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# Road Network Analysis with GIS and GRASS-GIS: A Probabilistic Approach

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ABSTRACT

In this paper we show how it can be useful to the probability of intersections in the determination of a classification rule for raster conversions in Geographical Information System (GIS) and GRASS GIS for the Road Network Analysis (RNA). We use a geometric probabilities approach for irregular path considering these results for transportation planning operations. We study two particular problems with irregular tessellations, in order to have a situation more realistic respect to map GIS and considering also the maximum value of probability to narrow the range of possible probability values.

## 1. Introduction

In the last years GIS represents the most fascinating tool for the transportation planning. That is the one most capable of performing the functions of collector between the available application potential and the numerous relational connections between the various branches of knowledge. These tools are technologically adequate to integrate knowledge from multiple sources and at the

same time able to create totally transversal environments at a collaborative level.

The analysis methodologies associated with their use have completely changed the strategic decision-making processes both in the organizational and scientific fields.

In summary, they can be defined as IT tools of a matrix, which allows the organization, storage and management of a large amount of mostly qualitative and quantitative data to create digital cartography, produce possible

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simulations and support planning processes for various reference levels. They are also tools characterized by the specific peculiarity of loading different overlapping information layers (overlays) within their system, realized through appropriately created adequately criteria.

The potential and the various functions to which they must fulfill with very rapid timing and high levels of absolute precision consecrate their role as a key element in the application processes.

Within a GIS software there are peculiar features that serve to perform a multiplicity of spatial analyzes based on geographical and statistical skills. The main fields of application are: the added value they determine in the field of representation and analysis, both overall and in detail, of the demographic-settlement and socio-economic data, allowing the identification of homogeneous territorial areas or with discontinuous distributions; allow to obtain fundamental elements relating to the analysis of the geophysical and anthropic components of an area for the possible management and planning of emergencies related to earthquakes and volcanic eruptions; the elaboration of statistical-cartographic models to determine the evolution of hydrogeological risks and territorial exposures to fires; in the tourism sector they determine the assessments of the overall offer as well as that for individual contexts in terms of endowments in order to determine possible specific characterizations and territorial vocations; even in urban planning studies they find their place by providing analysis on several levels and meticulous analysis on all the components that persist in a given area, providing elements of importance to better understand the different phenomena and plan new implementation phases.

## 2. Preliminaires and Context

A particular case of intersection occurs when one of the features is a cell in a regular lattice, and the other is a straight line or area because for short lines, the probability of intersecting a single cell boundary is useful and for long lines, the expected number of cells intersected with the standard deviation, is useful. In the case of cell-area intersection, either the probability of boundary intersection or the probability of overlapping the middle point of a cell may be important. Probabilities for lines are dependent upon both feature size and Sharpe relative to cell size.

An important operation on data stored is the calculation of distance between two points. In fact, if we do not consider the projection distortion, the operation is trivial when the positions are in the same zone. However, the calculation becomes more complex when the points are in different zones. When the zones have different

projection, the task of calculating between-zone distances may be even more onerous. For this situation, it is useful to compute the probability that two locations separated by a distance  $d$  are in the same zone. This probability, as a function of  $d$ , may be practically employed to calculate the expected time to compute the distance operation. This valuable became very important for real implementations when the number of operations to be performed is very large. A common GIS application is the conversion of vector landcover maps to a raster database. A variety of potential encoding rules exist; here the encoding rule chooses the class occupying the central point of each raster cell. Of course, there is a tradeoff between precision and data volume; coarse cell resolution leads to smaller and more tractable datasets at the cost of information loss. To what degree can this loss be estimated in advance of the procedure? For example, what is the probability that a landcover patch of a specific size is captured by raster encoding at a specific cell resolution? This is a specific case of a more general problem: the probability that a spatial feature will intersect at least one central point of a tile in a regular tessellation. Foreknowledge of such probabilities may be employed to identify ideal cell sizes for the application. Solution for these sorts of problems appear in recent GIS literature <sup>[11]</sup>, but their mathematical underpinnings are identifiable in texts of geometric probability <sup>[9,10,13,16]</sup>. In fact, integral geometry provides instruments and methods to solve problems of this kind and there exists an active research area working in this direction <sup>[12,15]</sup>. In general, Earth and its features are located and evolve in 3D space and time. However, for most applications a projection of geospatial data to a flat plane is sufficient; therefore, two-dimensional representation of geographical features (with data georeferenced by their horizontal coordinates) is the most common. GIS provides the most comprehensive support for 2D data. Recently a new Geographic Information System, commonly referred to as GRASS GIS <sup>[4]</sup>, is developing; it is used for data management processing, graphics production, spatial modeling. Recent versions of GRASS GIS include a 3D raster model for volume data. In view of these new recent developments the study of the statistics of intersections in the three-dimensional case becomes a powerful instrument for eventual encoding rules for 3D raster data model. In <sup>[2]</sup> the authors illustrated the solution given by <sup>[8]</sup> and they show that such solution is a special case of a more general result of integral geometry, given by <sup>[6]</sup> Stoka and Duma extended the results of <sup>[6]</sup> to the three-dimensional case applying the solution determinate <sup>[7]</sup> for 2D raster conversion is extended for the analogue problem in 3D case. In fact, starting from <sup>[12]</sup> and <sup>[15]</sup>, which had great

importance for these studies, between the late nineties and the beginning of the present century, the Stoka research team made a significant contribution to the development of research of geometric probabilities. In [5] and [16] the authors introduced in the Buffon-Laplace type problems so-called obstacles. In [1] they studied a Buffon Needle problem for an irregular lattice determining the maximum value probability, i.e. managing to reduce the usual probability range.

### 3. Probabilistic Approach

The variations of the classic Buffon's Needle problem are of particular interest. In this variation we consider a tile  $\mathcal{R}(a, b, \alpha)$  composed by irregular fundamental cells  $C_0$  represented as in Figure 1.

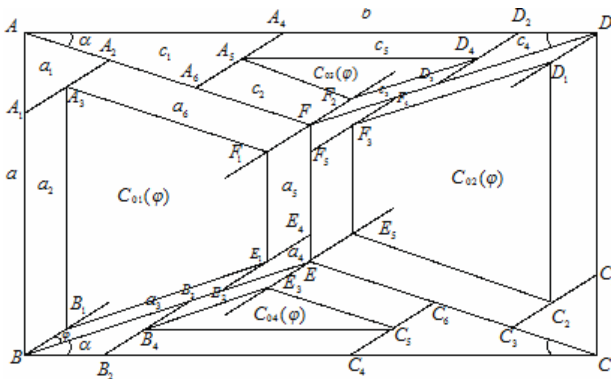


Figure 1. Fundamental cell with segment limited position

where  $\alpha \in [0, \frac{\pi}{4}]$  is an angle and  $a > btga$ .

Denoting by  $M1$  and with  $M2$  the set of all segments  $s$  that their center in  $C_{01}$  and  $C_{03}$ . Denote likewise by  $N1$  and  $N2$  the set of all segment  $s$  completely contained in  $C_{01}$  and  $C_{03}$ . We compute the the intersection probability between the sides of the cell and the segments by [16]:

$$P = \frac{\mu(N_1) + \mu(N_3)}{\mu(M_1) + \mu(M_3)} \tag{1}$$

where  $\mu$  denote the Lebesgue measure in the Euclidean plane.

To compute the above measures, we use the Poincaré kinematic measure [12]:  $dK = dx \wedge dy \wedge d\varphi$ , where  $x, y$  are the coordinates of the point  $\theta$  and  $\varphi$  the angle of  $s$ . By Figure1,  $0 \leq \varphi \leq \frac{\pi}{2} - \alpha$ , and we have

$$\mu(M_1) = \int_0^{\frac{\pi}{2}-\alpha} d\varphi \iint_{\{(x,y) \in C_{01}\}} dx dy = \int_0^{\frac{\pi}{2}-\alpha} (area C_{01})$$

$$d\varphi = \left(\frac{\pi}{2} - \alpha\right) area C_{01},$$

$$\mu(M_3) = \int_0^{\frac{\pi}{2}-\alpha} d\varphi \iint_{\{(x,y) \in C_{03}\}} dx dy = \int_0^{\frac{\pi}{2}-\alpha} (area C_{03})$$

$$d\varphi = \left(\frac{\pi}{2} - \alpha\right) area C_{03},$$

$$\mu(M_1) + \mu(M_3) = \left(\frac{\pi}{2} - \alpha\right) \frac{ab}{2}, \tag{2}$$

and

$$\mu(N_1) = \int_0^{\frac{\pi}{2}-\alpha} d\varphi \iint_{\{(x,y) \in \hat{C}_{01}\}} dx dy = \int_0^{\frac{\pi}{2}-\alpha} [area \hat{C}_{01}(\varphi)] d\varphi, \tag{3}$$

$$\mu(N_3) = \int_0^{\frac{\pi}{2}-\alpha} d\varphi \iint_{\{(x,y) \in \hat{C}_{03}\}} dx dy = \int_0^{\frac{\pi}{2}-\alpha} [area \hat{C}_{03}(\varphi)] d\varphi. \tag{4}$$

Integrating relations (3) and (4) and substituting in (1) we obtain

$$P = 1 - \frac{4}{(\pi - 2\alpha)ab} \left\{ al(1 - \sin\alpha) - \frac{bl}{2}(1 - \sin\alpha + 3\cos\alpha) - \frac{l^2}{2} \left[ \cos 2\alpha + \left(\frac{\pi}{2} - \alpha\right)(1 + ctg\alpha) \right] \right\}. \tag{5}$$

Denoting with

$$f(\alpha) = \frac{2al(1 - \sin\alpha) - bl(1 - \sin\alpha + 3\cos\alpha) - l^2 \left[ \cos 2\alpha + \left(\frac{\pi}{2} - \alpha\right)(1 + ctg\alpha) \right]}{\pi - 2\alpha},$$

we can write

$$P = 1 - \frac{2}{ab} f(\alpha). \tag{6}$$

We prove that there exist a system values for  $\alpha, a, b, l$  for which the probability  $P$  is maximum. In fact, for  $\alpha = \frac{\pi}{4}$  is easy to verify that  $f'(\alpha) = 0$  and  $f''(\alpha) > 0$  then the probability  $P$  is maximum.

For our considered lattice we have:

**Example 1.** What is the probability that the body test is missed during a 2D conversion to raster?

**Theorem 1.** The probability  $P_{int}$  that a random segment  $s$  of fixed length  $l$ , fulfilling the relation  $l < \frac{a}{2}$ , uniformly distributed in a bounded region of the plane, intersects a side of the lattice  $\mathcal{R}(a)$  is:

$$P = \frac{4}{(\pi - 2\alpha)ab} \left\{ al(1 - \sin\alpha) - \frac{bl}{2}(1 - \sin\alpha + 3\cos\alpha) - \frac{l^2}{2} \left[ \cos 2\alpha + \left(\frac{\pi}{2} - \alpha\right)(1 + ctg\alpha) \right] \right\}.$$

**Corollary 1.** For  $\alpha = \frac{\pi}{4}$ , by (6) we have that  $0 \leq P_{int} \leq \frac{2}{ab} f(\alpha)$ .

Figure 2 represents trip distance respect to the probability that the trip crosses a cell boundary.

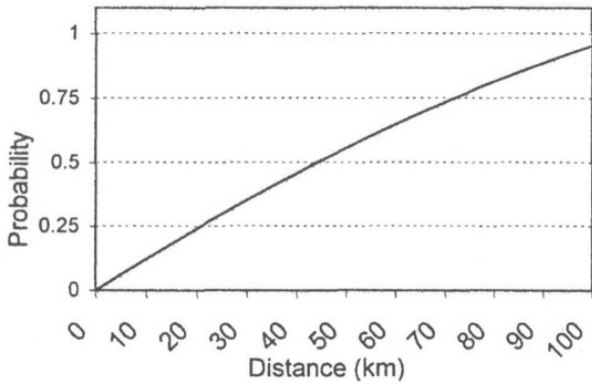


Figure 2. Probability of two points a given distance apart falling in different UK National Grid zones

### 4. Geometric Probabilities for Rectangle

We consider a tile  $\mathcal{R}(a, \alpha, \beta)$  composed by an irregular fundamental cell  $C_0 = C_{01} \cup C_{02}$  represented in the Figure 3 where  $\alpha$  and  $\beta$  are angles with  $\frac{\pi}{4} < \alpha \leq \frac{\pi}{3}$  and  $\beta \leq \alpha$ :

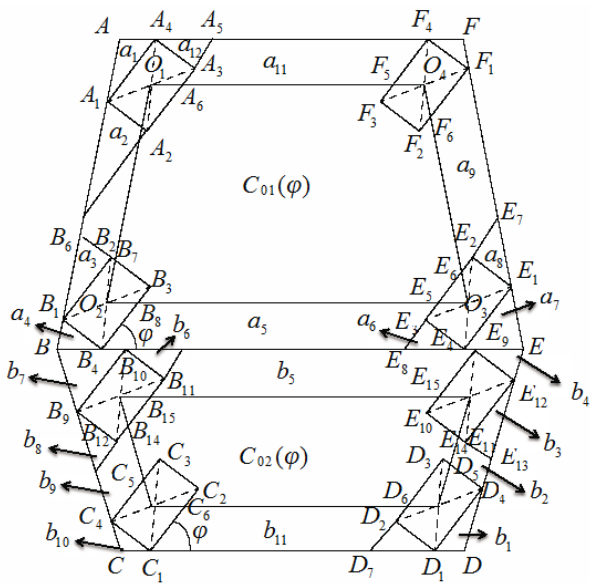


Figure 3. Fundamental cell with segment limited position

In the same way of the section 2, we have:

**Example 2.** What is the probability that the body test rectangle is missed during a 2D conversion to raster?

**Theorem 2.** The probability  $P_{int}$  that a random rectangle  $r$  of side  $a$  and  $b$ , uniformly distributed in a bounded region of the plane, intersects a side of the lattice  $\mathcal{R}(a, \alpha, \beta)$  is:

$$P_{int} = \frac{1}{3a^2(\alpha \tan \alpha + \beta \tan \beta)} \left\{ al \left[ 6 - 4 \cos \alpha - 4 \cos \beta \right] - \right.$$

$$\left. \left( s\beta + \frac{1}{\cos \alpha} + \frac{1}{\cos \beta} \right) \right\} -$$

$$\left( 2s\alpha + 2s\beta - \frac{l^2}{2} [s\alpha(s\alpha + c\alpha) + s\beta(s\beta + c\beta) - ac\alpha - \beta c\beta] - \right.$$

$$\left. \frac{m^2}{2} (s\alpha^2 + s\beta^2) - ac\alpha - \beta c\beta \right\}. \quad (7)$$

For  $m=0$  the rectangle  $r$  became a segment of length  $l$  and we find the probability determined in [1].

### 5. Conclusions

In this paper we highlight even more the relevance of geometric probabilities for RNA applications.

The GIS and GRASS-GIS application is the conversion of vector landcover maps to a raster database.

In order to estimate the number of maps required to analyze an area of interest and to determine the probability of “missing” converting vector data to raster grids in a transportation planning problems, in this paper we studied a probabilistic approach for the road network analysis with GIS and GRASS-GIS. For some fundamental GIS operations, the mathematical approach showed descriptive, with little or no notion whether observed quantities or relationships are significant. Respect to the previous studies of other authors, we consider two new aspects, in fact we introduce an irregular lattice also considering the maximum value of probability obtaining situation and results more realistic.

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