (1) Development of new mathematical tools; (2) Analysis the hierarchy dividing sciences; (3) General issues of behavioral economics.

**1. New mathematical tools.** Our paper [35] essentially refers to the contribution that mathematical sciences can give to social sciences and economics, with the aim of building a bridge between hard and soft sciences. The driving rationale is in [9], where the first step of the approach consists in deriving a general mathematical structure that is suitable to capture the main features of living complex systems, specifically focusing on social science and economics.

A number of replies have suggested to investigate the possibility of developing a mathematical structure derived from the mathematical theory of swarms.

We do share this suggestion which essentially means developing a new approach to swarming dynamics suitable to define the conceptual framework to derive models in social science and economics. Applied mathematicians are already working at this objective, see [4,29], where interactions are allowed to be asymmetric; [47], where the concept of internal variable is introduced; and [13], that proposes a more general structure which includes short and long range interactions, as well as asymmetric interactions involving a fixed number of individual entities as conjectured by a team of physicists [5].

Various comments have further stressed the need of studying systems where different dynamics simultaneously occur. Of course, we agree with these comments and we have already reviewed some papers where this feature is accounted for [14,33,34,36]. We have also suggested this topic as being a fundamental research perspective. Within this framework, some comments [6,40] have also mentioned the possibility of modeling hybrid systems, where the modeling approach includes both deterministic and stochastic models, so that the analytic complexity is technically reduced. While social dynamics is stochastic, some aspects of population dynamics can be taken, for example, as being deterministic.

**2. About the hierarchy from soft to hard sciences.** The idea of a “*Hierarchy*” of the Sciences, as observed in [37], is about 200 years old. It is a controversial topic, which has motivated various speculations [27,51], beginning from the famous treatise by August Comte [26]. Paper [37] provides data supporting that idea that the concept of hierarchy is made more weak by successful achievements. The point raised by [46] highlights the social role that a successful modeling approach potentially have. Therefore, future research activity should be addressed to the specific objective of overcoming the concept of “hierarchy” among sciences.

The aim above is precisely described in Chapter 7 of [9], where the authors propose to replace the concept of “soft versus hard sciences” by the concept of *science of living systems*. This means that the interaction between economics and mathematics should lead to a specific theory of a mathematical economics derived within the general framework of a general theory of living and complex, systems. The interpretation of this idea is supported by the third author of [35] who is also author of [9]. Indeed, any achievement can be considered successful if the whole general theory is scientifically supported.

The theory proposed in [9] appears to have shown successful achievements in several fields, see [7,10,12,21,22,36]; however a fully consistent theory in economics still should be fully developed. Individual behaviors play an important role towards this challenging objective. Accordingly, such behaviors should not be the output of an ideological decision, but rather should come out of a true and effective interaction among researchers.

### 3. Interactions between politics, social sciences and economy.

Finally, we wish to mention that we take the three questions in [49] as a gift and a challenge for an interdisciplinary project involving mathematicians and economists. The key problem consists in modeling interactions based on the theoretical tools that game theory can offer, subsequently specific models can be derived and simulations can provide a quantitative answer to the three questions. Some recent results, for instance [14], have been able to discover precursors of an event that might be called “Black Swan”. Here the most challenging demand appears to be the “measure of turbulence”. Indeed, this means providing a quantitative description of a “soft” variable and subsequently deriving models suitable to describe this specific variable. As a matter of fact, mathematicians have already been involved in the quantification of soft variables [52]. At present, we can only state that we will base our future research programs on these questions, trusting that we will be able to reach interesting research results.

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