

MAGILL-TYPE THEOREMS FOR MAPPINGS

GIORGIO NORDO and BORIS A. PASYNKOV

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Magill's and Rayburn's theorems on the homeomorphism of Stone-Čech remainders and some of their generalizations to the remainders of arbitrary Hausdorff compactifications of Tychonoff spaces are extended to some class of mappings.

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1. Introduction. In 1968, Magill [8] proved the following theorem.

THEOREM 1.1. *For two locally compact Tychonoff spaces X and Y , the Stone-Čech remainders (i.e., the remainders of the Stone-Čech compactifications) $\beta X \setminus X$ and $\beta Y \setminus Y$ are homeomorphic if and only if the posets $K(X)$ and $K(Y)$ of all Hausdorff compactifications of X and Y are isomorphic.*

In 1973, Rayburn [12] gave the following definition.

DEFINITION 1.2. A Tychonoff space X is called k -absolute if $\beta X \setminus X$ is a k -space.

It is proved in [12] that a Tychonoff space X is k -absolute if and only if $cX \setminus X$ is a k -space for some $cX \in K(X)$ and if and only if $cX \setminus X$ is a k -space for any $cX \in K(X)$.

All locally compact Tychonoff spaces are k -absolute because their Stone-Čech remainders are compact. Consequently, the following Rayburn's theorem generalizes one half of [Theorem 1.1](#).

THEOREM 1.3. *For any pair of k -absolute spaces X, Y , if the posets $K(X)$ and $K(Y)$ are isomorphic, then the Stone-Čech remainders $\beta X \setminus X$ and $\beta Y \setminus Y$ are homeomorphic.*

We note that the second half of [Theorem 1.1](#) cannot be generalized to k -absolute spaces (see [12, Example (B)]).

In [2], both [Theorems 1.1](#) and [1.3](#) are generalized to arbitrary compactifications of two Tychonoff spaces.

This paper is devoted to an extension of [Theorems 1.1](#) and [1.3](#) and their generalizations (given in [2]) to the class of WZ-mappings (in particular, closed mappings) from a locally compact Tychonoff space or a k -absolute space to a compact Hausdorff space. Although, even for this rather narrow class of mappings, the formulations of corresponding theorems look rather complicated, the examples presented in [Section 4](#) show that more simple approaches are not sufficient.

We note that results concerning the extension of [Theorem 1.1](#) to mappings are also contained in [5], but they are different from ours (see the remark at the end of this paper).

2. Preliminaries. Throughout this paper, space will mean a topological space and mapping will mean a continuous function. Terms and undefined concepts are used as in [4]. In this section, we recall some definitions and results from [2]. Some additional notions concerning fibrewise general topology (FGT) can be found in [9, 10].

DEFINITION 2.1. Let X, Y, Z be spaces and $\lambda : X \rightarrow Y, \mu : X \rightarrow Z$ mappings. We say that λ is equivalent to μ and we will write $\lambda \equiv \mu$ if there exists some homeomorphism $h : Y \rightarrow Z$ such that $\mu = h \circ \lambda$.

Evidently, the homeomorphism h is unique.

We will identify equivalent mappings, and so we can consider the set $\mathcal{C}(X)$ of all the continuous maps from a fixed space X onto other spaces.

DEFINITION 2.2. Let $\lambda, \mu \in \mathcal{C}(X)$. We say that λ follows μ and we will write $\lambda \geq \mu$ if there exists some continuous mapping $h : Y \rightarrow Z$ such that $\mu = h \circ \lambda$.

It is evident that $(\mathcal{C}(X), \geq)$ is a poset.

DEFINITION 2.3. Let $\lambda \in \mathcal{C}(X)$. We say that

- (i) λ is *simple* if there exists a unique point $t_\lambda \in \lambda(X)$ such that $|\lambda^{-1}(\{t_\lambda\})| > 1$ and $|\lambda^{-1}(\{t\})| = 1$ for every $t \in \lambda(X) \setminus \{t_\lambda\}$;
- (ii) λ is *finite simple* if there exists a nonempty finite set $T \subset \lambda(X)$ of points such that $|\lambda^{-1}(\{t\})| > 1$ for every $t \in T$ and $|\lambda^{-1}(\{t\})| = 1$ for every $t \in \lambda(X) \setminus T$.

We will suppose from this moment that X is a Hausdorff space and that $\mathcal{P}(X)$ denotes the poset (as a subset of $\mathcal{C}(X)$) of all perfect onto mappings of X . Clearly, $\lambda(X)$ is Hausdorff for any $\lambda \in \mathcal{P}(X)$.

DEFINITION 2.4. A mapping $\lambda \in \mathcal{P}(X)$ is called a *dual point* if it is simple and $|\lambda^{-1}(\{t_\lambda\})| = 2$.

Let $\mathcal{D} = \mathcal{D}(X)$ denote the set of all dual points of $\mathcal{P}(X)$ and $\mathcal{FS}(X) = \{\lambda \in \mathcal{P}(X) : \lambda \text{ is finite simple}\} \cup \{\text{id}_X\}$.

DEFINITION 2.5. A family $\mathcal{F} \subset \mathcal{D}$ is said to be a *3-vertex family* if for any distinct $\alpha, \beta \in \mathcal{F}$ there exists some $\gamma \in \mathcal{D} \setminus \mathcal{F}$ such that $\gamma > \inf\{\alpha, \beta\}$.

DEFINITION 2.6. A 3-vertex family $\mathcal{F} \subset \mathcal{D}$ is called a *point family* if it is maximal (i.e., if there is no 3-vertex family properly containing \mathcal{F}).

LEMMA 2.7. If \mathcal{F} is a 3-vertex family consisting of more than one element, then the set $X_{\mathcal{F}} = \bigcap \{\lambda^{-1}(\{t_\lambda\}) : \lambda \in \mathcal{F}\}$ is a single point (which will be denoted by $J_{\mathcal{F}}(\mathcal{F})$).

DEFINITION 2.8. Let $\mathcal{J} \subset \mathcal{P}(X)$ such that $\mathcal{FS}(X) \subset \mathcal{J}$ and $x \in X$. We put $K_{\mathcal{J}}(x) = \{\delta \in \mathcal{D} : x \in \delta^{-1}(\{t_\delta\})\}$.

LEMMA 2.9. If $|X| > 2$, then $K_{\mathcal{J}}(x)$ is a point family.

Evidently, for $|X| > 2$,

$$J_{\mathcal{F}}(K_{\mathcal{F}}(x)) = x \quad \text{for any } x \in X, \tag{2.1}$$

$$K_{\mathcal{F}}(J_{\mathcal{F}}(\mathcal{F})) = \mathcal{F} \quad \text{for any point family } \mathcal{F} \subset \mathcal{D}. \tag{2.2}$$

In [2], dual points are characterized only by means of the order in \mathcal{F} . It follows from this that if, for Hausdorff spaces X_j , we have $\mathcal{F}\mathcal{P}(X_j) \subset \mathcal{F}_j \subset \mathcal{P}(X_j)$ with $j = 1, 2$ and $i : \mathcal{F}_1 \rightarrow \mathcal{F}_2$ is a poset isomorphism, then $i(K_{\mathcal{F}_1}(x))$ is a point family in \mathcal{F}_2 .

We recall [2] that a one-to-one mapping $f : X \rightarrow Y$ between Hausdorff spaces is called a k -homeomorphism if f is continuous on compact subsets of X and its inverse f^{-1} is continuous on compact subsets of Y . Clearly, a k -homeomorphism between k -spaces is a homeomorphism.

THEOREM 2.10. *Let X_j be a Hausdorff space and $\mathcal{P}(X_j)$ the set of all perfect onto mappings of X_j (with $j = 1, 2$). If X_1 and X_2 are k -homeomorphic and they are k -spaces, then $\mathcal{P}(X_1)$ and $\mathcal{P}(X_2)$ (and so $\mathcal{F}\mathcal{P}(X_1)$ and $\mathcal{F}\mathcal{P}(X_2)$) are isomorphic. Let $\mathcal{F}\mathcal{P}(X_j) \subset \mathcal{F}_j \subset \mathcal{P}(X_j)$ for $j = 1, 2$. If \mathcal{F}_1 and \mathcal{F}_2 are poset isomorphic, then X_1 and X_2 are k -homeomorphic, and if, additionally, X_1 and X_2 are k -spaces, then they are homeomorphic. More precisely, if $i : \mathcal{F}_1 \rightarrow \mathcal{F}_2$ is a poset isomorphism and $|X_1| > 2$, then the function $h_i : X_1 \rightarrow X_2$, such that $h_i(x) = J_{\mathcal{F}_2}(i(K_{\mathcal{F}_1}(x)))$ for any $x \in X_1$, is a k -homeomorphism.*

Now, let X be a Tychonoff space and let $K(X)$ denote the poset of all Hausdorff compactifications of X (see, e.g., [3]).

For any $cX, dX \in K(X)$ such that $cX < dX$, let

$$\pi_{dc} = \pi_{dcX} : dX \rightarrow cX \tag{2.3}$$

be the canonical map (i.e., it is continuous and $\pi_{dc|X} = \text{id}_X$).

Then $\pi_{dc}^{-1}(cX \setminus X) = dX \setminus X$ and the mapping

$$r\pi_{dc} = r\pi_{dcX} \stackrel{\text{def}}{=} \pi_{dc} : dX \setminus X \rightarrow cXX \tag{2.4}$$

is perfect and onto, that is, $r\pi_{dc} \in \mathcal{P}(dX \setminus X)$.

Fix $eX \in K(X)$ and let

$$K(eX) = \{cX \in K(X) : cX \leq eX\} \tag{2.5}$$

(in particular, $K(\beta X) = K(X)$).

In [2], the function

$$\sigma_{eX} : K(eX) \rightarrow \mathcal{P}(eX \setminus X) \tag{2.6}$$

was defined by $\sigma_{eX}(cX) = r\pi_{ec}$ and the following lemma was proved.

LEMMA 2.11. *σ_{eX} is an isomorphism of the posets $K(eX)$ and*

$$\mathcal{L}(eX) \stackrel{\text{def}}{=} \sigma_{eX}(K(eX)). \tag{2.7}$$

The following two lemmas were also proved in [2].

LEMMA 2.12. $\mathcal{F}\mathcal{P}(eX \setminus X) \subset \mathcal{L}(eX)$.

LEMMA 2.13. *If X is a locally compact space, then $\mathcal{L}(eX) = \mathcal{P}(eX \setminus X)$.*

3. On the homeomorphisms of two pairs of spaces

DEFINITION 3.1. A space X and a closed subset A are called a *pair of spaces* and denoted by (X, A) .

DEFINITION 3.2. Suppose (X, A) and (Y, B) are pairs of spaces, where X, Y are Hausdorff. Then a homeomorphism (a k -homeomorphism) $h : X \rightarrow Y$ is called a homeomorphism (a k -homeomorphism) of the pair (X, A) onto the pair (Y, B) if $h(A) = B$.

Let X be a Hausdorff space and A a closed subset of X .

For any $\lambda \in \mathcal{P}(X)$, let

$$\text{res}_{XA}(\lambda) = \lambda : A \rightarrow \lambda(A), \tag{3.1}$$

that is, $\text{res}_{XA}(\lambda)$ is the corestriction of λ to A .

Evidently, $\text{res}_{XA}(\mathcal{P}(X)) \subset \mathcal{P}(A)$ and $\text{res}_{XA} : \mathcal{P}(X) \rightarrow \mathcal{P}(A)$ is monotonous.

It is not difficult to prove the following lemma.

LEMMA 3.3. $\text{res}_{XA}(\mathcal{P}(X)) = \mathcal{P}(A)$, $\text{res}_{XA}(\mathcal{F}\mathcal{P}(X)) = \mathcal{F}\mathcal{P}(A)$, and $\text{res}_{XA}(\mathcal{D}(X) \cup \{\text{id}_X\}) = \mathcal{D}(A) \cup \{\text{id}_A\}$.

Let $\mathcal{F}\mathcal{P}(X) \subset \mathcal{F}(X) \subset \mathcal{P}(X)$ and $\mathcal{F}\mathcal{P}(A) \subset \mathcal{F}(A) \subset \mathcal{P}(A)$. Then, clearly,

$$\text{res}_{XA}(K_{\mathcal{F}(X)}(x)) \setminus \{\text{id}_A\} = K_{\mathcal{F}(A)}(x) \quad \forall x \in A. \tag{3.2}$$

THEOREM 3.4. *Let X and Y be Hausdorff spaces, let A be a closed subset of X , and let B be a closed subset of Y ,*

$$\begin{aligned} \min\{|X|, |Y|\} \geq 3, & \quad \min\{|A|, |B|\} \geq 2, \\ \mathcal{F}\mathcal{P}(X) \subset \mathcal{F}(X) \subset \mathcal{P}(X), & \quad \mathcal{F}\mathcal{P}(A) \subset \mathcal{F}(A) \subset \mathcal{P}(A), \\ \mathcal{F}\mathcal{P}(Y) \subset \mathcal{F}(Y) \subset \mathcal{P}(Y), & \quad \mathcal{F}\mathcal{P}(B) \subset \mathcal{F}(B) \subset \mathcal{P}(B), \end{aligned} \tag{3.3}$$

and let

$$i_{XY} : \mathcal{F}(X) \rightarrow \mathcal{F}(Y), \quad i_{AB} : \mathcal{F}(A) \rightarrow \mathcal{F}(B) \tag{3.4}$$

be poset isomorphisms such that

$$i_{AB} \circ \text{res}_{XA} = \text{res}_{YB} \circ i_{XY}. \tag{3.5}$$

Then

- (i) *in the case $\min\{|A|, |B|\} \geq 3$ for k -homeomorphisms $h_{i_{AB}} : A \rightarrow B$ and $h_{i_{XY}} : X \rightarrow Y$ (see [Theorem 2.10](#)),*

$$h_{i_{AB}}(x) = h_{i_{XY}}(x) \quad \text{for every } x \in A; \tag{3.6}$$

(ii) in the case $\min\{|A|, |B|\} = 2$, there exists a homeomorphism $h_{i_{AB}}(x) : A \rightarrow B$ such that (3.6) is also true.

Thus

$$h_{i_{XY}}(A) \subset B, \quad h_{i_{AB}} = h_{i_{XY}} : A \rightarrow B, \tag{3.7}$$

and so $h_{i_{XY}}$ is a k -homeomorphism of (X, A) onto (Y, B) .

If, additionally, X and Y are k -spaces, then $h_{i_{XY}}$ and $h_{i_{AB}}$ are homeomorphisms, and so $h_{i_{XY}}$ is a homeomorphism of (X, A) onto (Y, B) .

PROOF. First, let $\min\{|A|, |B|\} \geq 3$.

Let $x \in A$. Then, by Theorem 2.10, $h_{i_{XY}}$ and $h_{i_{AB}}$ are k -homeomorphisms and

$$\begin{aligned} h_{i_{AB}}(x) &= J_{\mathcal{F}(B)}(i_{AB}(K_{\mathcal{F}(A)}(x))) \quad (\text{by (3.2)}) \\ &= J_{\mathcal{F}(B)}(i_{AB}(\text{res}_{XA}(K_{\mathcal{F}(X)}(x)) \setminus \{\text{id}_A\})) \\ &\quad (\text{by (3.5) and since } i_{AB} \text{ is a poset isomorphism}) \\ &= J_{\mathcal{F}(B)}(\text{res}_{YB}(i_{XY}(K_{\mathcal{F}(X)}(x))) \setminus \{\text{id}_B\}) \\ &\quad (\text{by (2.2) and since } i_{XY} \text{ is a poset isomorphism}) \\ &= J_{\mathcal{F}(B)}(\text{res}_{YB}(K_{\mathcal{F}(Y)}(J_{\mathcal{F}(Y)}(i_{XY}(K_{\mathcal{F}(X)}(x)))))) \setminus \{\text{id}_B\}) \\ &\quad (\text{by the definition of } h_{i_{XY}}) \\ &= J_{\mathcal{F}(B)}(\text{res}_{YB}(K_{\mathcal{F}(Y)}(h_{i_{XY}}(x))) \setminus \{\text{id}_B\}) \quad (\text{by (3.2)}) \\ &= J_{\mathcal{F}(B)}(K_{\mathcal{F}(B)}(h_{i_{XY}}(x))) \quad (\text{by (2.1)}) \\ &= h_{i_{XY}}(x). \end{aligned} \tag{3.8}$$

Now, let $|A| = 2$. Since i_{AB} is a poset isomorphism, $|\mathcal{F}(B)| = |\mathcal{F}(A)| = 2$ and so $|B| = 2$.

There is a unique dual point $\lambda \in \mathcal{F}(X)$ such that $\lambda^{-1}(\{t_\lambda\}) = A$. Let $A = \{x_1, x_2\}$. Evidently, $K_{\mathcal{F}(X)}(x_1) \cap K_{\mathcal{F}(X)}(x_2) = \{\lambda\}$. Then, by (2.2),

$$\begin{aligned} K_{\mathcal{F}(Y)}(h_{i_{XY}}(x_i)) &= K_{\mathcal{F}(Y)}(J_{\mathcal{F}(Y)}(i_{XY}(K_{\mathcal{F}(X)}(x_i)))) \\ &= i_{XY}(K_{\mathcal{F}(X)}(x_i)) \quad \text{for } i = 1, 2. \end{aligned} \tag{3.9}$$

Hence

$$\begin{aligned} i_{XY}(\{\lambda\}) &= i_{XY}(K_{\mathcal{F}(X)}(x_1) \cap K_{\mathcal{F}(X)}(x_2)) \\ &= i_{XY}(K_{\mathcal{F}(X)}(x_1)) \cap i_{XY}(K_{\mathcal{F}(X)}(x_2)) \\ &= K_{\mathcal{F}(Y)}(h_{i_{XY}}(x_1)) \cap K_{\mathcal{F}(Y)}(h_{i_{XY}}(x_2)). \end{aligned} \tag{3.10}$$

Thus, for $\eta = i_{XY}(\lambda)$, $\eta^{-1}(\{t_\eta\}) = \{h_{i_{XY}}(x_1), h_{i_{XY}}(x_2)\} = h_{i_{XY}}(A)$. But, by (3.5), $\xi = \text{res}_{YB}(\eta) = \text{res}_{YB}(i_{XY}(\lambda)) = i_{AB}(\text{res}_{XA}(\lambda))$. Since $\text{res}_{XA}(\lambda)$ is a dual point in $\mathcal{F}(A)$ and i_{AB} is a poset isomorphism, ξ is also a dual point in $\mathcal{F}(B)$. Thus, $B = \xi^{-1}(\{t_\xi\}) = \eta^{-1}(\{t_\eta\}) = h_{i_{XY}}(A)$. \square

4. Extensions of Magill's and Rayburn's theorems to mappings

DEFINITION 4.1. For mappings $f_j : X_j \rightarrow Y$ of (Hausdorff) spaces X_j (with $j = 1, 2$), a (k -) homeomorphism $h : X_1 \rightarrow X_2$ is called a (k -) homeomorphism of f_1 onto f_2

if $f_1 = f_2 \circ h$. The mappings $f_j : X_j \rightarrow Y$ (for $j = 1, 2$) are (k -) homeomorphic if there exists a (k -) homeomorphism of f_1 onto f_2 .

It is not difficult to prove the following lemma.

LEMMA 4.2. *For mappings $f_j : X_j \rightarrow Y$ (with $j = 1, 2$) and for (k -) homeomorphism $h : X_1 \rightarrow X_2$ of spaces X_1 and X_2 , the following conditions are equivalent:*

- (i) h is a homeomorphism of f_1 onto f_2 ;
- (ii) $h(f_1^{-1}(\{y\})) \subset f_2^{-1}(\{y\})$ for every $y \in Y$;
- (iii) there is a function $h_y : f_1^{-1}(\{y\}) \rightarrow f_2^{-1}(\{y\})$ such that $h_y = h : f_1^{-1}(\{y\}) \rightarrow f_2^{-1}(\{y\})$ for every $y \in Y$.

Given a Tychonoff space X and a closed subset A of X , we may define a function

$$\text{kres}_{XA} : K(X) \rightarrow K(A) \tag{4.1}$$

such that

$$\text{kres}_{XA}(cX) = cl_{cX}(A) \quad \text{for every } cX \in K(X). \tag{4.2}$$

If $eX, cX \in K(X)$ and $cX < eX$, then $\pi_{ecX}(cl_{eX}(A)) = cl_{cX}(A)$, $\pi_{ecX}(A) = A$, and $(\pi_{ecX} : A \rightarrow A) = \text{id}_A$.

Consequently, $(eA = \text{kres}_{XA}(eX)) > (cA = \text{kres}_{XA}(cX))$ and $\pi_{ecA} = \pi_{ecX} : eA \rightarrow cA$. Thus, kres_{XA} is monotone and $r\pi_{ecA} = r\pi_{ecX} : eA \setminus A \rightarrow cA \setminus A$, that is, $r\pi_{ecA} = \text{res}_{eX \setminus X, eA \setminus A}(r\pi_{ecX})$. It follows from this that $(\sigma_{eA} \circ \text{kres}_{XA})(cX) = \sigma_{eA}(cA) = r\pi_{ecA} = \text{res}_{eX \setminus X, eA \setminus A}(r\pi_{ecX}) = \text{res}_{eX \setminus X, eA \setminus A}(\sigma_{eX}(cX))$.

We have then proven the following lemma.

LEMMA 4.3. *If $eX \in K(X)$ and $eA = \text{kres}_{XA}(eX)$, then*

$$\text{res}_{eX \setminus X, eA \setminus A} \circ \sigma_{eX} = \sigma_{eA} \circ \text{kres}_{XA} \upharpoonright_{K(eX)}. \tag{4.3}$$

Let $f : X \rightarrow Y$ be a mapping to a Tychonoff space Y and let $\beta f^+ : \beta X \rightarrow \beta Y$ be the (usual) continuous extension of f over the Stone-Ćech compactifications βX , $\beta_f X = (\beta f^+)^{-1}(Y)$, and $\beta f = \beta f^+ : \beta_f X \rightarrow Y$. Evidently, the mapping βf is perfect. We note that X is C^* -embedded in $\beta_f X$ because $X \subset \beta_f X \subset \beta X$. Recall that f is called a *WZ-mapping* [6] (resp., a *Z-mapping*) if $(\beta f)^{-1}(\{y\}) = cl_{\beta_f X}(f^{-1}(\{y\}))$ for every $y \in Y$ (resp., if $f(Z)$ is closed for any zero-set Z in X). It is clear that $(\beta f)^{-1}(\{y\}) = \beta(f^{-1}(\{y\}))$ if the space X is normal and f is a WZ-mapping. It is known [6] that every Z-mapping is a WZ-mapping.

THEOREM 4.4. *Let X_j be a Tychonoff space, let Y be a compact Hausdorff space, let $f_j : X_j \rightarrow Y$ be a WZ-mapping, let eX_j be a Hausdorff compactification of X_j , and let $ef_j : eX_j \rightarrow Y$ be a continuous extension of f_j (thus, ef_j is a compactification of f_j) for $j = 1, 2$. Let also $X_{jy} = f_j^{-1}(\{y\})$, $eX_{jy} = cl_{eX_j}(X_{jy})$ (i.e., $eX_{jy} = \text{kres}_{X_j X_{jy}}(eX_j)$) for $j = 1, 2$, and suppose that there exist poset isomorphisms $i : K(eX_1) \rightarrow K(eX_2)$ and $i_y : K(eX_{1y}) \rightarrow K(eX_{2y})$ such that*

$$i_y \circ \text{kres}_{X_1 X_{1y}} = \text{kres}_{X_2 X_{2y}} \circ i \quad \text{for every } y \in Y. \tag{4.4}$$

Then the remainders $ef_j \setminus f_j \stackrel{\text{def}}{=} ef_j : eX_j \setminus X_j \rightarrow Y$ of ef_j for $j = 1, 2$ are k -homeomorphic (more exactly, if $\min\{|eX_1 \setminus X_1|, |eX_2 \setminus X_2|\} \geq 3$, then the function $h_{\sigma_{eX_2} \circ i \circ \sigma_{eX_1}^{-1}} : eX_1 \setminus X_1 \rightarrow eX_2 \setminus X_2$ is a k -homeomorphism of $ef_1 \setminus f_1$ onto $ef_2 \setminus f_2$). If, additionally, X_1 and X_2 are k -absolute spaces, then the remainders $ef_1 \setminus f_1$ and $ef_2 \setminus f_2$ are homeomorphic.

PROOF. Fix $y \in Y$. Let $R_{e_j} = eX_j \setminus X_j$ and $R_{e_{jy}} = eX_{jy} \setminus X_{jy}$ for $j = 1, 2$.

By Lemmas 2.11 and 2.12, the mappings $\sigma_{eX_j} : K(eX_j) \rightarrow (\mathcal{F}_j \stackrel{\text{def}}{=} \sigma_{eX_j}(K(eX_j)))$ and $\sigma_{eX_{jy}} : K(eX_{jy}) \rightarrow (\mathcal{F}_{jy} \stackrel{\text{def}}{=} \sigma_{eX_{jy}}(K(eX_{jy})))$ are poset isomorphisms and $\mathcal{F}\mathcal{G}(R_{e_j}) \subset \mathcal{F}_j$, $\mathcal{F}\mathcal{G}(R_{e_{jy}}) \subset \mathcal{F}_{jy}$ for $j = 1, 2$.

Hence, $i_{12} = \sigma_{eX_2} \circ i \circ \sigma_{eX_1}^{-1} : \mathcal{F}_1 \rightarrow \mathcal{F}_2$ and $i_{12y} = \sigma_{eX_{2y}} \circ i_y \circ \sigma_{eX_{1y}}^{-1} : \mathcal{F}_{1y} \rightarrow \mathcal{F}_{2y}$ are poset isomorphisms too.

We consider the following diagram:

$$\begin{array}{ccc}
 K(eX_1) & \xrightarrow{i} & K(eX_2) \\
 \downarrow \text{kres}_{X_1 X_{1y}} & \swarrow \sigma_{eX_1}^{-1} & \searrow \sigma_{eX_2} \\
 & \mathcal{F}_1 & \xrightarrow{i_{12}} & \mathcal{F}_2 \\
 & \downarrow \text{res}_{R_{e_1} R_{e_{1y}}} & & \downarrow \text{res}_{R_{e_2} R_{e_{2y}}} \\
 & \mathcal{F}_{1y} & \xrightarrow{i_{12y}} & \mathcal{F}_{2y} \\
 & \swarrow \sigma_{eX_{1y}}^{-1} & & \searrow \sigma_{eX_{2y}} \\
 K(eX_{1y}) & \xrightarrow{i_y} & K(eX_{2y})
 \end{array} \tag{4.5}$$

By (4.4), the external part of it is commutative. We prove that its internal part is commutative too, that is, that

$$\text{res}_{R_{e_2} R_{e_{2y}}} \circ i_{12} = i_{12y} \circ \text{res}_{R_{e_1} R_{e_{1y}}} \tag{4.6}$$

By (4.3), for $j = 1, 2$, we have (see the diagram)

$$\text{res}_{R_{e_j} R_{e_{jy}}} \circ \sigma_{eX_j} = \sigma_{eX_{jy}} \circ \text{kres}_{X_j X_{jy}} \upharpoonright_{K(eX_j)} \tag{4.7}$$

Thus, since $\sigma_{eX_j}, \sigma_{eX_{jy}}$ are isomorphisms,

$$\begin{aligned}
 \text{kres}_{X_j X_{jy}} \upharpoonright_{K(eX_j)} \circ \sigma_{eX_j}^{-1} &= \sigma_{eX_{jy}}^{-1} \circ \text{res}_{R_{e_j} R_{e_{jy}}}, \\
 \text{res}_{R_{e_j} R_{e_{jy}}}(\mathcal{F}_j) &\subset \mathcal{F}_{jy}.
 \end{aligned} \tag{4.8}$$

Hence,

$$\begin{aligned}
 \text{res}_{R_{e2}R_{e2y}} \circ i_{12} &= \text{res}_{R_{e2}R_{e2y}} \circ \sigma_{eX_2} \circ i \circ \sigma_{eX_1}^{-1} \\
 &= \sigma_{eX_{2y}} \circ \text{kres}_{X_2X_{2y}} \circ i \circ \sigma_{eX_1}^{-1} \\
 &= \sigma_{eX_{2y}} \circ i_y \circ \text{kres}_{X_1X_{1y}} \circ \sigma_{eX_1}^{-1} \\
 &= \sigma_{eX_{2y}} \circ i_y \circ \sigma_{eX_{1y}}^{-1} \circ \text{res}_{R_{e1}R_{e1y}} \\
 &= i_{12y} \circ \text{res}_{R_{e1}R_{e1y}}.
 \end{aligned}
 \tag{4.9}$$

By [Theorem 3.4](#), in the case $\min\{|R_{e1}|, |R_{e2}|, |R_{e1y}|, |R_{e2y}|\} \geq 3$, the homeomorphisms $h_{i_{12}} : R_{e1} \rightarrow R_{e2}$ and $h_{i_{12y}} : R_{e1y} \rightarrow R_{e2y}$ are such that

$$h_{i_{12y}} = h_{i_{12}} : R_{e1y} \rightarrow R_{e2y}, \tag{4.10}$$

and in the case $\min\{|R_{e1}|, |R_{e2}|\} \geq 3, \min\{|R_{e1y}|, |R_{e2y}|\} \geq 2$, there exists a homeomorphism $h_{i_{12y}} : R_{e1y} \rightarrow R_{e2y}$ such that [\(4.10\)](#) is also true.

Finally, in the case $\min\{|R_{e1}|, |R_{e2}|\} \geq 3, \min\{|R_{e1y}|, |R_{e2y}|\} \leq 1$, the existence of a homeomorphism $h_{i_{12y}} : R_{e1y} \rightarrow R_{e2y}$ such that [\(4.10\)](#) holds is evident.

Thus, in the case $\min\{|R_{e1}|, |R_{e2}|\} \geq 3$, [\(4.10\)](#) holds for every $y \in Y$.

In the case $\min\{|R_{e1}|, |R_{e2}|\} \leq 2$, the existence of homeomorphisms $h_{i_{12}} : R_{e1} \rightarrow R_{e2}$ and $h_{i_{12y}} : R_{e1y} \rightarrow R_{e2y}$ such that [\(4.10\)](#) holds is evident.

Since f_j is a WZ-mapping, $R_{e_jy} = (ef_j)^{-1}(\{y\}) \setminus f_j^{-1}(\{y\})$ for $j = 1, 2$, and so

$$h_{i_{12y}} = h_{i_{12}} : (ef_1)^{-1}(\{y\}) \setminus f_1^{-1}(\{y\}) \rightarrow (ef_2)^{-1}(\{y\}) \setminus f_2^{-1}(\{y\}) \quad \text{for every } y \in Y. \tag{4.11}$$

By [Lemma 4.2](#), $h_{i_{12}}$ is a k -homeomorphism of $ef_1 \setminus f_1$ onto $ef_2 \setminus f_2$. □

COROLLARY 4.5 [2]. *Let X_1, X_2 be Tychonoff spaces and let eX_1, eX_2 be their Hausdorff compactifications. If $K(eX_1)$ and $K(eX_2)$ are poset isomorphic, then the remainders $eX_1 \setminus X_1$ and $eX_2 \setminus X_2$ are k -homeomorphic, and they are homeomorphic if, additionally, X_1, X_2 are k -absolute spaces.*

PROOF. It is sufficient to apply [Theorem 4.4](#) to (the simplest) mappings f_j of X_j to the single point space Y for $j = 1, 2$. □

In particular, in [Corollary 4.5](#), for $eX_j = \beta X_j$ ($j = 1, 2$), we have Rayburn's [Theorem 1.3](#). Thus, [Theorem 4.4](#) is a generalization of this theorem to mappings.

THEOREM 4.6. *Let X_j be a locally compact Tychonoff space, let Y be a compact Hausdorff space, let $f_j : X_j \rightarrow Y$ be a WZ-mapping, let eX_j be a Hausdorff compactification of X_j , and let $ef_j : eX_j \rightarrow Y$ be a continuous extension of f_j for $j = 1, 2$. Let also $X_{jy} = f_j^{-1}(\{y\})$, $eX_{jy} = cl_{eX_j}(X_{jy})$ (i.e., $eX_{jy} = \text{kres}_{X_jX_{jy}}(eX_j)$) (for $j = 1, 2$). Then the remainders $ef_j \setminus f_j = ef_j : eX_j \setminus X_j \rightarrow Y$ of ef_j for $j = 1, 2$ are homeomorphic if and only*

if there exist poset isomorphisms $i : K(eX_1) \rightarrow K(eX_2)$ and $i_y : K(eX_{1y}) \rightarrow K(eX_{2y})$ such that (4.4) holds.

PROOF. One half of the theorem follows from [Theorem 4.4](#).

Let $R_{e_j} = eX_j \setminus X_j$ and $R_{e_{jy}} = eX_{jy} \setminus X_{jy}$ for all $y \in Y$ and $j = 1, 2$. Now, suppose that the remainders $ef_1 \setminus f_1$ and $ef_2 \setminus f_2$ are homeomorphic. Then there exists a homeomorphism $h : R_{e_2} \rightarrow R_{e_1}$ such that $ef_2 \setminus f_2 = (ef_1 \setminus f_1) \circ h$. Hence, the mappings $h_y = h : R_{e_{2y}} \rightarrow R_{e_{1y}}$ are homeomorphisms for all $y \in Y$. Since f_j is a WZ-mapping, $(ef_j)^{-1}(\{y\})$ is a compactification of $f_j^{-1}(\{y\})$, and so $eX_{jy} = (ef_j)^{-1}(\{y\})$ for all $y \in Y$ and $j = 1, 2$. Evidently, $i_{12} : \mathcal{P}(R_{e_1}) \rightarrow \mathcal{P}(R_{e_2})$ and $i_{12y} : \mathcal{P}(R_{e_{1y}}) \rightarrow \mathcal{P}(R_{e_{2y}})$, such that $i_{12}(\lambda) = \lambda \circ h$ for $\lambda \in \mathcal{P}(X_1)$ and $i_{12y}(\lambda) = \lambda \circ h_y$ for $\lambda \in \mathcal{P}(X_{1y})$ and all $y \in Y$, are poset isomorphisms.

We prove that (4.4) holds for all $y \in Y$.

Indeed, for every $\lambda \in \mathcal{P}(R_{e_1})$ and $y \in Y$, we have

$$\begin{aligned} i_{12y} \circ \text{res}_{R_{e_1}R_{e_{1y}}}(\lambda) &= i_{12y}(\lambda|_{R_{e_{1y}}}) = \lambda|_{R_{e_{1y}}} \circ h_y \\ &= (\lambda \circ h)|_{R_{e_{2y}}} = (i_{12}(\lambda))|_{R_{e_{2y}}} \\ &= \text{res}_{R_{e_2}R_{e_{2y}}} \circ i_{12}(\lambda). \end{aligned} \tag{4.12}$$

By [Lemmas 2.11](#) and [2.13](#), $\sigma_{eX_j} : K(eX_j) \rightarrow \mathcal{P}(R_{e_j})$ and $\sigma_{eX_{jy}} : K(eX_{jy}) \rightarrow \mathcal{P}(R_{e_{jy}})$, for $j = 1, 2$ and $y \in Y$, are poset isomorphisms. Hence, $i = \sigma_{eX_2}^{-1} \circ i_{12} \circ \sigma_{eX_1}$ and $i_y = \sigma_{eX_{2y}}^{-1} \circ i_{12y} \circ \sigma_{eX_{1y}}$ are poset isomorphisms for $y \in Y$ and $j = 1, 2$.

We consider the diagram obtained from the previous one by replacing \mathcal{F}_j and \mathcal{F}_{jy} by $\mathcal{P}(R_{e_j})$ and $\mathcal{P}(R_{e_{jy}})$ for $j = 1, 2$ and $\sigma_{eX_1}^{-1}, \sigma_{eX_{1y}}^{-1}, \sigma_{eX_2}, \sigma_{eX_{2y}}$ by $\sigma_{eX_1}, \sigma_{eX_{1y}}, \sigma_{eX_2}^{-1}, \sigma_{eX_{2y}}^{-1}$, respectively. By (4.12), its internal part is commutative. As above, in the proof of [Theorem 4.4](#), we can prove that its external part is commutative too, that is, that (4.4) holds. □

COROLLARY 4.7 [2]. *Let X_1, X_2 be locally compact Tychonoff spaces and let eX_1, eX_2 be Hausdorff compactifications of X_1 and X_2 , respectively. Then the remainders $eX_1 \setminus X_1$ and $eX_2 \setminus X_2$ are homeomorphic if and only if $K(eX_1)$ and $K(eX_2)$ are poset isomorphic.*

PROOF. It is sufficient to apply [Theorem 4.6](#) to the simplest mappings f_j of X_j and ef_j of eX_j to the single point space Y for $j = 1, 2$. □

In particular, when in [Corollary 4.7](#), $eX_j = \beta X_j$ for $j = 1, 2$, we have Magill's theorem from [8]. Thus, [Theorem 4.6](#) is a generalization of this theorem to mappings.

5. Reformulations of results obtained above and some examples. Some readers may find that [Theorems 4.4](#) and [4.6](#) do not sound very natural. The reformulations, in the framework of FGT, sound better to us.

We will start with some definitions and results of FGT.

A mapping is called *compact* if it is perfect.

The following is evident.

LEMMA 5.1. For a compact Hausdorff space Y , a mapping $f : X \rightarrow Y$ is compact if and only if X is compact.

DEFINITION 5.2. A mapping $f : X \rightarrow Y$ is said to be T_0 [11] if for every $x, x' \in X$ such that $x \neq x'$ and $f(x) = f(x')$, there exists a neighbourhood of x in X which does not contain x' or a neighbourhood of x' in X not containing x .

DEFINITION 5.3. A mapping $f : X \rightarrow Y$ is said to be *completely regular* [11] if for every closed set F of X and $x \in X \setminus F$, there exist a neighbourhood O of $f(x)$ in Y and a continuous function $\varphi : f^{-1}(O) \rightarrow [0, 1]$ such that $\varphi(x) = 0$ and $\varphi(F \cap f^{-1}(O)) \subseteq \{1\}$. A completely regular T_0 -mapping is called *Tychonoff* (or $T_{3(1/2)}$).

It is not difficult to prove the following lemma.

LEMMA 5.4 [11]. For a Tychonoff space Y , a mapping $f : X \rightarrow Y$ is Tychonoff if and only if X is Tychonoff.

DEFINITION 5.5. A compact Tychonoff mapping $ef : e_f X \rightarrow Y$ is called a *Tychonoff compactification* of a Tychonoff mapping $f : X \rightarrow Y$ if $X \subset e_f X$, X is dense in $e_f X$, and $e_f X|_X = f$ (more precisely, if some embedding $e : X \rightarrow e_f X$ is fixed so that $e(X)$ is dense in $e_f X$ and $f = ef \circ e$, but usually, X and $e(X)$ are identified by means of e).

Throughout the rest of the paper, we fix a space Y and we will consider only Tychonoff mappings to Y and their Tychonoff compactifications.

DEFINITION 5.6. A mapping $\lambda : d_f X \rightarrow c_f X$ between two compactifications $cf : c_f X \rightarrow Y$ and $df : d_f X \rightarrow Y$ is called *canonical* if $df = cf \circ \lambda$ and $\lambda|_X = \text{id}_X$. In this case, one says that we have a canonical morphism $\lambda : df \rightarrow cf$ (and we write that $df > cf$).

It is not difficult to prove that df and cf are homeomorphic if and only if $df > cf$ and $cf > df$ (see, e.g., [1]).

It is proved in [11] (see also [1]) that all compactifications of a mapping to Y form a set up to canonical homeomorphisms. This set will be denoted by $TK(f)$. Evidently, with respect to the just defined relation $>$, $TK(f)$ is a poset.

In [11], it is also proved that there exists the maximal element $\beta f : \beta_f X \rightarrow Y$ in $TK(f)$ and that, if Y is Tychonoff, βf may be obtained in the following way.

By Lemma 5.4, X is Tychonoff. Hence, there exists the unique continuous extension $\beta f^+ : \beta X \rightarrow \beta Y$ of f . Then $\beta_f X = (\beta f^+)^{-1}(Y)$ and $\beta f = \beta f^+ : \beta_f X \rightarrow Y$.

For a compactification $ef : e_f X \rightarrow Y$ of a mapping $f : X \rightarrow Y$, the mapping $ef \setminus f = ef : e_f X \setminus X \rightarrow Y$ is called the *remainder* of ef .

A mapping $f : X \rightarrow Y$ is called *locally compact* [7] if for any point $x \in X$, there exists a neighbourhood O of x in X such that $f|_{cl_X(O)} : cl_X(O) \rightarrow Y$ is compact.

It is not difficult to prove that

- (i) for a locally compact Tychonoff space Y , a mapping $f : X \rightarrow Y$ is locally compact if and only if X is locally compact;

(ii) a mapping $f : X \rightarrow Y$ is locally compact if and only if X is open in $\beta_f X$ or, equivalently, if X is open in $e_f X$ for any compactification $e_f : e_f X \rightarrow Y$ of f .

Now, [Theorem 4.4](#) can be reformulated in the following way.

THEOREM 5.7. *Let Y be a compact Hausdorff space and $e_{f_j} : e_{f_j} X_j \rightarrow Y$ a Tychonoff compactification of a locally compact Tychonoff WZ-mapping $f_j : X_j \rightarrow Y$ (for $j = 1, 2$). Let also $X_{jY} = f_j^{-1}(\{y\})$, $eX_{jY} = cl_{eX_j}(X_{jY})$ for $j = 1, 2$. Then the remainders $e_{f_1} \setminus f_1$ and $e_{f_2} \setminus f_2$ are homeomorphic if and only if there exist poset isomorphisms $i : K(e_{f_1} X_1) \rightarrow K(e_{f_2} X_2)$ and $i_y : K(eX_{1Y}) \rightarrow K(eX_{2Y})$ such that (4.4) holds.*

[Theorem 3.4](#) may be reformulated analogously. But, even in the style of [Theorem 5.7](#), the formulation of [Theorem 4.4](#) (and [Theorem 3.4](#)) seems too complicated, but this complexity may not be avoided. In order to explain why, for any fixed space Y , consider the category \mathbf{Top}_Y , where

$$\text{Ob}(\mathbf{Top}_Y) = \{f \in C(X, Y) : X \in \text{Ob}(\mathbf{Top})\} \tag{5.1}$$

is the class of the *objects* and, for every pair $f : X \rightarrow Y, g : Z \rightarrow Y$ of objects,

$$M(f, g) = \{\lambda \in C(X, Z) : g \circ \lambda = f\} \tag{5.2}$$

is the class of the *morphisms* from f to g , whose generic representation is denoted in short by $\lambda : f \rightarrow g$.

So, the question is: may the passage from the category \mathbf{Top} to the category \mathbf{Top}_Y allow us to give simpler variants of [Theorems 4.4](#) and [4.6](#) which can generalize [Magill's](#) and [Rayburn's](#) theorems?

In this connection, we will give two examples which demonstrate that in the framework of \mathbf{Top}_Y , such generalizations are impossible.

EXAMPLE 5.8. Let $Y = [0, 2], X_1 = X_2 = \mathbb{N}$, and $f_j : X_j \rightarrow Y$ be such that $f_j(X_j) = \{j\}$ for $j = 1, 2$. Then $\beta_{f_j} X_j = \beta\mathbb{N}, \beta f_j(\beta_{f_j} X_j) = \{j\}, \beta f_j \setminus f_j = \beta f_j : \beta_{f_j} X_j \setminus X_j \rightarrow Y$, and so $\beta f_j \setminus f_j = \beta f_j : \beta\mathbb{N} \setminus \mathbb{N} \rightarrow Y$, with $(\beta f_j \setminus f_j)(\beta\mathbb{N} \setminus \mathbb{N}) = \{j\}$ for $j = 1, 2$. Thus, the remainders $\beta f_1 \setminus f_1$ and $\beta f_2 \setminus f_2$ are not homeomorphic, but $TK(f_1)$ and $TK(f_2)$ are poset isomorphic because they, in fact, coincide with $K(\mathbb{N})$.

This example shows that an extension of [Magill's](#) Theorem to the category \mathbf{Top}_Y must take into consideration fibres of objects of \mathbf{Top}_Y .

EXAMPLE 5.9. Let $I = \{(x, 0) \in \mathbb{R}^2 : 0 \leq x \leq 1\}, L = \{(0, y) \in \mathbb{R}^2 : -1 \leq y \leq 1\}, J = \{(x, 1) \in \mathbb{R}^2 : 0 \leq x \leq 1\}, Y = I, S_1 = I \cup L$, and $S_2 = J \cup L$. Let $j = 1, 2$. Put $pr_j(x, y) = x$ for any $(x, y) \in S_j$. Let ω_1 be the space of all finite and countable ordinal numbers. Then $\beta\omega_1 = \omega_1 + 1$. Put $X_j = S_j \times \omega_1$ and let π_j be the projection of the product X_j onto its factor S_j . Take $f_j = pr_j \circ \pi_j$. Evidently, f_j is either closed or open and all its fibres are countable compact. Then $\beta X_j = S_j \times (\omega_1 + 1)$ and $R_j = \beta X_j \setminus X_j$ is homeomorphic to S_j . Let p_j be the projection of the product βX_j onto its factor S_j . Then $\beta f_j = pr_j \circ p_j$ and $\beta f_j \setminus f_j = \beta f_j : R_j \rightarrow Y$ is homeomorphic to pr_j .

Thus, *the remainders $\beta f_1 \setminus f_1$ and $\beta f_2 \setminus f_2$ are not homeomorphic.*

For any $t = (x, 0) \in Y$, $(\beta f_j)^{-1}(\{t\}) = \beta(f_j^{-1}(\{t\}))$, and, for any $t = (x, 0) \in Y$ with $x > 0$, the remainders $R_{jt} = (\beta f_j)^{-1}(\{t\}) \setminus f_j^{-1}(\{t\}) = \beta(f_j^{-1}(\{t\})) \setminus f_j^{-1}(\{t\})$ are single points. Hence, $K(f_j^{-1}(\{t\}))$ consists of only one element for $t = (x, 0) \in Y$ with $x > 0$. For $t = (0, 0)$, the remainder R_{jt} is homeomorphic to L . Evidently, $TK(f_j)$ and $K(f_j^{-1}(\{(0, 0)\}))$ are poset isomorphic to $\mathcal{P}(L)$.

Now, it is evident that *there exist poset isomorphisms* $i : TK(f_1) \rightarrow TK(f_2)$, $i_t : K(f_1^{-1}(\{t\})) \rightarrow K(f_2^{-1}(\{t\}))$ for $t \in Y$ and *monotone functions* $m_{tj} : TK(f_j) \rightarrow K(f_j^{-1}(\{t\}))$ for $t \in Y$ and $j = 1, 2$ such that

- (i) for any $cf_j : c_{f_j}X_j \rightarrow Y$ with $cf_j \in TK(f_j)$, $m_{tj}(cf_j) = (cf_j)^{-1}(\{t\})$, that is, $m_{tj}(cf_j)$ is the closure of $f_j^{-1}(\{t\})$ in $c_{f_j}X_j$;
- (ii) $m_{t2} \circ i = i_t \circ m_{t1}$ for every $t \in Y$.

Example 5.9 shows that the use of the posets $K(eX_1)$ and $K(eX_2)$ (in particular $K(X_1)$ and $K(X_2)$) instead of $TK(f_1)$ and $TK(f_2)$ is justified.

REMARK 5.10. Let $f : X \rightarrow Y$ be a mapping between Tychonoff spaces $M(f) = \{y \in Y : f \text{ is not closed at } y \text{ or } f^{-1}(\{y\}) \text{ is not compact}\}$ and $M_1(f) = \{y \in Y : |(\beta f)^{-1}(\{y\}) \setminus f^{-1}(\{y\})| = 1\}$. Evidently, $M_1(f) \subset M(f)$ and $\beta f(\beta_f X \setminus X) = M(f)$. The following proposition is proved in [5].

PROPOSITION 5.11. *Let $f : X \rightarrow Y$ and $g : Z \rightarrow T$ be locally compact mappings between Tychonoff spaces and $|M_1(f)| = |M_1(g)| = 1$. Then there exists a one-to-one correspondence $\Psi : M(f) \rightarrow M(g)$ such that $(\beta f \setminus f)^{-1}(\{y\})$ and $(\beta g \setminus g)^{-1}(\{\Psi(y)\})$ are homeomorphic for any $y \in Y$. If, additionally, $M(f)$ and $M(g)$ are discrete, then $\beta f \setminus f$ and $\beta g \setminus g$ are homeomorphic in the sense of [5], that is, there exist homeomorphisms $\varphi : \beta_f X \setminus X \rightarrow \beta_g Z \setminus Z$ and $\psi : M(f) \rightarrow M(g)$ such that $\psi \circ (\beta f \setminus f) = (\beta g \setminus g) \circ \varphi$ if and only if $TK(f)$ and $TK(g)$ are poset isomorphic.*

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Giorgio Nardo: Dipartimento di Matematica, Università di Messina, Contrada Papardo, Salita Sperone 31, 98166 Sant'Agata, Messina, Italy

E-mail address: nordo@dipmat.unime.it

Boris A. Pasynkov: Chair of General Topology and Geometry, Faculty of Mechanics and Mathematics, Moscow State University, Moscow 119899, Russia

E-mail address: pasynkov@mech.math.msu.su