Sohag Journal of Mathematics
An International Journal

http://dx.doi.org/10.18576/sjm/070302

Neutrosophic Soft s-Open Sets

Arif Mehmood [1,*], *Giorgio Nordo* [2], *Fawad Nadeem* [3], *Humaira Kalsoom* [4], *Shamoona Jabeen*[5] *and Mehak Gul*[6]

[1]Department of Mathematics and Statistics, Riphah International University Sector I-14, Islamabad, Pakistan
[2]MIFT - Dipartimento di Scienze Matematiche e Informatiche, Scienze Fisiche e Scienze della Terra, Università di Messina, Italy
[3]Department of Mathematics, University of Science and Technology Bannu, Khyber Pakhtunkhwa, Pakistan
[4]School of Mathematical Sciences, Zhejiang University, Hangzhou 310027, China
[5]School of Mathematics and Systems Science, Beihang University Beijing, China
[6]Department of Mathematics, University of Science and Technology Bannu, Khyber Pakhtunkhwa, Pakistan

Received: 23 Mar. 2020, Revised: 23 Apr. 2020, Accepted: 3 May 2020
Published online: 1 Sep. 2020

Abstract: This paper concerns the study of the notions of neutrosophic soft s-open set, neutrosophic soft s-neighborhood and neutrosophic soft s-separation axioms in neutrosophic soft topological spaces. By using such notions and that of neutrosophic soft point, we study the separation axioms s-T^i (with $i = 0, \ldots 4$), the s-regular and the s-normal neutrosophic soft topological spaces by proving some relationships between these classes of spaces and other results concerning hereditary and subspaces.

Keywords: Neutrosophic soft set, Neutrosophic soft point, Neutrosophic soft s-open set, Neutrosophic soft s-neighborhood, Neutrosophic soft s-separation axioms

1 Introduction

The traditional fuzzy sets is characterized by the membership value or the grade of membership value. Some times it may be very difficult to assign the membership value for a fuzzy sets. Consequently, the concept of interval valued fuzzy sets was proposed to capture and grip the uncertainty of grade of membership value. In some real-life problems in expert system, belief system, information fusion and so on, we must consider the truth-membership as well as the falsity-membership for a proper description of an object in an uncertain, ambiguous environment.
Neither the fuzzy sets nor the interval valued fuzzy sets are able to represent and manage these kinds of scenarios. The importance of intuitionistic fuzzy sets has come into play in such a dangerous situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership or association (or simply membership) and non-membership values. It does not handle the indeterminate and inconsistent information which exists in belief system.
Smarandache [1] introduced the new concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. This theory is a straightforward generalization of crisp sets, fuzzy set theory [2], intuitionistic fuzzy set theory [3], etc. Numerous researchers from different areas of mathematics have contributed to the development of neutrosophic sets theory with their work (see for example [4,5]). Many practical problems in economics, engineering, environment, social science, medical science, etc. can not be dealt by classical techniques, because such methods have heritable complexities. These complexities may be taking birth due to the insufficiency of the theories of parametrization tools. Each of these theories has its intrinsic difficulties, as was noted by Molodtsov [6].
Molodtsov originated an absolutely modern access to cope with uncertainty and vagueness and applied it progressively in different directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and so on. Honestly, Theory of soft sets is free from the parameterization meagerness syndrome of fuzzy set theory and is a great tool for dealing with uncertainty.
Shabir and Naz [7] first floated the notion of soft topological spaces, which are defined over an initial universe of discourse with a fixed set of parameters, and showed that a soft topological space gives a

* Corresponding author e-mail: mehdaniyal@gmail.com

parameterized family of topological structures. Other theoretical studies on soft topological spaces have been carried out by numerous authors [8,9,10,11].
The combination of neutrosophic set with soft sets was first introduced by Maji [12] who defined a new mathematical notion called Neutrosophic soft set, which was later improved by Deli and Broumi [13]. Further development of the theory consisted of the introduction of a new mathematical structure known as neutrosophic soft topological spaces, which were first defined and investigated by Bera in [14].
Guzide [15] attempted to bring together the areas of spheres,soft real numbers and soft points. Relating spheres to soft real numbers and soft points provides a natural and intrinsic construction of soft spheres. Guzide [16] introduced and investigated the theory of soft topological space generated by L-soft sets by using some restricted and extended intersections on L-soft sets. Guzide [17] discussed soft point's soft matrix form which were not described before for each set of parameters. The matrix representation of soft points is useful for storing all soft points that can be obtained in all different parameters. That soft matrix provides every soft point that changes with each parameter that takes place in a soft set and enables detailed examination in application of soft set theory. Guzide [18] discussed comparative research on the definition of Soft Point.
A. M. Khattak et al. [19,20] continued the investigation on soft bi-topological structures. by discussing weak separation axioms and other separation axioms soft bi-topological space with respect to crisp points and soft points of the spaces respectively. A. M. Khattak et al. [21, 22,23] continued the study by introducing the notions of generalized and most generalized neutrosophic soft open sets in neutrosophic soft topological structures and discussing their properties with respect to soft point of the space.
The first aim of this article concerns the study of the notions of neutrosophic soft s-open set, neutrosophic soft s-neighborhood, neutrosophic soft s-closed set and neutrosophic soft s-separation axioms in neutrosophic soft topological spaces. Later on, are discussed some interesting results related to these newly defined concepts with respect to soft points. Finally, we use the notions of s-separation axioms in order to obtain some results on neutrosophic soft topological spaces with respect to soft points. Furthermore, properties of neutrosophic soft s-T^i-space $(i = 0,1,2,3,4)$, some relationships between them, as well as issues concerning hereditary and subspaces are discussed.
We hope that these results will best fit for future study on neutrosophic soft topology to carry out a general framework for practical applications.

2 Preliminaries

In this section we present some basic definitions concerning the theories of soft sets, neutrosophic sets and neutrosophic soft sets.

Definition 2.1. [1] A neutrosophic set A on the universe set X is defined as:

$$A = \{\langle x, T^A(x), I^A(x), F^A(x)\rangle : x \in X\},$$

where T, I, $F : X \to]^-0, 1^+[$ and $^-0 \leqq T^A(x) + I^A(x) + F^A(x) \leqq 3^+$.

Definition 2.2. [6] Let X be an initial universe, E be a set of all parameters, and P(x) denote the power set of X. A pair (F,E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. In other words, the soft set is a parameterized family of subsets of the set X. For $\lambda \in E$, $F(\lambda)$ may be considered as the set of λ-elements of the soft set (F,E), or as the set of λ-approximate element of the set, i.e.

$$(F,E) = \{(\lambda, F(\lambda)) : \lambda \in E, F : E \to P(X)\}.$$

The notion of neutrosophic soft set first defined by Maji [12], was later modified by Deli and Broumi [13] as given below:

Definition 2.3. [13] Let X be an initial universe set and E be a set of parameters. Let P(X) denote the set of all neutrosophic sets of X. Then a neutrosophic soft set $(\widetilde{F},E)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$, where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set $(\widetilde{F},E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set P(X) and therefore it can be written as a set of ordered pairs:

$$(\widetilde{F},E) = \{(\lambda, \langle x, T^{\widetilde{F}(\lambda)}(x), I^{\widetilde{F}(\lambda)}(x), F^{\widetilde{F}(\lambda)}(x)\rangle : x \in X) : \lambda \in E\},$$

where $T^{\widetilde{F}(\lambda)}(x)$, $I^{\widetilde{F}(\lambda)}(x)$, $F^{\widetilde{F}(\lambda)}(x) \in [0, 1]$ are respectively called the truth-membership, indeterminacy-membership, and falsity-membership function of $\widetilde{F}(\lambda)$. Since the supremum of each T, I, F is 1, the inequality $0 \leqq T^{\widetilde{F}(\lambda)}(x) + I^{\widetilde{F}(\lambda)}(x) + F^{\widetilde{F}(\lambda)}(x) \leqq 3$ is obvious.

Definition 2.4. [14] Let $(\widetilde{F},E)$ be a neutrosophic soft set over the universe set X. The complement of $(\widetilde{F},E)$ is denoted by $(\widetilde{F},E)^c$ and is defined by:

$$(\widetilde{F},E)^c = \{(\lambda, \langle x, F^{\widetilde{F}(\lambda)}(x), 1 - I^{\widetilde{F}(\lambda)}(x), T^{\widetilde{F}(\lambda)}(x)\rangle : x \in X) : \lambda \in E\}.$$

It is obvious that $((\widetilde{F},E)^c)^c = (\widetilde{F},E)$.

Definition 2.5. [12] Let $(\widetilde{F},E)$ and $(\widetilde{G},E)$ be two neutrosophic soft sets over the universe set X. $(\widetilde{F},E)$ is said to be a neutrosophic soft subset of $(\widetilde{G},E)$ if $T^{\widetilde{F}(\lambda)}(x) \leqq T^{\widetilde{G}(\lambda)}(x)$, $I^{\widetilde{F}(\lambda)}(x) \leqq I^{\widetilde{G}(\lambda)}(x)$, $F^{\widetilde{F}(\lambda)}(x) \geqq F^{\widetilde{G}(\lambda)}(x)$,$\forall\lambda \in E$, $\forall x \in X$. It is denoted by $(\widetilde{F},E) \subseteq (\widetilde{G},E)$. $(\widetilde{F},E)$ is said to be neutrosophic soft equal to $(\widetilde{G},E)$ if $(\widetilde{F},E)$ is a neutrosophic soft subset of $(\widetilde{G},E)$ and $(\widetilde{G},E)$ is a neutrosophic soft subset of $(\widetilde{F},E)$. It is denoted by $(\widetilde{F},E) = (\widetilde{G},E)$.

3 Neutrosophic Soft Points and Related Characteristics

In this section, some important definitions related to the special class of entities called as neutrosophic soft points as well as the concerned results are addressed. This part is a starting point for the content of the next section.

Definition 3.1. Let $(\widetilde{F}^1,E)$ and $(\widetilde{F}^2,E)$ be two neutrosophic soft sets over universe set X. Then, their union is denoted by $(\widetilde{F}^1,E) \sqcup (\widetilde{F}^2,E) = (\widetilde{F}^3,E)$ and is defined by:

$$(\widetilde{F}^3,E) = \{(\lambda,\langle x,T^{\widetilde{F}^3(\lambda)}(x),I^{\widetilde{F}^3(\lambda)}(x),F^{\widetilde{F}^3(\lambda)}(x)\rangle : x \in X) : \lambda \in E\},$$

where:

$$\begin{aligned} T^{\widetilde{F}^3(\lambda)}(x) &= \max\{T^{\widetilde{F}^1(\lambda)}(x),T^{\widetilde{F}^2(\lambda)}(x)\},\\ I^{\widetilde{F}^3(\lambda)}(x) &= \max\{I^{\widetilde{F}^1(\lambda)}(x),I^{\widetilde{F}^2(\lambda)}(x)\},\\ F^{\widetilde{F}^3(\lambda)}(x) &= \max\{F^{\widetilde{F}^1(\lambda)}(x),F^{\widetilde{F}^2(\lambda)}(x)\}. \end{aligned}$$

Definition 3.2. Let $(\widetilde{F}^1,E)$ and $(\widetilde{F}^2,E)$ be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $(\widetilde{F}^1,E) \sqcap (\widetilde{F}^2,E) = (\widetilde{F}^3,E)$ and is defined by:

$$(\widetilde{F}^3,E) = \{(\lambda,\langle x,T^{\widetilde{F}^3(\lambda)}(x),I^{\widetilde{F}^3(\lambda)}(x),F^{\widetilde{F}^3(\lambda)}(x)\rangle : x \in X) : \lambda \in E\},$$

where:

$$\begin{aligned} T^{\widetilde{F}^3(\lambda)}(x) &= \min\{T^{\widetilde{F}^1(\lambda)}(x),T^{\widetilde{F}^2(\lambda)}(x)\},\\ I^{\widetilde{F}^3(\lambda)}(x) &= \max\{I^{\widetilde{F}^1(\lambda)}(x),I^{\widetilde{F}^2(\lambda)}(x)\},\\ F^{\widetilde{F}^3(\lambda)}(x) &= \max\{F^{\widetilde{F}^1(\lambda)}(x),F^{\widetilde{F}^2(\lambda)}(x)\}. \end{aligned}$$

Definition 3.3. A neutrosophic soft set $(\widetilde{F},E)$ over the universe set X is said to be a null neutrosophic soft set if $T^{\widetilde{F}(\lambda)}(x) = 0$, $I^{\widetilde{F}(\lambda)}(x) = 0$, $F^{\widetilde{F}(\lambda)}(x) = 1$; $\forall\lambda \in E$, $\forall x \in X$. It is denoted by $0^{(X,E)}$.

Definition 3.4. A neutrosophic soft set $(\widetilde{F},E)$ over the universe set X is said to be an absolute neutrosophic soft set if $T^{\widetilde{F}(\lambda)}(x) = 1$, $I^{\widetilde{F}(\lambda)}(x) = 1$, $F^{\widetilde{F}(\lambda)}(x) = 0$; $\forall\lambda \in E$, $\forall x \in X$. It is denoted by $1^{(X,E)}$.

Clearly, $0^c(X,E) = 1^{(X,E)}$ and $1^c(X,E) = 0^{(X,E)}$.

Definition 3.5. Let NSS(X,E) be the family of all neutrosophic soft sets over the universe set X and $\Im \sqsubset$ NSS(X,E). Then $\Im$ is said to be a neutrosophic soft topology on X if:

1. $0^{(X,E)}$ and $1^{(X,E)}$ belong to $\Im$,
2. the union of any number of neutrosophic soft sets in $\Im$ belongs to $\Im$,
3. the intersection of a finite number of neutrosophic soft sets in $\Im$ belongs to $\Im$.

Then $(X,\Im,E)$ is said to be a neutrosophic soft topological space over X. Each member of $\Im$ is said to be a neutrosophic soft open set.

Definition 3.6. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X and $(\widetilde{F},E)$ be a subset of neutrosophic soft topological space over X. Then $(\widetilde{F},E)$ is said to be a neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Definition 3.7. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X and $(\widetilde{F},E)$ be a subset of neutrosophic soft topological space over X. Then $(\widetilde{F},E)$ is said to be a neutrosophic soft s-open (NSSO) set if $(\widetilde{F},E) \subseteq NScl(NSint((\widetilde{F},E)))$

Definition 3.8. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X and $(\widetilde{F},E)$ be a subset of neutrosophic soft topological space over X. Then $(\widetilde{F},E)$ is said to be a neutrosophic soft s-closed (NSSC) set if $(\widetilde{F},E) \supseteq NSint(NScl((\widetilde{F},E)))$

Definition 3.10. Let NS be the family of all neutrosophic sets over the universe set X and x $\in$ X. The neutrosophic set $x^{(\alpha,\beta,\gamma)}$ is called a neutrosophic point, for $0 < \alpha,\beta,\gamma \leqq 1$, and is defined as follow:

$$x^{(\alpha,\beta,\gamma)}(y) = \begin{cases} (\alpha,\beta,\gamma) & \text{if } y = x \\ (0,0,1) & \text{if } y \neq x. \end{cases} \quad (1)$$

It is clear that every neutrosophic set is the union of its neutrosophic points.

Example 3.1. Suppose that X = $\{x^1,x^2\}$. Then the neutrosophic set

$$A = \{\langle x^1,0.1,0.3,0.5\rangle,\langle x^2,0.5,0.4,0.7\rangle\}$$

is the union of the neutrosophic points $x^1(0.1,0.3,0.5)$ and $x^2(0.5,0.4,0.7)$.

Now, we define the concept of neutrosophic soft points for neutrosophic soft sets.

Definition 3.11. Let NSS(X,E) be the family of all neutrosophic soft sets over the universe set X. The neutrosophic soft set $x^{\lambda}(\alpha,\beta,\gamma)$ is called a neutrosophic soft point if, for every x ∈ X, $0<\alpha,\beta,\gamma\leqq 1$, λ ∈ E, it is defined as follows:

$$x^{\lambda^{(\alpha,\beta,\gamma)}}(\lambda')(y)=\begin{cases}(\alpha,\beta,\gamma) & \text{if } \lambda'=\lambda \text{ and } y=x\\ (0,0,1) & \text{if } \lambda'\neq\lambda \text{ or } y\neq x.\end{cases} \quad (2)$$

Example 3.2. Suppose that the universe set X is given by $X=\{x^1,x^2\}$ and the set of parameters by $E=\{\lambda^1,\lambda^2\}$. Let us consider neutrosophic soft sets $(\widetilde{F},E)$ over the universe X as follows:

$$(\widetilde{F},E)=\left\{\begin{array}{l}\lambda^1=\{\langle x^1,0.3,0.7,0.6\rangle,\langle x^2,0.4,0.3,0.8\rangle\}\\ \lambda^2=\{\langle x^1,0.4,0.6,0.8\rangle,\langle x^2,0.3,0.7,0.2\rangle\}\end{array}\right\} \quad (3)$$

It is clear that $(\widetilde{F},E)$ is the union of its neutrosophic soft point $x^1\lambda^{1(0.3,0.7,0.6)}$, $x^1\lambda^{2(0.4,0.6,0.8)}$, $x^2\lambda^1$, and $x^2\lambda^{2(0.3,0.7,0.6)}$. Here:

$$x^1\lambda^{1(0.3,0.7,0.6)}=\left\{\begin{array}{l}\lambda^1=\{\langle x^1,0.3,0.7,0.6\rangle,\langle x^2,0,0,1\rangle\}\\ \lambda^2=\{\langle x^1,0,0,1\rangle,\langle x^2,0,0,1\rangle\}\end{array}\right\} \quad (4)$$

$$x^1\lambda^{2(0.4,0.6,0.8)}=\left\{\begin{array}{l}\lambda^1=\{\langle x^1,0.3,0.7,0.6\rangle,\langle x^2,0,0,1\rangle\}\\ \lambda^2=\{\langle x^1,0.4,0.6,0.8\rangle,\langle x^2,0,0,1\rangle\}\end{array}\right\} \quad (5)$$

$$x^2\lambda^{1(0.4,0.3,0.8)}=\left\{\begin{array}{l}\lambda^1=\{\langle x^1,0,0,1\rangle,\langle x^2,0.4,0.3,0.8\rangle\}\\ \lambda^2=\{\langle x^1,0,0,1\rangle,\langle x^2,0,0,1\rangle\}\end{array}\right\} \quad (6)$$

$$x^2\lambda^{2(0.3,0.7,0.2)}=\left\{\begin{array}{l}\lambda^1=\{\langle x^1,0,0,1\rangle,\langle x^2,0,0,1\rangle\}\\ \lambda^2=\{\langle x^1,0,0,1\rangle,\langle x^1,0.3,0.7,0.2\rangle\}\end{array}\right\} \quad (7)$$

Definition 3.12. Let $(\widetilde{F},E)$ be a neutrosophic soft set over the universe set X. We say that the neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}$ belongs to the neutrosophic soft set $(\widetilde{F},E)$ and we write that $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{F},E)$ whenever $\alpha\leqq T^{\widetilde{F}(\lambda)}(x),\beta\leqq I^{\widetilde{F}(\lambda)}(x)$ and $\gamma\geqq F^{\widetilde{F}(\lambda)}(x)$.

Definition 3.13. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X. A neutrosophic soft set $(\widetilde{F},E)$ in $(X,\Im,E)$ is called a neutrosophic soft s-neighborhood of the neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{F},E)$ if there exists a neutrosophic soft s-open set $(\widetilde{G},E)$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{G},E)\sqsubset(\widetilde{F},E)$.

Theorem 3.1. Let $(X,\Im,E)$ be a neutrosophic soft topological space and $(\widetilde{F},E)$ be a neutrosophic soft set over X. Then $(\widetilde{F},E)$ is a neutrosophic soft s-open set if and only if $(\widetilde{F},E)$ is a neutrosophic soft s-neighborhood of its neutrosophic soft points.

Proof 3.1. Let $(\widetilde{F},E)$ be a neutrosophic soft s-open set and $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{F},E)$. Then $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{F},E)\sqsubset(\widetilde{F},E)$. Therefore, $(\widetilde{F},E)$ is a neutrosophic soft s-neighborhood of $x^{\lambda^{(\alpha,\beta,\gamma)}}$.

Conversely, let $(\widetilde{F},E)$ be a neutrosophic soft s-neighborhood of its neutrosophic soft points. Let $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{F},E)$. Since $(\widetilde{F},E)$ is a neutrosophic soft s-neighborhood of the neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}$, there exists $(\widetilde{G},E)\in\Im$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{G},E)\sqsubset(\widetilde{F},E)$. Since $(\widetilde{F},E)=\sqcup\{x^{\lambda^{(\alpha,\beta,\gamma)}}:x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{F},E)\}$, it follows that $(\widetilde{F},E)$ is a union of neutrosophic soft s-open sets and hence $(\widetilde{F},E)$ is a neutrosophic soft s-open set. □

The s-neighborhood system of a neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}$, denoted by $U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$, is the family of all its s-neighborhoods.

Theorem 3.2. The neighborhood system $U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$ at $x^{\lambda^{(\alpha,\beta,\gamma)}}$ in a neutrosophic soft topological space $(X,\Im,E)$ has the following properties.

1) If $(\widetilde{F},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$, then $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{F},E)$;
2) If $(\widetilde{F},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$ and $(\widetilde{F},E)\sqsubset(\widetilde{H},E)$, then $(\widetilde{H},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$;
3) If $(\widetilde{F},E),(\widetilde{G},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$, then $(\widetilde{F},E)\sqcap(\widetilde{G},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$;
4) If $(\widetilde{F},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$, then there exists a $(\widetilde{G},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$ such that $(\widetilde{G},E)\in U(y^{\lambda'^{(\alpha',\beta',\gamma')}},E)$, for each $y^{\lambda'^{(\alpha',\beta',\gamma')}}\in(\widetilde{G},E)$.

Proof 3.2. The proof of 1), 2), and 3) are obvious from definition 3.12.
4) If $(\widetilde{F},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$, then there exists a neutrosophic soft s-open set $(\widetilde{G},E)$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}}\in(\widetilde{G},E)\sqsubset(\widetilde{F},E)$. From Proposition 3.1, $(\widetilde{G},E)\in U(x^{\lambda^{(\alpha,\beta,\gamma)}},E)$, so for each $y^{\lambda'^{(\alpha',\beta',\gamma')}}\in(\widetilde{G},E)$, $(\widetilde{G},E)\in U(y^{\lambda'^{(\alpha',\beta',\gamma')}},E)$ is obtained. □

Definition 3.14. We say that two neutrosophic soft points $x^{\lambda^{(\alpha,\beta,\gamma)}}$ and $y^{\lambda'^{(\alpha',\beta',\gamma')}}$ over a common universe X, are distinct if $x^{\lambda^{(\alpha,\beta,\gamma)}}\sqcap y^{\lambda'^{(\alpha',\beta',\gamma')}}=0^{(X,E)}$.

It is clear that $x^{\lambda^{(\alpha,\beta,\gamma)}}$ and $y^{\lambda'^{(\alpha',\beta',\gamma')}}$ are distinct neutrosophic soft points if and only if x ≠ y or λ′ ≠ λ.

4 Neutrosophic Soft Separation Axioms by means of Neutrosophic Soft s-Open Sets

In this section we study the first three separation axioms s-T^i (with $i = 0,1,2$) for the neutrosphic soft topological spaces by using soft points and the notion of neutrosophic soft s-open set, we give some peculiar examples about them and we prove some relationships between them. Subsequently, further separation axioms for neutrosophic soft spaces such as s-regular, s-T^3, s-normal and s-T^4 are introduced and investigated also in relation to the previous three separation axioms.

Definition 4.1. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X. We say that $(X,\Im,E)$ is:

a) a neutrosophic soft s-T^o-space if, for every pair of distinct neutrosophic soft points $x^{\lambda^{(\alpha,\beta,\gamma)}}$ and $y^{\lambda'^{(\alpha',\beta',\gamma')}}$, there exist two neutrosophic soft s-open sets $(\widetilde{F},E)$ and $(\widetilde{G},E)$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F},E)$ and $x^{\lambda^{(\alpha,\beta,\gamma)}} \sqcap (\widetilde{G},E) = 0^{(X,E)}$ or $y^{\lambda'^{(\alpha',\beta',\gamma')}} \in (\widetilde{G},E)$ and $y^{\lambda'^{(\alpha',\beta',\gamma')}} \sqcap (\widetilde{F},E) = 0^{(X,E)}$;

b) a neutrosophic soft s-T^1-space if, for every pair of distinct neutrosophic soft points $x^{\lambda^{(\alpha,\beta,\gamma)}}$ and $y^{\lambda'^{(\alpha',\beta',\gamma')}}$, there exist two neutrosophic soft s-open sets $(\widetilde{F},E)$ and $(\widetilde{G},E)$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F},E)$, $x^{\lambda^{(\alpha,\beta,\gamma)}} \sqcap (\widetilde{G},E) = 0^{(X,E)}$ or $y^{\lambda'^{(\alpha',\beta',\gamma')}} \in (\widetilde{G},E)$, $y^{\lambda'^{(\alpha',\beta',\gamma')}} \sqcap (\widetilde{F},E) = 0^{(X,E)}$;

c) a neutrosophic soft s-T^2-space if, for every pair of distinct neutrosophic soft points $x^{\lambda^{(\alpha,\beta,\gamma)}}$ and $y^{\lambda'^{(\alpha',\beta',\gamma')}}$, there exist two neutrosophic soft s-open sets $(\widetilde{F},E)$ and $(\widetilde{G},E)$ such that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\widetilde{F},E)$, $y^{\lambda'^{(\alpha',\beta',\gamma')}} \in (\widetilde{G},E)$ and $(\widetilde{F},E) \sqcap (\widetilde{G},E) = 0^{(X,E)}$.

Example 4.1. Let X = $\{x^1,x^2\}$ be a universe set, E = $\{\lambda^1,\lambda^2\}$ be a parameters set, and $(x^1)\lambda^{1(0.1,0.4,0.7)}$, $(x^1)\lambda^{2(0.2,0.5,0.6)}$, $(x^2)\lambda^{1(0.3,0.3,0.5)}$, and $(x^2)\lambda^{2(0.4,0.4,0.4)}$ be neutrosophic soft points. Then the family $\Im$ = $\{0^{(X,E)},1^{(X,E)},(\widetilde{F}^1,E),(\widetilde{F}^2,E),(\widetilde{F}^3,E),$ $(\widetilde{F}^4,E),(\widetilde{F}^5,E),(\widetilde{F}^6,E),(\widetilde{F}^7,E),(\widetilde{F}^8,E)\}$, where

$(\widetilde{F}^1,E) = \{x^{1^{\lambda^{1(0.1,0.4,0.7)}}}\}$,
$(\widetilde{F}^2,E) = \{(x^1)\lambda^{2(0.2,0.5,0.6)}\}$,
$(\widetilde{F}^3,E) = \{(x^2)\lambda^1(0.3,0.3,0.5)\}$,
$(\widetilde{F}^4,E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^2,E)$,
$(\widetilde{F}^5,E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^3,E)$,
$(\widetilde{F}^6,E) = (\widetilde{F}^2,E) \sqcup (\widetilde{F}^3,E)$,
$(\widetilde{F}^7,E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^2,E) \sqcup (\widetilde{F}^3,E)$,
$(\widetilde{F}^8,E) = \{(x^1)\lambda^{1(0.1,0.4,0.7)},(x^1)\lambda^{2(0.2,0.5,0.6)},$
$(x^2)\lambda^{2(0.3,0.3,0.5)},(x^2)\lambda^2(0.4,0.4,0.4)\}$,

is a neutrosophic soft topology over X. Hence, $(X,\Im,E)$ is a neutrosophic soft topological space over X. Also, $(X,\Im,E)$ is a neutrosophic soft s- T^0-space but not a neutrosophic soft s-T^1-space because for neutrosophic soft points $(x^1)\lambda^1(0.1,0.4,0.7)$ and $(x^2)\lambda^2(0.4,0.4,0.4)$,$(X,\Im,E)$ is not a neutrosophic soft s-T^1-space.

Example 4.2. Let X = $\mathbb{N}$ be a natural numbers set and E = $\{\lambda\}$ be a parameters set. Here $n^{\lambda^{(\alpha n,\beta n,\gamma n)}}$ are neutrosophic soft points. Here we can give $(\alpha n,\beta n,\gamma n)$ appropriate values and the neutrosophic soft points $n^{\lambda^{(\alpha n,\beta n,\gamma n)}}$,$m^{\lambda^{(\alpha n,\beta n,\gamma n)}}$ are distinct neutrosophic soft points if and only if n $\neq$ m. It is clear that there is one-to-one compatibitily between the set of natural numbers and the set of neutrosophic soft points N^λ = $\{n^{\lambda^{(\alpha n,\beta n,\gamma n)}}\}$.
Now, we define a cofinite topology on X as the family of all neutrosophic soft set $(\widetilde{F},E)$ which complement respect to N^λ is a finite set of neutrosophic soft points. Hence, $(X,\Im,E)$ is a neutrosophic soft s-T^1-space but not a neutrosophic soft s-T^2-space.

Example 4.3. Let X = $\{x^1,x^2\}$ be a universe set, E = $\{\lambda^1,\lambda^2\}$ be a parameters set, and $x^1\lambda^{1(0.1,0.4,0.7)}$,$x^1\lambda^{2(0.2,0.5,0.6)}$,$x^2\lambda^{1(0.3,0.3,0.5)}$, and $x^2\lambda^{2(0.4,0.4,0.4)}$, be neutrosophic soft points. Then the family $\Im$ = $\{0^{(X,E)},1^{(X,E)},(\widetilde{F}^1,E),(\widetilde{F}^2,E),...,(\widetilde{F}^{15},E)\}$, where

$(\widetilde{F}^1,E) = \{x^1\lambda^{1(0.1,0.4,0.7)}\}$,
$(\widetilde{F}^2,E) = \{(x^1)\lambda^{2(0.2,0.5,0.6)}\}$,
$(\widetilde{F}^3,E) = \{(x^2)\lambda^{1(0.3,0.3,0.5)}\}$,
$(\widetilde{F}^4,E) = \{(x^2)\lambda^{2(0.4,0.4,0.4)}\}$,
$(\widetilde{F}^5,E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^2,E)$,
$(\widetilde{F}^6,E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^3,E)$,
$(\widetilde{F}^7,E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^4,E)$,
$(\widetilde{F}^8,E) = (\widetilde{F}^2,E) \sqcup (\widetilde{F}^3,E)$,
$(\widetilde{F}^9,E) = (\widetilde{F}^2,E) \sqcup (\widetilde{F}^4,E)$,
$(\widetilde{F}^{10},E) = (\widetilde{F}^3,E) \sqcup (\widetilde{F}^4,E)$,
$(\widetilde{F}^{11},E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^2,E) \sqcup (\widetilde{F}^3,E)$,
$(\widetilde{F}^{12},E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^2,E) \sqcup (\widetilde{F}^4,E)$,
$(\widetilde{F}^{13},E) = (\widetilde{F}^2,E) \sqcup (\widetilde{F}^3,E) \sqcup (\widetilde{F}^4,E)$,
$(\widetilde{F}^{14},E) = (\widetilde{F}^1,E) \sqcup (\widetilde{F}^3,E) \sqcup (\widetilde{F}^4,E)$,
$(\widetilde{F}^{15},E) = \{(x^1)\lambda^{1(0.1,0.4,0.7)},(x^1)\lambda^{2(0.2,0.5,0.6)},$
$(x^2)\lambda^{2(0.3,0.3,0.5)},(x^2)\lambda^{2(0.4,0.4,0.4)}\}$,

is a neutrosophic soft topology over X. Hence, $(X,\Im,E)$ is a neutrosophic soft topological space over X. Also, $(X,\Im,E)$ is a neutrosophic soft s-T^2-space.

Theorem 4.1. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X. Then $(X,\Im,E)$ is a neutrosophic soft s-T^1-space if and only if each neutrosophic soft point is a neutrosophic soft s-closed set.
Proof 4.1. Let$(X,\Im,E)$ be a neutrosophic soft s-T^1-space

and $x^{\lambda(\alpha,\beta,\gamma)}$ be an arbitrary neutrosophic soft point. We show that $(x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$ is a neutrosophic soft s-open set. Let $y^{\lambda'(\alpha',\beta',\gamma')} \in (x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$; then $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points. Hence, $x \neq y$ or $\lambda' \neq \lambda$.
Since $(X,\Im,E)$ is a neutrosophic soft s-T^1-space, there exists a neutrosophic soft s-open set $(\widetilde{G},E)$ such that $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G},E)$ and $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{G},E) = 0^{(X,E)}$.
Then, since $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{G},E) = 0^{(X,E)}$, we have $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G},E) \sqsubset (x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$. This implies that $(x^{\lambda(\alpha,\beta,\gamma)})^{\lambda}$ is a neutrosophic soft s-open set, i.e. $x^{\lambda(\alpha,\beta,\gamma)}$ is a neutrosophic soft s-closed set.

Conversely, suppose that each neutrosophic soft point is a neutrosophic soft s-closed set and let $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ be a pair of distinct neutrosophic soft points. Then $(x^{\lambda(\alpha,\beta,\gamma)})^{c}$ is a neutrosophic soft open set and it results $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Furthemore, we have that $y^{\lambda'(\alpha',\beta',\gamma')} \in (x^{\lambda(\alpha,\beta,\gamma)})^{c}$ and $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (x^{\lambda(\alpha,\beta,\gamma)})^{c} = 0^{(X,E)}$ and this proves that $(X,\Im,E)$ is a neutrosophic soft s-T^1-space over X. □

Theorem 4.2. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X. Then $(X,\Im,E)$ is a neutrosophic soft s-T^2-space iff for distinct neutrosophic soft points $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$, there exists a neutrosophic soft s-open set $(\widetilde{F},E)$ containing $x^{\lambda}(\alpha,\beta,\gamma)$ but not $y^{\lambda'}(\alpha',\beta',\gamma')$ such that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $\overline{(\widetilde{F},E)}$.

Proof 4.2. Let $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ be two distinct neutrosophic soft points in neutrosophic soft s-T^2-space $(X,\Im,E)$. Then there exist disjoint neutrosophic soft s-open set $(\widetilde{F},E)$, $(\widetilde{G},E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F},E)$ and $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{G},E)$. Since $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$ and $(\widetilde{F},E) \sqcap (\widetilde{G},E) = 0^{(X,E)}$, we have that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $(\widetilde{F},E)$ and this implies that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $\overline{(\widetilde{F},E)}$.

Next suppose that, for distinct neutrosophic soft points $x^{\lambda(\alpha,\beta,\gamma)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$, there exists a neutrosophic soft s-open set $(\widetilde{F},E)$ containing $x^{\lambda(\alpha,\beta,\gamma)}$ but not $y^{\lambda'(\alpha',\beta',\gamma')}$ such that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $\overline{(\widetilde{F},E)}$. Then $y^{\lambda'(\alpha',\beta',\gamma')} \in (\overline{(\widetilde{F},E)})^{c}$, i.e. $(\widetilde{F},E)$ and $(\overline{(\widetilde{F},E)})^{c}$ are disjoint neutrosophic soft s-open sets containing $x^{\lambda(\alpha,\beta,\gamma)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$ respectively. □

Theorem 4.3. Let $(X,\Im,E)$ be a neutrosophic soft s-T^1-space. If, for every neutrosophic soft point $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F},E) \in \Im$, there exists a neutrosophic soft s-open set $(\widetilde{G},E)$ such that

$$x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset \overline{(\widetilde{G},E)} \sqsubset (\widetilde{F},E)$$

then $(X,\Im,E)$ is a neutrosophic soft s-T^2-space.

Proof 4.3. Suppose that $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Since $(X,\Im,E)$ is a neutrosophic soft T^1-space, $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are neutrosophic soft s-closed sets in $\Im$. Thus $x^{\lambda(\alpha,\beta,\gamma)} \in (y^{\lambda'}(\alpha',\beta',\gamma'))^{c} \in \Im$. Then there exists a neutrosophic soft s-open set $(\widetilde{G},E)$ in $\Im$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset \overline{(\widetilde{G},E)} \sqsubset (y^{\lambda'}(\alpha',\beta',\gamma'))^{c}$. Hence, we have $y^{\lambda'(\alpha',\beta',\gamma')} \in (\overline{(\widetilde{G},E)})^{c}$, $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E)$, and $(\widetilde{G},E) \sqcap (\overline{(\widetilde{G},E)})^{c} = 0^{(X,E)}$, that is $(X,\Im,E)$ is a neutrosophic soft s- T^2-space. □

Example 4.1. Let $(X,\Im,E)$ be a neutrosophic soft s-T^1-space for $i = 0,1,2$. For each $x \neq y$, the neutrosophic points $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ have neighborhoods satisfying conditions of s-T^i-space in neutrosophic topological space $(X,\Im^{\lambda})$, for each $\lambda \in E$.

Definition 4.2. A neutrosophic soft topological space $(X,\Im,E)$ over X is called a neutrosophic soft s-regular space if, for every neutrosophic soft s-closed set $(\widetilde{F},E)$ and every $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{F},E) = 0^{(X,E)}$, there exist a pair of neutrosophic soft open sets $(\widetilde{G}^1,E),(\widetilde{G}^2,E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G}^1,E)$, $(\widetilde{F},E) \sqsubset (\widetilde{G}^2,E)$, and $(\widetilde{G}^1,E) \sqcap (\widetilde{G}^2,E) = 0^{(X,E)}$.
Additionally, $(X,\Im,E)$ is said to be a neutrosophic soft s-T^3-space if it is both a neutrosophic soft s-regular and a neutrosophic soft s-T^1-space.

Theorem 4.4. A neutrosophic soft topological space $(X,\Im,E)$ over X is a neutrosophic soft s-T^3-space if and only if for every $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F},E) \in \Im$, with $(\widetilde{F},E)$ neutrosophic soft s-open set, there exists $(\widetilde{G},E) \in \Im$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset \overline{(\widetilde{G},E)} \sqsubset (\widetilde{F},E)$.

Proof 4.4. Let $(X,\Im,E)$ be a neutrosophic soft s-T^3-space and $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F},E) \in \Im$. Since $(X,\Im,E)$ is a neutrosophic soft s-T^3-space for the neutrosophic soft point $x^{\lambda(\alpha,\beta,\gamma)}$ and neutrosophic soft s-closed set $(\widetilde{F},E)^{c}$, there exist a pair of neutrosophic soft s-open set $(\widetilde{G},E)$, $(\widetilde{G}^2,E) \in \Im$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E)$, $(\widetilde{F},E)^{c} \sqsubset (\widetilde{G}^2,E)$ and $(\widetilde{G},E) \sqcap (\widetilde{G}^2,E) = 0^{(X,E)}$. Thus, $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset (\widetilde{G}^2,E)^{c} \sqsubset (\widetilde{F},E)$. Since $(\widetilde{G}^2,E)^{c}$ is a neutrosophic soft s-closed set, we also have that $\overline{(\widetilde{G},E)}$

$\sqsubset (\widetilde{G}^2,E)^c$ and this proves that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset \overline{(\widetilde{G},E)} \sqsubset (\widetilde{F},E)$.

Conversely, let $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\widetilde{H},E) = 0^{(X,E)}$ and $(\widetilde{H},E)$ be a neutrosophic soft s-closed set. Thus, $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{H},E)^c$ and from the hypothesis, it follows that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E) \sqsubset \overline{(\widetilde{G},E)} \sqsubset (\widetilde{H}{\cdot}E)^c$. So, we have that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{G},E)$, $(\widetilde{H},E) \sqsubset (\overline{(\widetilde{G},E)})^c$ and $(\widetilde{G},E) \sqcap \overline{(\widetilde{G},E)}^c = 0^{(X,E)}$ are satisfied, i.e. $(X,\Im,E)$ is a neutrosophic soft s-T^3-space. □

Definition 4.3. A neutrosophic soft topological space $(X,\Im,E)$ over X is called a neutrosophic soft s-normal space if, for every pair of neutrosophic soft s-closed sets $(\widetilde{F}^1,E)$, $(\widetilde{F}^2,E)$, there exist a pair of disjoint neutrosophic soft s-open sets $(\widetilde{G}^1,E),(\widetilde{G}^2,E)$ such that $(\widetilde{F}^1,E) \sqsubset (\widetilde{G}^1,E)$ and $(\widetilde{F}^2,E) \sqsubset (\widetilde{G}^2,E)$.
Additionally, $(X,\Im,E)$ is said to be a neutrosophic soft s-T^4-space if it is both a neutrosophic soft s-normal and a neutrosophic soft s-T^1-space.

Theorem 4.5. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X. Then $(X,\Im,E)$ is a neutrosophic soft s-T^4-space if and only if, for each neutrosophic soft s-closed set $(\widetilde{F},E)$ and neutrosophic soft s-open set $(\widetilde{G},E)$ with $(\widetilde{F},E) \sqsubset (\widetilde{G},E)$, there exists a neutrosophic soft s-open set $(\widetilde{D},E)$ such that $(\widetilde{F},E) \sqsubset (\widetilde{D},E) \sqsubset \overline{(\widetilde{D},E)} \sqsubset (\widetilde{G},E)$.
Proof 4.5. Let $(X,\Im,E)$ be a neutrosophic soft s-T^4-space, $(\widetilde{F},E)$ be a neutrosophic soft s-closed set and $(\widetilde{F},E) \sqsubset (\widetilde{G},E) \in \Im$. Then $(\widetilde{G},E)^c$ is a neutrosophic soft s-closed set and $(\widetilde{F},E) \sqcap (\widetilde{G},E)^c = 0^{(X,E)}$. By hypothesis there exist neutrosophic soft s-open sets $(\widetilde{D}^1,E)$ and $(\widetilde{D}^2,E)$ such that $(\widetilde{F},E) \sqsubset (\widetilde{D}^1,E)$, $(\widetilde{G},E)^c \sqsubset (\widetilde{D}^2,E)$, and $(\widetilde{D}^1,E) \sqcap (\widetilde{D}^2,E) = 0^{(X,E)}$. This implies that $(\widetilde{F},E) \sqsubset (\widetilde{D}^1,E) \sqsubset (\widetilde{D}^2,E)^c \sqsubset (\widetilde{G},E)$. Since $(\widetilde{D}^2,E)^c$ is a neutrosophic soft s-closed set, it follows that $\overline{(\widetilde{D}^1,E)} \sqsubset (\widetilde{D}^2,E)^c$ and hence that $(\widetilde{F},E) \sqsubset (\widetilde{D}^1,E) \sqsubset \overline{(\widetilde{D}^1,E)} \sqsubset (\widetilde{G},E)$.

Conversely, suppose that $(\widetilde{F}^1,E)$, $(\widetilde{F}^2,E)$ are two disjoint neutrosophic soft s-closed sets and, consequently, that $(\widetilde{F}^1,E) \sqsubset (\widetilde{F}^2,E)^c$. So, applying the hypothesis, there exists a neutrosophic soft s-open set $(\widetilde{D},E)$ such that $(\widetilde{F}^1,E) \sqsubset (\widetilde{D},E) \sqsubset \overline{(\widetilde{D}^1,E)} \sqsubset (\widetilde{F}^2,E)^c$. Hence, $(\widetilde{D},E)$ and $(\overline{(\widetilde{D},E)})^c$ are two disjoint neutrosophic soft s-open sets such that $(\widetilde{F}^1,E) \sqsubset (\widetilde{D},E)$ and $(\widetilde{F}^2,E) \sqsubset (\overline{(\widetilde{D},E)})^c$. This proves that $(X,\Im,E)$ is a neutrosophic soft s-T^4-space and concludes our proof. □

Definition 4.4. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X and let $(\widetilde{F},E)$ be an arbitrary neutrosophic soft set. Then, the family $\Im^{(\widetilde{F},E)} = \{(\widetilde{F},E) \sqcap (\widetilde{H},E) : (\widetilde{H},E) \in \Im\}$ of all the intersections of $(\widetilde{F},E)$ with the members of $\Im$ is said to be the neutrosophic soft topology on $(\widetilde{F},E)$ and $((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$ is said to be a neutrosophic soft topological subspace of $(X,\Im,E)$.

Theorem 4.6. Let $(X,\Im,E)$ be a neutrosophic soft topological space over X. If $(X,\Im,E)$ is a neutrosophic soft s-T^i-space (for $i = 0,1,2,3$), then the neutrosophic soft topological subspace $((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$ is a neutrosophic soft s-T^i-space too.
Proof 4.6. Suppose that $(X,\Im,E)$ is a neutrosophic soft s-T^i-space, for some fixed $i = 0,1,2,3$, and let $x^{\lambda(\alpha,\beta,\gamma)}, y^{\lambda'(\alpha',\beta',\gamma')} \in ((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Thus, there exist two neutrosophic soft s-open set $(\widetilde{F}^1,E)$ and $(\widetilde{F}^2,E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}^1,E)$, $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{F}^2,E)$ satisfying the specific condition of neutrosophic soft s-T^i-space. From here it follows that $(\widetilde{F}^1,E) \sqcap (\widetilde{F},E)$ and $(\widetilde{F}^2,E) \sqcap (\widetilde{F},E)$ are two disjoint neutrosophic soft s-open sets of $\Im^{(\widetilde{F},E)}$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\widetilde{F}^1,E) \sqcap (\widetilde{F},E)$, $y^{\lambda'(\alpha',\beta',\gamma')} \in (\widetilde{F}^2,E) \sqcap (\widetilde{F},E)$ and satisfying the corresponding condition of neutrosophic soft s-T^i-space respect to the neutrosophic soft topology on $(\widetilde{F},E)$. This proves that $((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$ is a neutrosophic soft s-T^i-space. □

Theorem 4.7. If $(X,\Im,E)$ is a neutrosophic soft s-T^4 topological space and $(\widetilde{F},E)$ is a neutrosophic soft s-closed set in $(X,\Im,E)$, then the neutrosophic soft topological subspace $((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$ is a neutrosophic soft s-T^4-space.
Proof 4.7. Suppose that $(X,\Im,E)$ is a neutrosophic soft s-T^4-space, $(\widetilde{F},E)$ is a neutrosophic soft s-closed set in $(X,\Im,E)$ and let us consider two disjoint neutrosophic soft s-closed sets $(\widetilde{F}^1,E)$, $(\widetilde{F}^2,E)$ of $((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$. Since $(\widetilde{F},E)$ is a neutrosophic soft s-closed set in $(X,\Im,E)$, it follows that $(\widetilde{F}^1,E)$ and $(\widetilde{F}^2,E)$ are neutrosophic soft s-closed sets also in $(X,\Im,E)$. Hence, by the hypotesis, there exist two disjoint neutrosophic soft s-open sets $(\widetilde{G}^1,E)$ and $(\widetilde{G}^2,E)$ such that $(\widetilde{F}^1,E) \sqsubset (\widetilde{G}^1,E)$ and $(\widetilde{F}^2,E) \sqsubset (\widetilde{G}^2,E)$. Thus $(\widetilde{G}^1,E) \sqcap (\widetilde{F},E)$ and $(\widetilde{G}^2,E) \sqcap (\widetilde{F},E)$ represent two disjoint neutrosophic soft s-open sets of $((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$ such that $(\widetilde{F}^1,E) \sqsubset (\widetilde{G}^1,E) \sqcap (\widetilde{F},E)$ and $(\widetilde{F}^2,E) \sqsubset (\widetilde{G}^2,E) \sqcap (\widetilde{F},E)$ and this proves that $((\widetilde{F},E),\Im^{(\widetilde{F},E)},E)$ is a neutrosophic soft s-T^4-space. □

5 Conclusion

Neutrosophic Soft topology is a theory which contains both the characters of applied mathematics and pure mathematics and this is why it is supposed to be a very important branch of mathematics. In this our particular study we have first defined the notion of neutrosophic soft s-open set and, by means of soft points, we have given some peculiar examples. Furthermore, using such a new notion, we studied the first three separation axioms s-T^i (with $i = 0,1,2$) for the neutrosphic soft topological spaces and we have also extended the investigation to other separation axioms such as s-regular, s-T^3, s-normal and s-T^4, by proving some relationships between them and some other relevant results concerning hereditary and subspaces.

Acknowledgement

The authors would like to express their sincere gratitude to the Anonymous Referee for his/her help in bringing the manuscript in a meaningful form.

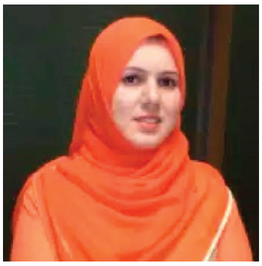
References

[1] F. Smarandache, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, Int J Pure Appl Math. **24**, 287–297 (2005).
[2] L.A. Zadeh, Fuzzy sets, Inform Control. **8**, 338–353 (1965).
[3] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets Syst. **20**, 87–96 (1986).
[4] T. Bera, N.K. Mahapatra, On neutrosophic soft function, Ann fuzzy Math Inform. **12**, 101–199 (2016).
[5] A.A. Salma, S.A. Alblowi, Neutrosophic set and Neutrosophic topological spaces, IOSR J Math. **3**, 31–35 (2012).
[6] D. Molodtsov, Soft set theory-first results, Comput Math Appl. **37**, 19–31 (1999).
[7] M. Shabir, M. Naz, On soft topological spaces, IOSR J Math. **61**, 1786–1799 (2011).
[8] S. Bayramov, C. Gunduz, On intuitionistic fuzzy soft topological spaces, TWMS J Pure Appl Math. **5**, 66–79 (2014).
[9] S. Bayramov, C. Gunduz, A new approach to separability and compactness in soft topological spaces, TWMS J Pure Appl Math. **9**, 82–93 (2018).
[10] N. Cagman, S. Karatas, S. Enginoglu, Soft topology, Comput Math Appl. **62** (2011) 351–358.
[11] A.C. Gunduz, S. Bayramov, On the Tietze extension theorem in soft topological spaces, Proceedings of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan. **43**, 105–115 (2017).
[12] P.K. MajI, Neutrosophic soft set, Ann Fuzzy Math Inform. **5**, 157–168 (2013).
[13] I. Deli, S. Broumi, Neutrosophic soft relations and some properties, Ann Fuzzy Math Inform. **9**, 169–182 (2015).
[14] T. Bera, N.K. Mahapatra, Introduction to neutrosophic soft topological space, Opsearch. **54**, 841–867 (2017).
[15] S. Guzide, A New Construction of Spheres Via Soft Real Numbers and Soft Points, Mathematics Letters. **4**(3), 39–43 (2018).
[16] S. Guzide, Soft Topology Generated by L-Soft Sets, Journal of New Theory. **4**(24), 88–100 (2018).
[17] S. Guzide, The Parameterizations Reduction of Soft Point and its Applications with Soft Matrix, International Journal of Computer Applications. **164**(1), 1–6 (2017).
[18] S. Guzide, A Comparative Research on the definition of Soft Point, International Journal of Computer Applications. **163**(2), 1–4 (2017).
[19] A. M. Khattak, M. Zamir, M. Alam, F. Nadeem, S. Jabeen, A. Zaighum,'Weak Soft Open Sets in Soft Bi Topological Spaces', Journal of Mathematical Extension, **14**, 85-116 (2020).
[20] A. M. Khattak, F. Nadeem, M. Zamir, G. Nordo, C. Park, M. Gul,'Other separation axioms in soft bi-topological space', Mayala Journal of Matematik, **7**(4), 724–734 (2019).
[21] A. M. Khattak, N. Hanif, F. Nadeem, M. Zamir, C. Park, G. Nordo, S. Jabeen,'Soft b-Separation Axioms in Neutrosophic Soft Topological Structures', Ann. Fuzzy Math. Inform. **18**(1), 93–105 (2019).
[22] A. Mehmood, F. Nadeem, G. Nordo, M. Zamir, C. Park, H. Kalsoom, S. Jabeen, M. I. Khan,'Generalized Neutrosophic Separation Axioms in Neutrosophic Soft Topological Spaces', Neutrosophic Sets and Systems, **32**(1), 38–51 (2020).
[23] A. Mehmood, F. Nadeem, C. Park, G. Nordo, S. Abdullah, Neutrosophic soft topological structure relative to most generalized neutrosophic soft open set, Communication in Mathematics and Applications, [In press]

Arif Mehmood is Ph.D from Riphah International University Sector I-14, Islamabad, Pakistan with specialization in Soft Topology, Neutrosophic Soft Topology, Vague Soft Topology, Bi-Polar Soft Topological Spaces and fixed points theory. He is faculty member of mathematics department in university of science and technology Bannu, Pakistan. He has strong research background. He is in affiliation with different Universities researchers in Pakistan and outside Pakistan. He supervised more than fifty five M. Phil students inside and outside the country. He has published more than seventy research papers in national and international journals.

Giorgio Nordo is a faculty member of the MIFT (Mathematics, Informatics, Physics and Earsth sciences) Department of the Messina University (Italy) where he works as Researcher and Aggregate Professor since 1997. His main research interests are General Topology, Fibrewise Topology, Digital Geometry, Soft Topology and Neutrosophic Spaces. He has taught courses in Topology, Digital Geometry and Computational Geometry, has written over forty scientific articles, has been a member of numerous scientific committees of international workshops and conferences, has been a visiting researcher in the United States, Spain and Russia and is accredited as a referee for many scientific journals.

Fawad Nadeem (M.Phil). Visiting Lecturer at University of Science and Technology Bannu, Pakistan. His area of research is Mathematical Biology (Applied Mathematics). In addition, he has strong collaboration with renowned researchers in the theory of soft sets, soft topology, Neutrosophic Soft topology and bi-polar Soft topology. He has command on Fixed Point Theory, Matlab and differential Equations. So far he has published more than 11 research articles in well reputed international journals and five research articles are in press.



Humaira Kalsoom is Ph.D. Scholar at School of Mathematical Sciences, Zhejiang University department of Mathematics (Pure Maths), China. Her area of research is Quantum Integral Inequalities and Post Quantum Calculus Simpsons Type Inequalities for Coordinates. In continuation, she has strong collaboration with prominent scholars (mathematicians) in the theory of soft sets, Soft Topological Structures, Neutrosophic Soft Topological Structures, Bi-polar Neutrosophic Soft Topological Spaces and their applications in practical life. In addition to this, she has good command on Functional Analysis and Real Analysis. She has published more than seventeen research articles in vigorous journals which are almost of impact factors.

Shamoona Jabeen is Ph.D. Scholar at School of Mathematics and Systems Science, Beihang University Beijing, China. Her area of research is Fixed point results in Fuzzy cone metric spaces, Generalization of Fuzzy graphs and their applications. In addition, she has collaboration with renowned researchers in the theory of soft sets, Community detection in Complex networks and Machine learning applications. She is motivated not only by the ”pleasure of finding things out” but also explaining them to others. She has published more than 10 research articles in well reputed international journals so far.

Mehak Gul qualified her master degree from post graduate college Bannu, Pakistan in Maths. Completed her Mphil from University of science and technology Bannu, Pakistan. Her area of research is soft topology (Pure Mathematics). She has good command on soft sets, soft topological structures and soft bi-topology. She published several research papers in national and international journals. Presently she is engaged in teaching at College level.