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MHD as a driver for Mixing in AGB Stars

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ABSTRACT: We present analytical exact 2D and 3D MHD computations for the layers of an AGB star known to be affected by deep mixing phenomena, in order to verify previous suggestions that magnetic buoyancy may provide a sound explaination for the isotopic changes observed in AGB stars and in presolar grains. The structure of the relevant layers is similar to a polytrope of index 3 (a bubble of radiation), containing little mass. Due to this, the material is close to be unstable for expansion. Addition of any extra *engine* under the form of a magnetic dynamo generating toroidal structures unstable for buoyancy yields plasma phenomena that closely resemble those of the solar wind, in which almost ideal, non-resistive MHD allows for an easy analytical integration of the model equations. The results show that a further expansion occurs for magnetized domains (flux tubes). These last form close to thermonuclear shells and transport outward nucleosynthesis products with a velocity $v \sim r^2$, faster than for diffusion but slower than for convection, adequate to give a physical interpretation to extra-mixing processes in evolved stars.

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1. Introduction

Deep mixing in evolved stars, sometimes also called *cool bottom process*[1] still challenges modelers. The operation of extensive circulations with non-convective nature is known to be necessary at least after the so-called Bump of the Luminosity Function of RGB stars [2]. Thermohaline diffusion [3] was for a few years considered as the most appropriate candidate, but increasing difficulties due to its low speed shed many doubts on its effectiveness [4, 5].

Among alternative mechanisms, the transport of matter through buoyant magnetic structures was suggested by [6] (hereafter BWNC) and then considered also by [7, 8]. In order to verify quantitatively if this process is really as promising as claimed, here we abandoned parameterizations to start a systematic work on first principles, looking for exact analytical solutions of the MHD equations, exempt from the simplifications usually adopted in numerical codes, which are rich in free parameters.

From an analysis of various thermally-pulsing AGB models (where a knowledge of the mentioned elusive transport phenomena is most urgently needed) we recognized that the layers above the H-burning shell invariably look like bubbles of radiation, i.e. politropes of index close to 3, potentially subject to the Eddington instability, induced by the prevailing of the radiation pressure.

In such conditions we show, through 2D and 3D models, how addition of a dynamo mechanism induces rather fast buoyancy of magnetized structures linking the H-burning shell and the envelope. The necessary nature of buoyancy in stars with rotating cores suggests this to be probably the best candidate mechanism to drive deep mixing in evolved stars.

2. The model

The equations of the problem, expressed in Eulerian form and adopting cgs units are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} - c_d \mathbf{v} + \nabla \Psi \right] - \mu \Delta \mathbf{v} + \nabla P + \frac{1}{4\pi} \mathbf{B} \times (\nabla \times \mathbf{B}) = 0 \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) - \nu_m \Delta \mathbf{B} = 0 \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{4}$$

$$\rho \left[\frac{\partial \varepsilon}{\partial t} + (\mathbf{v} \cdot \nabla)\varepsilon \right] + P \nabla \cdot \mathbf{v} - \nabla \cdot (\kappa \nabla T) + \frac{\nu_m}{4\pi} (\nabla \times \mathbf{B})^2 = 0 \tag{5}$$

In the above equations, ε is the internal energy per unit mass. P, T, ρ are the pressure, temperature and density of the plasma, κ is the thermal conductivity. $\mathbf{B}$ is the magnetic induction field, $\mathbf{v}$ is the plasma velocity, μ is the *dynamic viscosity* (product of density and of the kinematic viscosity η) and $\mu \Delta \mathbf{v}$ is a simplified form often used for the viscous force per unit volume in stellar MHD (it would formally hold for incompressible fluids with constant μ). Ψ is the gravitational potential and ν_m is the *magnetic diffusivity*. The term $c_d \mathbf{v}$ represents the aerodynamic drag force per unit mass.

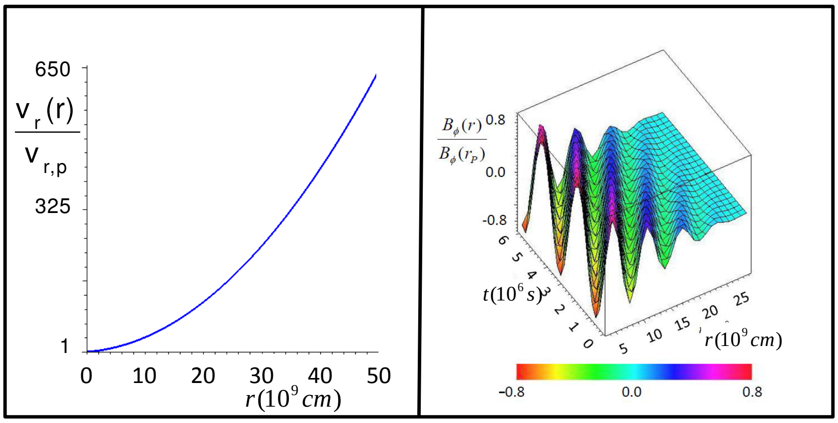

Figure 1: Left panel: the growth of the radial velocity v_r (normalized to the value at the bottom) in the radiative layers of our AGB star, where $w(t)$ is chosen as a linear function of t. Right panel: a 3D representation of the magnetic field, as a function of time and radius, for the same layers, choosing an oscillating form for $\Phi(\xi(r,t))$, i.e. a wave-like solution for B. Here $\omega = 2\pi/80$ yr^{-1} = 1.6 10^{-5} sec^{-1} was taken from [7]. The absolute value of the field at the level P is in this case a free paramter of the model.

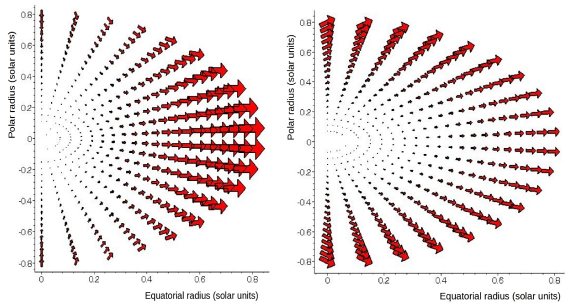

Figure 2: Illustrations of the velocity field over a stellar hemisphere for the 3D solutions discussed. The left panel combines solutions (8) and (9), the right panel solutions (10) and (11). See text for explanations. The axes show the equatorial and polar extension of the radiative layer, in units of the solar radius.

TABLE I
Parameters of the AGB-Star Layers of Interest
M=1.5 $M_{\odot}$, Z=0.01

Parameter	Value
r_P	$1.97 \cdot 10^9$
ρ_P	4.13
T_P	$4.92 \cdot 10^7$
P_P	$4.24 \cdot 10^{16}$
r_{env}	$5.19 \cdot 10^{10}$
ρ_{env}	$2.48 \cdot 10^{-4}$
k_{rad}	-3
T_{env}	$2.17 \cdot 10^6$
P_{env}	$1.29 \cdot 10^{11}$
k_{con}	$-3/2$
r_{sur}	$2.32 \cdot 10^{13}$
T_{sur}	$3 \cdot 10^3$
P_{sur}	$\approx 5 \cdot 10^{-2}$
ρ_{sur}	$\approx 10^{-9}$

Notes: units are cgs. Concerning the subscripts, “P" refers to the maximum mixing penetration; “env" refers to the border between the radiative layer and the convective envelope; “sur" indicates the surface values. The physical parameters T, ρ, P, r are from BWNC.

Indicating with r the radial coordinate and with φ the azimuthal angle in the equatorial plane, we assume that $\mathbf{B} = (B_r(t,r,\varphi), B_\varphi(t,r,\varphi), 0)$ is such that $B_r = 0$. This describes an azimuthal field as a function of r, φ and time t. We also consider pure circular symmetry in the equatorial plane, so that the velocity components do not depend on the azimuthal angle; let also the velocity field be parallel to the equator. Hence: $\mathbf{v} = (v_r(t,r,\varphi), v_\varphi(t,r,\varphi), 0)$ is such that $v_\varphi = v_\varphi(t,r)$ and $v_r = v_r(t,r)$.

In several astrophysical dynamo scenarios, the third term of equation (3) is much smaller than the second one, as the conductivity of a ionized medium is very large. Magnetic diffusivity is actually negligible if we consider transport phenomena occurring through advection, by the fields described in the second term of the equation. It is common to say that, in this situation, the field and the transported fluid are mutually “frozen".

With the above assumption, we can solve the equations using, for the stellar structure, the AGB model studied by BWNC, whose parameters are summarized in Table 1. That model shows how the density distribution as a function of the radius is very close to a power law, $\rho = \rho_0 (r/r_0)^k$, with $k \simeq -3$ in the radiative layers. In such conditions the solution of the MHD equations (1), (2), (3) and (4) describes the equilibrium of the stellar plasma in an inertia frame. An outline of how this solution is obtained is presented in [9].

3. Results and discussion

The solution for the MHD equations in the above conditions, as computed in 2D (neglecting latitude effects) is described in [9]. It is the following:

$$v_r = v_{r,P}\left(\frac{r}{r_P}\right)^2 \tag{6}$$

$$B_\varphi(r,t) = B_{\varphi,P}\cos(\omega t + r_P/r)\left(\frac{r_P}{r}\right)^2 \tag{7}$$

where $v_{r,P} = \omega r_P$, and $B_{\varphi,P} = A r_P^{-2}$. This solution satisfies dimensional constraints on v_r and B_φ if A has the dimensions of a magnetic flux [units Mx] and ω is measured in $\sec^{-1}$ (i.e. is a pulsation). We can take this parameter from [7]. Figure 1 shows the ensuing behavior for v_r and B_φ. This solution describes a magnetic field that oscillates in time and space and is spatially distorted by a phase shift varying as $1/r$.

In order to compute its equivalent in three dimensions, one can impose the validity of the continuity equation in 3D spherical coordinates. Possible solutions with this property are:

$$v_r(r,\vartheta) = \frac{1}{2}\frac{v_{r,P}}{{r_P}^2}r^2\left[2\cos^2\vartheta - \sin^2\vartheta\right] \tag{8}$$

$$v_\vartheta(r,\vartheta) = -\frac{1}{4}\frac{v_{r,P}}{{r_P}^2}r^2\sin 2\vartheta \tag{9}$$

Notice that the third component v_φ enters the continuity equation only through the derivative of (ρv_φ) with respect to φ, which is zero in our hypotheses. It will depend on magnetic fields in a complex way. However, the simple form of MHD holding in our case makes the behavior of v_φ irrelevant for v_r and v_ϑ, so that we can discuss our generalization that includes meridional motions without the need of invoking the explicit form of the azimuthal velocity. What is important for us is to verify if it is true that v_r is proportional to r^2. We can deduce this, in two or in three dimensions, without referring directly to v_φ. This also tells that any form of differential rotation possibly descending by an exact derivation of this last function would not alter our basic conclusions for the expansion.

At the equator, where v_ϑ vanishes, the solution for v_r reduces to the one derived in the 2D case. Also its average over the latitude (from $-\pi/2$ to $\pi/2$) differs from our solution (6) only by a numerical factor α (with α = 0.25). This is obviously irrelevant: it can be included in the multiplying coefficient of equation (6), which depends on the free parameter $v_{r,p}$. Notice that v_ϑ vanishes also at the poles.

Another, a bit more sophisticated, solution (describing in this case a double toroidal structure, with a radial velocity that achieves its maxima at intermediate latitudes, north and south of the equator, like for the Sun and many active stars) is for example provided by the following relation for v_r:

$$v_r = \frac{1}{2}\frac{v_{r,p}}{r_p^2}r^2(2\cos\vartheta\cosh\vartheta + \sin\vartheta\sinh\vartheta) \tag{10}$$

In order to satisfy the continuity equation this requires that:

$$v_\vartheta = -\frac{1}{2}\frac{v_{r,p}}{r_p^2} r^2 \cosh\vartheta \sin\vartheta \tag{11}$$

Again, the above formulae fulfil the requirements at $\vartheta = 0$, where they reduce to our 2D solution found before, because there $v_\vartheta = 0$ and $v_r = (v_{r,p}/r_p^2) \times r^2$, which is again formula (6). The velocity v_r is the product of two terms, depending one on r and the second on ϑ; this second function (see equation 10) does not diverge in the interval of interest (from $-\pi/2$ to $+\pi/2$, i.e. from the south to the north pole). It is symmetric with respect to the equator and its average corresponds to the form (6) multiplied by a factor α (which is, in this case, $\alpha = 1.198$).

The two example solutions discussed above are represented in Figure 2.

When the above framework is applied to the study of real AGB stars, with model parameters like those described in Table 1, one finds that the buoyancy velocity is always effective enough to provide an efficient mixing mechanism, with velocities smaller than for convection, but much larger than for thermohaline mixing and in general of the order of meters per second. This is sufficient to account for essentially all the requirements of extra-mixing, including the most sophisticated ones, accounting for the formation of a ^{13}C reservoir in the He shell, suitable to activate the $^{13}\mathrm{C}(\alpha,\mathrm{n})^{16}O$ reaction, thus inducing slow neutron capture nucleosynthesis [10].

As a consequence we can state that, with a procedure based on simple, realistic hypotheses in two/three-dimensions, the buoyancy of magnetized structures provides an exact solution to the problem of extra-mixing in the radiative zones of AGB stars. In it, pure advection (on time scales much faster than any diffusion process) can yield the rapid crossing of the layer up to the convective envelope base, thanks to the fact that the radial velocity grows as the second power of the radius.

As the dependency of v_r on the second power of the radius remains the same in 2D and 3D, we believe this result to be quite robust.

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