DOI: 10.1478/AAPP.97S2A8

AAPP | Atti della Accademia Peloritana dei Pericolanti
Classe di Scienze Fisiche, Matematiche e Naturali
ISSN 1825-1242

Vol. 97, No. S2, A8 (2019)

WEAK DISCONTINUITY WAVES IN N-TYPE SEMICONDUCTORS WITH DEFECTS OF DISLOCATION

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ABSTRACT. In this paper a Boillat's methodology is applied to investigate discontinuity waves of a system of quasi-linear hyperbolic partial differential equations (PDEs), describing the interactions between the electronic and dislocation fields in extrinsic semiconductors with defects of dislocation. The thermodynamic model for the semiconductors under consideration was deduced in previous papers, in the frame of extended irreversible thermodynamics with internal variables, but here it is assumed that these semiconductors are not polarized. The solutions of the PDEs system considered are looked for in an approximate form, presenting a jump in the first order derivatives crossing the associated wave fronts. In particular, in the one-dimensional case, we study the propagation of one solution into a uniform unperturbed state, deriving the expression of the velocity along the characteristic rays, the associated wave front equation in the first approximation and Bernoulli's equation governing the propagation of the discontinuity amplitude.

1. Introduction

The theoretical interest in nonlinear waves was manifest as early as the years '50 and '60 of the last century and a lot of applications to various branches of physics were worked out (Lax 1954; Jeffrey 1963a,b; Jeffrey and Taniuti 1964; Boillat 1965; Choquet-Bruhat 1968; Boillat 1976; Jeffrey 1976; Boillat and Ruggeri 1979; Restuccia 1979; Donato 1980; Hunter and Keller 1983; Lax 1983; Donato and Greco 1986; Famà and Restuccia 2019). In Jeffrey (1976), the solution hypersurfaces of systems of PDEs are referred to as waves because they may be interpreted as representing propagating wavefronts. When physical problems are associated with such interpretation the solution on the side of the wavefront towards which propagation takes place may then regarded as being the undisturbed solution ahead of the wavefront, whilst the solution on the other side may be regarded as a propagating disturbance wave which is entering a region occupied by the undisturbed solution. Some of the solutions present various types of discontinuities, some others not. In the first case, the solution or/and its derivatives undergo a jump crossing the associated wave front. In this case it is said that the solution presents a *shock*, or it is a *shock wave* or that we are in presence of a *discontinuity wave* (jump of the first order derivatives)(see Jeffrey and Taniuti 1964; Boillat 1965; Jeffrey 1976; Boillat and Ruggeri 1979). In the second case, instead of the jump we have smooth solutions of the non linear PDEs, that present a steep variation in the normal direction to the associated wavefront and these solutions are called *asymptotic waves* (see Choquet-Bruhat 1968; Boillat 1976; Ciancio and Restuccia 1985a,b; Donato and Greco 1986; Ciancio and Restuccia 1987; Georgescu and Restuccia 2006; Restuccia and Georgescu 2008; Georgescu and Restuccia 2010, 2011; Mazzeo and Restuccia 2011a; Georgescu and Restuccia 2017; Restuccia 2018). Both these types of solutions are called *nonlinear waves* because they satisfy nonlinear PDEs and they are investigated because the closed-form solutions of nonlinear PDEs are rare. These solutions are looked for in approximated form, where a new variable is present related to the surface across which the solutions or/and some of their derivatives undergo a jump. In this paper the propagation of weak discontinuities in an elastic and isotropic n-type semiconductor with defects of dislocation is studied, taking into account a thermodynamic model (see Restuccia and Maruszewski 1995, and also Mazzeo and Restuccia 2009, Mazzeo and Restuccia 2011a and Restuccia 2019) developed in the frame of the extended irreversible thermodynamics with internal variables (Prigogine 1961; De Groot and Mazur 1962; Lax 1983; Kluitenberg 1984; Muschik 1993; Lebon *et al.* 2008; Jou and Restuccia 2011; Maugin 2015; Berezovski and Ván 2017), but here it is assumed that the semiconductor is not polarized. The system of non-linear PDEs that we consider is confined only to the electronic and dislocation fields (see Mazzeo and Restuccia 2011a). In Section **2** extrinsic semiconductors of n and p type without polarization are considered and a dislocation tensor is introduced, that describes the dislocation lines as a network of infinitesimally thin channels. In Section **3** the fundamental equations governing the behaviour of these media are presented. In Sections **4** and **5** the model, confined only to the electronic and dislocation fields, is illustrated as a non-linear PDEs system, and the methodology established by Boillat (1965) for weak discontinuity waves of quasi-linear and hyperbolic systems of the first order is illustrated. In Section **6** in a one-dimensional case, we study the propagation of a solution into an uniform unperturbed state and we obtain the velocity along the characteristic rays, the associated wave front in the first approximation and Bernoulli's equation governing the evolution equation of the amplitude of the weak discontinuity. The models for extrinsic semiconductors defective by dislocations and their solutions may have relevance in many fundamental technological sectors: in applied computer science, in the technology for integrated circuits VLSI (Very Large Scale Integration), in the realization of thin dielectric films in order to construct ”fixed memories”, in electronic microscopy, in nanotechnology and in other fields of applied sciences. Furthermore, dislocations reduce electrical conductivity in some range of dislocation densities ($10^6 cm^{-2} - 10^{10} cm^{-2}$). This makes that "dislocation engineering" is becoming increasingly useful in the optimization of semiconductor devices.

2. Extrinsic semiconductors with defects of dislocation

Semiconductor crystals, as germanium and silicon, are tetravalent elements with electrical conductivity in between that of a conductor and that of an insulator (Kireev 1975; Kittel 2005). They have a behavior of an insulator at a temperature of $0^\circ K$, but at room temperature, $300^\circ K$, electrons of the crystal can gain enough thermal energy to jump to the conduction band, creating holes (positive charges not neutralized) in their covalent bonds. To modify the electrical properties of intrinsic semiconductors, impurity atoms adding one electron or one hole are introduced inside semiconductor crystals, using different techniques of "doping". In this last case the semiconductors are called extrinsic semiconductors. By dopant pentavalent impurities, as antimony, a n-type extrinsic semiconductor is obtained, having more free electrons that may flow. Using dopant tetravalent impurities, as indium, a p-type extrinsic semiconductor crystal is obtained, having more holes that may flow freely. Furthermore, the defects acquired during the process of fabrication can cause a premature fracture because they can self propagate because of surrounding changed conditions. In this paper we will consider extrinsic semiconductors with defects of dislocation, whose structure resembles a network of infinitesimally thin pores or capillary tubes and disturbs the interatomic distances inside the crystal (see Nabarro 1967; Mataré 1971).

In Mazzeo and Restuccia (2011a) to describe the dislocation lines the authors used a dislocation core tensor à la Maruszewski (1991) as internal variable. Here, we use a dislocation tensor that describes the local structure of these dislocation defects, introduced by Jou and Restuccia (2018b) (see also Jou and Restuccia 2018a). The trace of this tensor is the dislocation density ρ_D (total length of dislocation lines per unit volume, which has units of (length)$^{-2}$), which is the simplest variable in the description of dislocations. The dislocation lines have their intrinsic orientation, which means, among other things, that two dislocations of opposite signs annihilate when lines critically approach to each other. Thus, let us consider a representative elementary volume of a semiconductor where the dislocations resemble a network of infinitesimally thin channels, large enough to provide a representation of all the statistical properties of this volume. All the microscopic quantities are described with respect to the coordinates ξ_i $(i = 1,2,3)$, describing the position along a given dislocation line, while the macroscopic quantities are described with respect to the x_i coordinates (i=1,2,3).

Thus, the *dislocation lines density*, ρ_D, is given by the average length of dislocation lines per unit volume $\rho_D = \frac{1}{\Omega}\int dl$, where dl is the elementary lenght element along the dislocation lines and the integration is carried out over the position $\boldsymbol{\xi}$ along the corresponding dislocation line, and over all dislocation lines present in a elementary local volume Ω at $\mathbf{x}$.

The dislocation field is described by the microscopic tensor

$$\mathbf{a}(\xi) \equiv \mathbf{n}(\xi) \otimes \mathbf{n}(\xi), \tag{1}$$

where $\mathbf{n}(\xi)$ is the tangent unit vector along the dislocation line at position ξ and the integration runs along all dislocation lines which are in the chosen integration volume Ω at $\mathbf{x}$. We define the macroscopic dislocation tensor $\mathbf{a}(\mathbf{x})$ as the local average of $\mathbf{a}(\xi)$ in the following way

$$a_{ij}(\mathbf{x}) = \langle \mathbf{a}(\xi) \rangle = \frac{1}{\Omega}\int_\Omega n_i(\xi) n_j(\xi) dl, \tag{2}$$

where $\langle ... \rangle$ represents the average calculated on the ensemble of vortex lines inside the elementary volume Ω, and the integration runs along all dislocation lines inside Ω at $\mathbf{x}$. a_{ij} is called *dislocation tensor*, models the anisotropy of the dislocation lines and has unit m^{-2}, because it is related to $\frac{dl}{\Omega}$ namely, length/(length)^3 =length^{-2}. Also in Jou and Restuccia (2018b) a macroscopic variable was introduced, the *polarity vector* $a_i(\mathbf{x})$, defining the direction and the orientation of dislocation lines by means the average of the microscopic tangent vectors $\mathbf{n}(\xi)$ along the dislocation lines inside the elementary volume Ω, and, then, the integration is carried out over the position ξ along the corresponding dislocation line, and over all dislocation lines present in the elementary volume Ω at $\mathbf{x}$, $a_i(\mathbf{x}) = \frac{1}{\Omega L}\int n_i(\xi)dl$.

3. Foundamental equations

In this Section we present a model for an isotropic, elastic semiconductor of n and p type with defects of dislocation, deduced in Restuccia and Maruszewski (1995) (see also Mazzeo and Restuccia 2011a,b), in the framework of extended thermodynamics with internal variables (Prigogine 1961; De Groot and Mazur 1962; Lax 1983; Kluitenberg 1984; Muschik 1993; Lebon *et al.* 2008; Jou and Restuccia 2011; Berezovski and Ván 2017), where it was assumed that the following fields interact with each other: the elastic field described by the nonsymmetric stress tensor τ_{ij} and the small-strain tensor ε_{ij}; the thermal field described by the temperature T, its gradient and the heat flux q_i; the electromagnetic field described by the electric field $\mathscr{E}_i$ referred to as an element of matter at time t (in the comoving frame) and the magnetic induction B_i; the charge carrier field described by the density of electrons n, its gradient and its flux j_i^n, the hole field p, its gradient and its flux j_i^p; the dislocation field described by the dislocation density tensor a_{ij}, the internal variable describing the defects, its gradient $a_{ij,k}$ and its dislocation flux $\mathscr{V}_{ijk}$.

The set of the independent variables is therefore

$$C = \left\{\mathscr{E}_i, B_i, |\varepsilon_{ij}, n, p, T, a_{ij}, |n_{,i}, p_{,i}, T_{,i}, a_{ij,k}, |j_i^n, j_i^p, q_i, \mathscr{V}_{ijk}\right\}. \tag{3}$$

The fluxes j_i^n, j_i^p, q_i and $\mathscr{V}_{ijk}$ are considered as independent variables to take into account the relaxation properties of the fields n, p, T and a_{ij}. The gradients of these fields take into consideration non-local effects. In Restuccia and Maruszewski (1995) (see also Mazzeo and Restuccia 2011a,b) the thermodynamic model was formulated for a polarized semiconductor, but, here, we will assume that the dielectric properties of the semiconductor may be disregarded (see also Restuccia 2019), so that the polarization of the body is null, and it was assumed that the physical processes occurring in the above-defined situation are governed by the following fundamental laws:
Maxwell's equations describing the electromagnetic field, which, in the Galilean approximation, have the form

$$\varepsilon_{ijk}E_{k,j} + \frac{\partial B_i}{\partial t} = 0, \qquad D_{i,i} - \rho Z = 0, \tag{4}$$

$$\varepsilon_{ijk}H_{k,j} - j_i^Z - \frac{\partial D_i}{\partial t} = 0, \qquad B_{i,i} = 0, \tag{5}$$

with

$$H_i = \frac{1}{\mu_0}B_i, \qquad E_i = \frac{1}{\varepsilon_0}D_i, \tag{6}$$

being ε_0 and μ_0 the permittivity and permeability of vacuum, respectively, and $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{H}$ denote the electric field, the magnetic induction, the electric displacement and the magnetic field per unit volume, respectively. The concentration of the total charge Z and the density of the total electric current $\mathbf{j}^Z$ are defined as follows

$$Z = n + p, \tag{7}$$

$$j_i^Z = \rho Z v_i + j_i^n + j_i^p, \tag{8}$$

where ρ denotes the mass density, n $(n < 0)$ is the concentration of the negative electric charge density (coming from the density of the free electrons given by doping the semiconductor by pentavalent impurities and the density of the intrinsic semiconductor base free electrons), p $(p > 0)$ is the concentration of total positive electric charge (coming from the concentration of the holes produced by doping the semiconductor by trivalent impurities and the concentration of the intrinsic semiconductor base holes), $\mathbf{j}^Z$ is the density of the total electric current, v_i is the velocity of the body. The sum of j_i^n and j_i^p gives the the conduction electric current, whereas $\rho Z v_i$ is the electric current due to convection.
The charge conservation laws are

$$\rho \dot{n} + j_{i,i}^n = g^n, \quad \rho \dot{p} + j_{i,i}^p = g^p, \quad g^n + g^p = 0, \tag{9}$$

where the dot over a quantity indicates the material time derivative, g^n and g^p are source terms of charge carriers, and Eq. $(9)_3$ describes the recombination of electrons and holes.

The *continuity equation* is given by

$$\dot{\rho} + \rho v_{i,i} = 0, \tag{10}$$

(in the following we assume that ρ is constant) ;
the *momentum balance* has the form

$$\rho \dot{v}_i = \tau_{ji,j} + \rho Z \mathscr{E}_i + \varepsilon_{ijk} \left(j_j^n + j_j^p \right) B_k + f_i, \tag{11}$$

where on the right-hand side ε_{ijk} is the Levi-Civita pseudo-tensor and the terms describe, respectively, the elastic force, the electric force, the magnetic force and the external body force;
and the *energy balance* is

$$\rho \dot{U} = -q_{i,i} + \tau_{ji} \frac{d\varepsilon_{ij}}{dt} + \left(j_j^n + j_j^p \right) \mathscr{E}_j + \rho r, \tag{12}$$

where U is the internal energy density, $\varepsilon_{ij} = 1/2(u_{i,j} + u_{j,i})$ is the small strain tensor, with $\mathbf{u}$ the displacement field, and on the right-hand side the terms correspond, respectively, to the heat exchange, the mechanical work, the electric work and the heat source.

The last group of laws concerns the transport equations, which in our case are expressed as *the rate equations for charges flux, heat flux, defects field and defects flux.* In the following the hole field p and its flux j^p will be disregarded. These laws are chosen in the form

$$\overset{*}{j}{}_i^n = J_i^n(C), \quad \overset{*}{q}_i = Q_i(C),$$

$$\overset{*}{a}_{ij} = -\mathscr{V}_{ijk,k} + A_{ij}(C), \quad \overset{*}{\mathscr{V}}_{ijk} = V_{ijk}(C), \tag{13}$$

where C refers to the whole set of variables given in (3), excluding the hole field and its flux, and the superimposed asterisk indicates the Zaremba-Jaumann derivative (see Truesdell and Toupin 1960; Hermann *et al.* 2004), i.e. $\overset{*}{j}{}_i^n = \dot{j}_i^n - \Omega_{ik} j_k^n$, $\overset{*}{q}_i = \dot{q}_i - \Omega_{ik} q_k$, $\overset{*}{a}_{ij} = \dot{a}_{ij} - \Omega_{ik} a_{kj} - \Omega_{jk} a_{ik}$, $\overset{*}{\mathscr{V}}_{ijk} = \dot{\mathscr{V}}_{ijk} - \Omega_{il} \mathscr{V}_{ljk} - \Omega_{jl} \mathscr{V}_{ilk} - \Omega_{kl} \mathscr{V}_{ijl}$, with $\Omega_{ij} = \frac{1}{2}(v_{i,j} - v_{j,i})$ the antisymmetric part of the velocity gradient $v_{i,j}$ and v_i the velocity field of body.

Furthermore, in (13) $J_i^n(C)$, $Q_i(C)$, $A_{ij}(C)$ and $V_{ijk}(C)$ are the sources of the fluxes of carriers, heat, defects and defects flux. The fluxes of electrons, heat and defects fluxes are not taken into consideration, because we have to obtain a balanced system of equations, where the number of equations is equal to the number of variables.

All the admissible solutions of the proposed evolution equations should be restricted by the following *entropy inequality*:

$$\rho \dot{S} + J^S_{k,k} - \frac{\rho r}{\theta} \geq 0, \tag{14}$$

where S denotes the entropy per unit mass and $\mathbf{J}^S$ is the entropy flux associated with the fields of the set C. In Restuccia and Maruszewski (1995) (see also Mazzeo and Restuccia 2009, 2011a; Restuccia 2019) the entropy inequality was analyzed by Liu's theorem (see (Liu 1972)), where all balance and evolution equations of the problem are considered as mathematical constraints for its physical validity, and, using the obtained results, by the help of Smith's theorem (Smith 1971), the constitutive theory was obtained. Here, the constitutive functions (dependent variables) τ_{ij}, U, g^n, J_i^n, Q_i, A_{ij}, V_{ijk}, S, ϕ_i, μ^n, π_{ij} , with μ^n the electrochemical potential for the electrons and π_{ij} a similar potential for the defects field, are expressed in terms of isotropic polynomial representations of suitable functions satisfying the objectivity and material frame indifference principles (see Truesdell and Toupin 1960; Muschik and Restuccia 2002; Hermann *et al.* 2004).

Assuming for the dislocation field, its flux and their sources the form

$$a_{ij} = a\delta_{ij}, \quad A_{ij} = A\delta_{ij}, \quad \mathscr{V}_{ijk} = \mathscr{V}_k \delta_{ij}, \quad V_{ijk} = V_k \delta_{ij}, \tag{15}$$

we derive, in a first approximation, the rate equations in the following form

$$\overset{*}{j}{}^n_k = \delta_n^1 \mathscr{E}_k + \delta_n^2 a_{,k} + \delta_n^3 n_{,k} + \delta_n^4 \theta_{,k} + \delta_n^5 \mathscr{V}_k + \delta_n^6 j_k^n + \delta_n^7 q_k, \tag{16}$$

$$\overset{*}{q}_k = \delta_q^1 \mathscr{E}_k + \delta_q^2 a_{,k} + \delta_q^3 n_{,k} + \delta_q^4 T_{,k} + \delta_q^5 \mathscr{V}_k + \delta_q^6 j_k^n + \delta_q^7 q_k, \tag{17}$$

$$\overset{*}{\mathscr{V}}_k = \delta_\upsilon^1 \mathscr{E}_k + \delta_\upsilon^2 a_{,k} + \delta_\upsilon^3 n_{,k} + \delta_\upsilon^4 \theta_{,k} + \delta_\upsilon^5 \mathscr{V}_k + \delta_\upsilon^6 j_k^n + \delta_\upsilon^7 q_k, \tag{18}$$

where δ_n^η, δ_q^η, δ_υ^η $(\eta = 1, 2, ..., 7)$ can depend on suitable invariants built on appropriate variables of the set C (see Eq. (3), excluding the hole field and its flux),

$$\overset{*}{a} + \mathscr{V}_{k,k} = \delta_a^1 n + \delta_a^2 a + \delta_a^3 \theta + \delta_a^4 \varepsilon_{kk}. \tag{19}$$

Also, we work out g^n (see Eq. $(9)_1$) as objective function, having, in a first approximation, the following expression

$$g^n = \beta_{g^n}^1 n + \beta_{g^n}^2 p + \beta_{g^n}^2 a + \beta_{g^n}^3 \theta + \beta_{g^n}^4 \varepsilon_{kk}. \tag{20}$$

In (19) and (20) δ_a^ε and $\beta_{g^n}^\varepsilon$ $(\varepsilon = 1, 2, 3, 4)$ can depend on suitable invariants built on appropriate variables of the set C (excluding the hole field and its flux). The laws (16)-(19) are very general, but it is possible to treat special problems describing the physical reality in several situations by some simplifications. These rate equations allow finite speeds for the disturbances and describe fast phenomena, whose relaxation time is comparable or higher than the relaxation time of the media under consideration. The equations (19) and (18) for the defects density and its flux are new, but the equation (16) is the generalized Ohm's law and equation (17) is the generalized Vernotte-Cattaneo relation.

4. Equation governing the evolution of electronic and dislocation coupling fields

In this Section we apply the results obtained to a problem of propagation of electronic-dislocation discontinuity waves in a n-type Ge, supposed at rest. Then, confining ourselves only to the electronic and dislocation fields and their fluxes, and assuming that the values of concentration of electrons n and the dislocation density a are low, i.e. the semiconductor is not degenerated and the influence of dislocations on the conductivity is relatively small Nabarro 1967), we can assume the equations governing the evolution of the electronic and dislocation fields and their fluxes (see (9), (20), (16), (19) and (18)) having the form (Mazzeo and Restuccia 2011a):

$$\begin{cases} \rho \dot{n} + j^n_{k,k} = \frac{\rho n}{\tau^+} - \kappa a, \\ \tau^n \dot{j}^n_k - \alpha_n a_{,k} + \rho D_n n_{,k} = -j^n_k, \\ \dot{a} + \mathscr{V}_{k,k} = 0 \\ \tau^a \dot{\mathscr{V}}_k + D_a a_{,k} - \alpha_\upsilon n_{,k} = -\mathscr{V}_k. \end{cases} \tag{21}$$

where, here, the superimposed dot denotes partial time derivative, the mass density ρ is constant, the interaction between the electronic and dislocation fluxes is disregarded and $\alpha_n = \alpha_n(a)$, $\alpha_\upsilon = \alpha_\upsilon(n)$ are coupling functions. Furthermore, τ^+ denotes the life time of electrons, τ^n is the relaxation time of electrons, D_n and D_a are the diffusion coefficients of electrons and dislocations, respectively, τ^a denotes the relaxation time of dislocations and k is the recombination constant due to dislocations. Moreover, we assume that $\tau^+ = \tau^n$ (see Kireev 1975). In Restuccia and Maruszewski 1995 the system (21) was considered in linear form with α_n and α_υ constant and the dispersion relation of the electronic-dislocation harmonic waves was studied, estimating the coefficients present in the system (21) following Nabarro (1967) and Mataré (1971) (see Table 1).

Coefficient	Measure unit	Value	Name
ρ	Kgm^{-3}	5.3×10^3	mass density
D_n	m^2s^{-1}	10^{-2}	electron diffusion coefficient
D_a	m^2s^{-1}	2.43×10^{-2}	dislocation diffusion coefficient
τ^n	s	$< 10^{-5}$	electron relaxation time
τ^a	s	0.3×10^{-8}	dislocation relaxation time
α_n	Cms^{-1}	< 1.8	cross-effects function
α_v	$Kg\,C^{-1}s^{-1}$	< 70	cross-effects function
κ	$Cm^{-1}s^{-1}$	$< 1.9 \times 10^{-4}$	recombination constant
c_n	ms^{-1}	10^3	$\sqrt{D_n/\tau^n}$
c_a	ms^{-1}	2846	$\sqrt{D_a/\tau^a}$

TABLE 1. The estimated coefficients, after Restuccia and Maruszewski (1995).

It is easy to see how the above mentioned system of equations takes the following matrix form:

$$\mathbf{A}^{\alpha}(\mathbf{U})\mathbf{U}_{\alpha} = \mathbf{B}(\mathbf{U}) \qquad (\alpha = 0,1,2,3). \tag{22}$$

where $x^0 = t$ (time), x^1, x^2, x^3 are the space coordinates, $\mathbf{U}_{\alpha} = \frac{\partial \mathbf{U}}{\partial x^{\alpha}}$, $\mathbf{U}$ is the vector of the unknown function (which depends on x^{α})

$$\mathbf{U} = (n, j_1^n, j_2^n, j_3^n, a, \mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3)^T, \tag{23}$$

$$\mathbf{B} = \left(\frac{\rho n}{\tau^n} - \kappa a, -j_1^n, -j_2^n, -j_3^n, 0, -\mathscr{V}_1, -\mathscr{V}_2, -\mathscr{V}_3\right)^T, \tag{24}$$

$$\mathbf{A}^0 = \begin{pmatrix} \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau^n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau^n & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau^n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tau^a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau^a & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau^a \end{pmatrix} \tag{25}$$

and $\mathbf{A}^i$ $(i = 1,2,3)$ are the following square matrices 8×8

$$\mathbf{A}^1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho D_n & 0 & 0 & 0 & -\alpha_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\alpha_v & 0 & 0 & 0 & D_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{26}$$

$$\mathbf{A}^2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho D_n & 0 & 0 & 0 & -\alpha_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha_v & 0 & 0 & 0 & D_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{27}$$

$$\mathbf{A}^3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho D_n & 0 & 0 & 0 & -\alpha_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha_v & 0 & 0 & 0 & D_a & 0 & 0 & 0 \end{pmatrix}. \tag{28}$$

In (23) and (24) the symbol $(...)^T$ means that $\mathbf{U}$ and $\mathbf{B}$ are column vectors. Equations (22) are semilinear (the highest order derivatives $\mathbf{U}_t$ and $\mathbf{U}_\alpha$ occur linearly) and, since $\mathbf{A}^\alpha = \mathbf{A}^\alpha(\mathbf{U})$, the PDEs system (22) is a quasi-linear system. We suppose that the system admits a known solution in the *uniform unperturbed state* $\mathbf{U}^0$, that satisfies the following condition

$$\mathbf{A}^\alpha(\mathbf{U}^0)\mathbf{U}^0_\alpha = \mathbf{B}(\mathbf{U}^0) \qquad (\alpha = 0,1,2,3). \tag{29}$$

Moreover, we admit that the system (22) describes a perturbation propagating into a state characterized by the vector $\mathbf{U}^0$ and $\varphi(x^\alpha) = 0$ is the surface, called *wavefront*, that separates the region perturbed, $\varphi(x^\alpha) = 0^+$, from the unperturbed, $\varphi(x^\alpha) = 0^-$, and moves in the Euclidean space E^{3+1} (when the time flows).

Remind that the wavefront $\varphi(x^\alpha) = 0$ is still an unknown function. In order to determine it, we recall that along the wavefront we have $\frac{d\varphi}{dt} = 0$, implying

$$\frac{\partial \varphi}{\partial t} + \mathbf{v} \cdot grad\varphi = 0,$$

or, equivalently,

$$\frac{\frac{\partial \varphi}{\partial t}}{|grad\varphi|} + \mathbf{v} \cdot \frac{grad\varphi}{|grad\varphi|} = 0.$$

with $(grad)_i = \frac{\partial}{\partial x^i}$

Obviously,

$$\frac{grad\varphi}{|grad\varphi|} = \mathbf{n}, \tag{30}$$

such that the previous equality reads

$$\frac{\frac{\partial \varphi}{\partial t}}{|grad\varphi|} + \mathbf{v} \cdot \mathbf{n} = 0. \tag{31}$$

Introduce the notation

$$\lambda = -\frac{\frac{\partial \varphi}{\partial t}}{|grad\varphi|}, \tag{32}$$

so that

$$\lambda(\mathbf{U}, \mathbf{n}) = \mathbf{v} \cdot \mathbf{n}, \tag{33}$$

where λ is called the *velocity normal to the progressive wave*, being $\mathbf{n} = (n_1, n_2, n_3)$ the unit vector normal to the wave front.

We suppose that the function $\mathbf{U}(x^\alpha)$ is piecewise continuous and presents a discontinuity across the surface $\varphi(x^\alpha) = 0$, i.e. the first derivatives of $\mathbf{U}$ present a jump across the front wave $\varphi(x^\alpha) = 0$ (the first derivatives are continue in the one and the other part of the wave front but they tend to two different limits).

Introducing the function $\varphi = \varphi(x^\alpha)$ as new variable, continuous together with its first and second derivatives, we have

$$\mathbf{U}_\alpha = \mathbf{U}_\varphi \; \varphi_\alpha, \tag{34}$$

where $\mathbf{U}_\varphi = \frac{\partial \mathbf{U}}{\partial \varphi}$ and $\varphi_\alpha = \frac{\partial \varphi}{\partial x^\alpha}$.

Moreover, we introduce the symbol denoting the *jump*

$$[\quad] = \lim_{\varphi \to 0^+} (\quad) - \lim_{\varphi \to 0^-} (\quad), \tag{35}$$

given by the difference between the value of a quantity assumed in the perturbed state and that one assumed in the unperturbed state, calculated on the surface $\varphi(x^\alpha) = 0$. Then, denoting by Π the jump of the normal derivative $\mathbf{U}_\varphi$, we have

$$[\mathbf{U}] = 0, \qquad \Pi = \left[\mathbf{U}_\varphi\right] = \lim_{\varphi \to 0^+} (\mathbf{U}_\varphi) - \lim_{\varphi \to 0^-} (\mathbf{U}_\varphi). \tag{36}$$

From Eq.s (22), (29) and (34) we obtain the following relations:

$$\mathbf{A}^\alpha(\mathbf{U})\varphi_\alpha \mathbf{U}_\varphi = \mathbf{B}(\mathbf{U}), \qquad \text{and} \qquad \mathbf{A}^\alpha(\mathbf{U}^0)\mathbf{U}^0_\varphi \varphi_\alpha = \mathbf{B}(\mathbf{U}^0). \tag{37}$$

Subtracting Eq. $(37)_2$ from Eq. $(37)_1$ and by computing on the surface $\varphi(x^\alpha) = 0$, where $\mathbf{U} = \mathbf{U}^0$ and $\mathbf{A}^\alpha(\mathbf{U}^0) = (\mathbf{A}^\alpha)_0(\mathbf{U}^0)$, we get

$$(\mathbf{A}^\alpha)_0 \varphi_\alpha \left[\mathbf{U}_\varphi\right] = 0, \quad \text{i.e.} \quad (\mathbf{A}^\alpha)_0 \varphi_\alpha \Pi = 0 \tag{38}$$

where the symbol "$(\ )_0$" indicates that the quantities are calculated in $\mathbf{U}_0$, $(\mathbf{A}^\alpha)_0 \varphi_\alpha$ represents a 8x8 matrix and Eq. $(38)_2$ is a homogeneous system in the 8 components of Π.

Introducing the quantities λ and $\mathbf{n}$, defined in Eq.s (30) and (32), the system $(38)_2$ takes the form

$$\left(\mathbf{A}^i n_i - \lambda \mathbf{A}^0\right) \Pi = \mathbf{0}. \tag{39}$$

In order to have a solution different from the zero solution, we have to impose

$$Det \|\mathbf{A}_n - \lambda \mathbf{A}^0\| = 0, \tag{40}$$

with $\mathbf{A}_n = \mathbf{A}^i n_i$. Eq. (39) shows that Π can be taken as equal to the right-eigenvector $\mathbf{r}$ of $\mathbf{A}_n$, corresponding to some eigenvalue λ. It follows that Π has the form

$$\Pi = \pi \mathbf{r}. \tag{41}$$

Then, in order to determine Π we have to determine the function $\pi = \pi(x^\alpha)$. Since $\mathbf{A}^0$ is a non-singular matrix, the system (22) can be written in the form

$$\mathbf{U}_t + (\mathbf{A}^0)^{-1}\mathbf{A}^i \mathbf{U}_i = (\mathbf{A}^0)^{-1}\mathbf{B}(\mathbf{U}) \qquad (i = 1, 2, 3), \tag{42}$$

where

$$(\mathbf{A}^0)^{-1} = \begin{pmatrix} \frac{1}{\rho} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\tau^n} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\tau^n} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\tau^n} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau^a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau^a} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau^a} \end{pmatrix} \tag{43}$$

is the inverse matrix of $\mathbf{A}^0$.

In the following we continue to call $\mathbf{A}^i$ the matrices $(\mathbf{A}^0)^{-1}\mathbf{A}^i$, and $\mathbf{B}$ the vector $(\mathbf{A}^0)^{-1}\mathbf{B}$. Then, the system assumes the following form

$$\mathbf{U}_t + \mathbf{A}^i \mathbf{U}_i = \mathbf{B}(\mathbf{U}) \qquad (i = 1, 2, 3), \tag{44}$$

with

$$\mathbf{U}_t = \frac{\partial \mathbf{U}}{\partial t}, \qquad \mathbf{U}_i = \frac{\partial \mathbf{U}}{\partial x^i} \qquad (i = 1,2,3). \tag{45}$$

From eq. (40) we have the following eigenvalues problem

$$Det\|\mathbf{A}_n - \lambda \mathbf{I}\| = 0, \tag{46}$$

with $\mathbf{A}_n = \mathbf{A}^i n_i$ $(i = 1,2,3)$.

5. First approximation of wave front

In this section we show how the wave front $\varphi(t,x_1,x_2,x_3) = 0$ can be determined. Following the general theory (see Boillat 1976) we introduce the quantity

$$\Psi(\mathbf{U},\Phi_\alpha) = \varphi_t + |grad\varphi|\lambda(\mathbf{U},\mathbf{n}), \tag{47}$$

with $\Phi_\alpha = \varphi_\alpha = \frac{\partial \varphi}{\partial x^\alpha}$ $(\alpha = 0,1,2,3)$, that, by virtue of the relations (30) and (32), becomes zero on the wavefront having velocity $\lambda = \lambda^0$, i.e.

$$\Psi(\mathbf{U}^0,\Phi_\alpha) = \varphi_t + |grad\varphi|\lambda^0(\mathbf{U}^0,\mathbf{n}^0) = \Psi^0 = 0. \tag{48}$$

To solve the above partial differential equation the *characteristic rays* are introduced, called characteristic curves of the system (22), given by the following differential equations

$$\frac{dx^\alpha}{d\sigma} = \frac{\partial \Psi^0}{\partial \Phi_\alpha} \quad (\alpha = 0,1,2,3), \tag{49}$$

$$\frac{d\Phi_\alpha}{d\sigma} = -\frac{\partial \Psi^0}{\partial x^\alpha} \quad (\alpha = 0,1,2,3), \tag{50}$$

where σ is the time along the characteristic rays.

From Eq. (50), considering the propagation in a uniform state $\mathbf{U}_0$, we have $\frac{\partial \Psi^0}{\partial x^\alpha} = 0$ and, then, Φ_α are constants along the characteristic rays.

Furthermore, Eq. (49) gives the components of a speed, called *radial velocity* Λ and defined by

$$\Lambda_i(\mathbf{U},\mathbf{n}) \equiv \frac{dx_i}{d\sigma} = \frac{\partial \Psi}{\partial \varphi_i} = \lambda n_i + \frac{\partial \lambda}{\partial n_i} - \left(\mathbf{n}\cdot\frac{\partial \lambda}{\partial \mathbf{n}}\right) n_i \quad (i = 1,2,3). \tag{51}$$

From Eq. (51) we have

$$\Lambda_i n_i = \lambda. \tag{52}$$

i.e. the velocity of propagation of the wavefront λ is the component of radial velocity Λ along the normal to the wavefront. By integration of Eq. (49) we obtain

$$x^0 = t = \sigma, \tag{53}$$

$$x^i = (x^i)^0 + \Lambda_i^0(\mathbf{U}^0,\mathbf{n}^0)t \quad (i = 1,2,3), \tag{54}$$

with

$$(x^i)^0 = (x^i)_{t=0} \quad (i = 1,2,3). \tag{55}$$

If we denote by φ^0 the given initial surface, we have $(\varphi)_{t=0} = \varphi^0((x^i)^0)$ and $\mathbf{n}^0$ represents the unit normal vector to the wavefront at the point $(x^i)^0$ defined by

$$\mathbf{n}^0 = \left(\frac{grad\varphi}{|grad\varphi|}\right)_{t=0} = \frac{grad^0\varphi^0}{|grad^0\varphi^0|}, \tag{56}$$

where

$$(grad^0)_i \equiv \frac{\partial}{\partial(x^i)^0} \quad (i = 1,2,3). \tag{57}$$

Thus, $\mathbf{x} = \mathbf{x}|_{t=0} + \Lambda^0 t$ and since the Jacobian J of the transformation $\mathbf{x} \to \mathbf{x}|_{t=0}$ is nonvanishing, i.e.

$$J = det\left| t\frac{\partial \Lambda^0_k}{\partial(x^i)^0} + \delta_{ik}\right| \neq 0 \quad (i,k = 1,2,3), \tag{58}$$

$(x^i)^0$ can be deduced from (53) and (54), and φ in the first approximation takes the following form

$$\varphi(t,x^i) = \varphi^0(x^i - \Lambda^0_i t). \tag{59}$$

Taking into account the initial conditions, we can deduce the phase $\varphi(x,t)$ of the considered wave. Then, developing by Taylor's formula the vector $\mathbf{U}$ up to first order in a neighborhood of the wavefront $\varphi(x^\alpha) = 0$ we have

$$\mathbf{U} = (\mathbf{U})_{\varphi=0^+} + \left(\frac{\partial \mathbf{U}}{\partial \varphi}\right)_{\varphi=0^+} + \mathscr{O}(\varphi^2), \tag{60}$$

$$\mathbf{U}^0 = \left(\mathbf{U}^0\right)_{\varphi=0^-} + \left(\frac{\partial \mathbf{U}^0}{\partial \varphi}\right)_{\varphi=0^-} + \mathscr{O}(\varphi^2). \tag{61}$$

Operating the difference between (60) and (61) we obtain

$$\mathbf{U} = \mathbf{U}_0 + \varphi\Pi + \mathscr{O}(\varphi^2), \tag{62}$$

where $\mathscr{O}(\varphi^2)$ is the Landau's notation and represents infinitesimals of higher order respect to φ.

In (62), following Boillat (1965), the amplitude of discontinuity π satisfies Bernoulli equation having the form

$$\left(\mathbf{l}^0 \cdot \mathbf{r}^0\right)\left\{\frac{d\pi}{d\sigma} + (\nabla\Psi \cdot \mathbf{r})_0\,\pi^2 + \frac{d}{dt}\ln\sqrt{J}\right\} = c^0\pi, \tag{63}$$

where

$$(\nabla\Psi \cdot \mathbf{r})_0 = (|grad\varphi|)_0\,(\nabla\lambda \cdot \mathbf{r})_0\,, \qquad c^0 = (\nabla(\mathbf{l}\cdot\mathbf{B})\cdot\mathbf{r})_0\,, \tag{64}$$

$$\nabla \equiv \left(\frac{\partial}{\partial n}, \frac{\partial}{\partial j^n_1}, \frac{\partial}{\partial j^n_2}, \frac{\partial}{\partial j^n_3}, \frac{\partial}{\partial a}, \frac{\partial}{\partial \mathscr{V}_1}, \frac{\partial}{\partial \mathscr{V}_2}, \frac{\partial}{\partial \mathscr{V}_3}\right) \tag{65}$$

and $\mathbf{r}^0$ is the right eigenvector corresponding to the eigenvalue λ^0. Eq.s (63), (49), (50) and (41) determine the discontinuity.

In Boillat (1965) it was seen that

$$\pi = \frac{h(\sigma)}{\Phi(\sigma)}, \quad \text{where} \quad h(\sigma) = h^0 exp\left[\int_0^\sigma -\frac{c^0}{(\mathbf{l}\cdot\mathbf{r})_0}d\tau\right], \quad \text{with} \quad h^0 = h(0) = \pi(0), \tag{66}$$

$$\Phi(\sigma) = 1 + \int_0^\sigma \frac{(\nabla\Psi \cdot \mathbf{r})_0}{\sqrt{J}}\, h d\tau. \tag{67}$$

From $(66)_1$ it follows that if there exists a time σ_c where $\Phi(\sigma_c) = 0$, then $\pi \to \infty$, and this may correspond to a shock wave (Boillat 1965).

6. One-dimensional case

Now, we consider the one-dimensional case. Assuming that the electronic-dislocation discontinuity wave propagation is along the x axis, the involved quantities depend on x_1, denoted by x, $x_2 = x_3 = 0$, the system (21) takes the following form:

$$\frac{\partial n}{\partial t} + \frac{1}{\rho}\frac{\partial j_1^n}{\partial x} = \frac{n}{\tau^n} - \frac{\kappa a}{\rho}, \tag{68}$$

$$\frac{\partial j_1^n}{\partial t} - \frac{\alpha_n}{\tau^n}\frac{\partial a}{\partial x} + \frac{\rho D_n}{\tau^n}\frac{\partial n}{\partial x} = -\frac{j_1^n}{\tau^n}, \tag{69}$$

$$\frac{\partial j_2^n}{\partial t} = -\frac{j_2^n}{\tau^n}, \tag{70}$$

$$\frac{\partial j_3^n}{\partial t} = -\frac{j_3^n}{\tau^n}, \tag{71}$$

$$\frac{\partial a}{\partial t} + \frac{\partial \mathscr{V}_1}{\partial x} = 0 \tag{72}$$

$$\frac{\partial \mathscr{V}_1}{\partial t} + \frac{D_a}{\tau^a}\frac{\partial a}{\partial x} - \frac{\alpha_\upsilon}{\tau^a}\frac{\partial n}{\partial x} = -\frac{\mathscr{V}_1}{\tau^a}, \tag{73}$$

$$\frac{\partial \mathscr{V}_2}{\partial t} = -\frac{\mathscr{V}_2}{\tau^a}, \tag{74}$$

$$\frac{\partial \mathscr{V}_3}{\partial t} = -\frac{\mathscr{V}_3}{\tau^a}. \tag{75}$$

where $\alpha_n = \alpha_n(a)$ and $\alpha_v = \alpha_v(n)$.

From the above system we have

$$j_2^n = j_2^{n0}(x)e^{-\frac{1}{\tau^n}t}, \qquad j_3^n = j_3^{n0}(x)e^{-\frac{1}{\tau^n}t},$$

$$\mathscr{V}_2 = \mathscr{V}_2^0(x)e^{-\frac{1}{\tau^a}}, \qquad \mathscr{V}_3 = \mathscr{V}_3^0(x)e^{-\frac{1}{\tau^a}}.$$

Then, we obtain

$$\mathbf{U}_t + \mathbf{A}(\mathbf{U})\mathbf{U}_x = \mathbf{B}(\mathbf{U}), \tag{76}$$

where

$$\mathbf{U} = (n, j_1^n, a, \mathscr{V}_1)^T, \tag{77}$$

$$\mathbf{B} = \left(\frac{n}{\tau^n} - \frac{\kappa a}{\rho}, -\frac{j_1^n}{\tau^n}, 0, -\frac{\mathscr{V}_1}{\tau^a},\right)^T \tag{78}$$

and

$$\mathbf{A} = \begin{pmatrix} 0 & \frac{1}{\rho} & 0 & 0 \\ \frac{\rho D_n}{\tau^n} & 0 & -\frac{\alpha_n}{\tau^n} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\alpha_v}{\tau^a} & 0 & \frac{D_a}{\tau^a} & 0 \end{pmatrix}. \tag{79}$$

Then, in Eq. (39) $\mathbf{A}_n(\mathbf{U}) = \mathbf{A}n_1$ has the form

$$\mathbf{A}_n(\mathbf{U}) = \mathbf{A}n_1 = \begin{pmatrix} 0 & \frac{1}{\rho}n_1 & 0 & 0 \\ \frac{\rho D_n}{\tau^n}n_1 & 0 & -\frac{\alpha_n}{\tau^n}n_1 & 0 \\ 0 & 0 & 0 & n_1 \\ -\frac{\alpha_v}{\tau^a}n_1 & 0 & \frac{D_a}{\tau^a}n_1 & 0 \end{pmatrix}, \tag{80}$$

being $n_1 = 1$.

The matrix $\mathbf{A}$ admits the following simple eigenvalues:

$$\lambda_1^{(\pm)} = \pm \frac{1}{\sqrt{2}} \sqrt{\frac{\rho D_n \tau^a + \rho D_a \tau^n - G}{\rho \tau^n \tau^a}}, \tag{81}$$

$$\lambda_2^{(\pm)} = \pm \frac{1}{\sqrt{2}} \sqrt{\frac{\rho D_n \tau^a + \rho D_a \tau^n + G}{\rho \tau^n \tau^a}}, \tag{82}$$

where

$$G = \sqrt{(\rho D_n \tau^a - \rho D_a \tau^n)^2 + 4\rho \alpha_n \alpha_v \tau^n \tau^a}. \tag{83}$$

The eigenvalues $\lambda_2^{(\pm)}$ are always real, whereas the eigenvalues $\lambda_1^{(\pm)}$ are real when the condition $\rho D_n \tau^a + \rho D_a \tau^n - G \geq 0$ is valid, i.e. for $\alpha_n \alpha_v \leq \rho D_n D_a$. In the case where we consider only

$$\rho D_n \tau^a + \rho D_a \tau^n - G > 0, \tag{84}$$

that is valid for $\alpha_n \alpha_v < \rho D_n D_a$, we have the propagation of discontinuty waves having normal velocities different than zero, represented by the obtained eigenvalues. Furthermore, G is different than zero, because from its definition (83) the relation $(\rho D_n \tau^a - \rho D_a \tau^n)^2 \neq -4\rho \alpha_n \alpha_v \tau^n \tau^a$ is always verified. This leads to the result $\lambda_1^{(\pm)} \neq \lambda_2^{(\pm)}$.

The left eigenvectors $\mathbf{l}_1^{(\pm)}$, $\mathbf{l}_2^{(\pm)}$, and the right eigenvectors $\mathbf{r}_1^{(\pm)}$, $\mathbf{r}_2^{(\pm)}$, corresponding to eigenvalues $\lambda_1^{(\pm)}$, $\lambda_2^{(\pm)}$, have the form

$$\mathbf{l}_1^{(\pm)} = \left(\frac{\lambda_1^{(\pm)} \mathscr{R}}{2\alpha_n \tau^a}, \frac{\mathscr{R}}{2\rho \alpha_n \tau^a}, \lambda_1^{(\pm)}, 1 \right), \quad \mathbf{l}_2^{(\pm)} = \left(\frac{\lambda_2^{(\pm)} \mathscr{S}}{2\alpha_n \tau^a}, \frac{\mathscr{S}}{2\rho \alpha_n \tau^a}, \lambda_2^{(\pm)}, 1 \right), \tag{85}$$

$$\mathbf{r}_1^{(\pm)} = \left(\frac{2\alpha_n (\tau^n)^2 \lambda_1^{(\pm)}}{\mathscr{C}}, -\frac{\alpha_n \tau^a \mathscr{P}}{\tau^n \mathscr{C}}, \frac{\lambda_1^{(\pm)} \tau^a \mathscr{S}}{\mathscr{C}}, 1 \right)^T, \tag{86}$$

$$\mathbf{r}_2^{(\pm)} = \left(\frac{2\alpha_n (\tau^n)^2 \lambda_2^{(\pm)}}{\mathscr{L}}, -\frac{\alpha_n \tau^a \mathscr{Q}}{\tau^n \mathscr{L}}, \frac{\lambda_2^{(\pm)} \tau^a \mathscr{R}}{\mathscr{L}}, 1 \right)^T, \tag{87}$$

with

$$\mathscr{R} = \rho D_a \tau^n - \rho D_n \tau^a + G, \quad \mathscr{S} = \rho D_a \tau^n - \rho D_n \tau^a - G, \tag{88}$$

$$\mathscr{P} = \rho D_a \tau^n + \rho D_n \tau^a - G, \quad \mathscr{Q} = \rho D_a \tau^n + \rho D_n \tau^a + G, \tag{89}$$

$$\mathscr{C} = D_a \left(\rho D_n \tau^a + G\right) - \rho D_a^2 \tau^n - 2\alpha_n \alpha_v \tau^a, \tag{90}$$

$$\mathscr{L} = D_a \left(\rho D_n \tau^a - G\right) - \rho D_a^2 \tau^n - 2\alpha_n \alpha_v \tau^a, \tag{91}$$

where $\mathscr{C}$, $\mathscr{L}$ are supposed different than zero and this assumption leads to the condition $\alpha_n \alpha_v \neq \rho D_n D_a$, compatible with the result (84).

The discontinuity waves which are propagating with the velocity given by $\lambda_1^{(\pm)}$ and $\lambda_2^{(\pm)}$ are not exceptional waves in the sense of Lax-Boillat (Boillat 1965), when

$$\nabla \lambda_1^{(\pm)} \cdot \mathbf{r}_1^{(\pm)} = \mp \frac{\tau^a}{2G\mathscr{C}} \left[2\rho \alpha_n^2 \tau^a \frac{\partial \alpha_v}{\partial n} + \alpha_v \mathscr{S} \frac{\partial \alpha_n}{\partial a} \right] \neq 0, \tag{92}$$

$$\nabla \lambda_2^{(\pm)} \cdot \mathbf{r}_2^{(\pm)} = \pm \frac{\tau^a}{2G\mathscr{L}} \left[2\rho \alpha_n^2 \tau^a \frac{\partial \alpha_v}{\partial n} + \alpha_v \mathscr{R} \frac{\partial \alpha_n}{\partial a} \right] \neq 0, \tag{93}$$

with

$$\nabla \equiv \left(\frac{\partial}{\partial n}, \frac{\partial}{\partial j_1^n}, \frac{\partial}{\partial a}, \frac{\partial}{\partial \mathscr{V}_1} \right).$$

Under this assumption, we fix our attention on $\lambda = \lambda_2^{(+)}$, which corresponds to a progressive fast wave traveling to the right. Analogous results are valid for the waves propagating with the other velocities.

The eigenvectors left and right $\mathbf{l} = \mathbf{l}_2^{(+)}$ and $\mathbf{r} = \mathbf{r}_2^{(+)}$, corresponding to $\lambda_2^{(+)}$, satisfy the following relation

$$\mathbf{l}_2^{(+)} \cdot \mathbf{r}_2^{(+)} = 1 + \frac{\mathscr{Q}(3\rho D_a \tau^n - 3\rho D_n \tau^a - G)}{2\rho \tau^n \mathscr{L}}, \tag{94}$$

whose value is supposed different than zero and this assumption ensures the hyperbolicity of the system (44) and the propagation of the weak discontinuity waves (see (63)).

The characteristic rays are (see (51), (53) and (54))

$$\frac{dt}{d\sigma} = 1 \qquad \frac{dx}{d\sigma} = \left(\lambda_2^{(+)} \right)_0. \tag{95}$$

Now, we consider an uniform unperturbed state in which $\mathbf{U}^0$, solution of the system (76), has the form

$$\mathbf{U}^0 = (n^0, 0, a^0, 0), \tag{96}$$

with n^0 and a^0 constants.

In $\mathbf{U}^0$ we have

$$\frac{dx}{d\sigma} = \sqrt{\frac{\rho D_a \tau^n + \rho D_n \tau^a + G^0}{2\rho \tau^n \tau^a}}, \tag{97}$$

$$\left(\mathbf{l}_2^{(+)}\right)_0 = \left(\frac{\left(\lambda_2^{(+)}\right)_0 \mathscr{S}^0}{2\alpha_n^0 \tau^a}, \frac{\mathscr{S}^0}{2\rho \alpha_n^0 \tau^a}, \left(\lambda_2^{(+)}\right)_0, 1 \right), \tag{98}$$

$$\left(\mathbf{r}_2^{(+)}\right)_0 = \left(\frac{2\alpha_n^0 (\tau^n)^2 \left(\lambda_2^{(+)}\right)_0}{\mathscr{L}^0}, -\frac{\alpha_n^0 \tau^a \mathscr{Q}^0}{\tau^n \mathscr{L}^0}, \frac{\left(\lambda_2^{(+)}\right)_0 \tau^a \mathscr{R}^0}{\mathscr{L}^0}, 1 \right), \tag{99}$$

$$\left(\mathbf{l}_2^{(+)} \cdot \mathbf{r}_2^{(+)}\right)_0 = 1 + \frac{\mathscr{Q}^0 (3\rho D_a \tau^n - 3\rho D_n \tau^a - G^0)}{2\rho \tau^n \mathscr{L}^0}, \tag{100}$$

and

$$\left(\nabla \lambda_2^{(+)} \cdot \mathbf{r}_2^{(+)}\right)_0 = \frac{\tau^a}{2G^0 \mathscr{L}^0} \left[2\rho (\alpha_n^0)^2 \tau^a \left(\frac{\partial \alpha_\nu}{\partial n}\right)^0 + \alpha_a^0 \mathscr{R}^0 \left(\frac{\partial \alpha_n}{\partial a}\right)^0 \right], \tag{101}$$

where the symbol " ^{0} " indicates that the quantities are calculated in $\mathbf{U}^0$.

The radial velocity along the characteristic rays is

$$\Lambda^0(\mathbf{U}^0, \mathbf{n}^0) = \left(\lambda_2^{(+)}\right)_0 \mathbf{n}^0 = \left(\frac{1}{\sqrt{2}} \sqrt{\frac{\rho D_n \tau^a + \rho D_a \tau^n + G^0}{\rho \tau^n \tau^a}}, 0, 0 \right). \tag{102}$$

By integration of $(95)_1$ one obtains

$$x^0 = \sigma = t, \qquad x = (x)^0 + \lambda_2^{(+)}(\mathbf{U}^0) t, \tag{103}$$

and the wave front in the first approximation is

$$\varphi(t,x) = \varphi^0 \left(x(t) - \left(\lambda_2^{(+)}\right)_0 t \right), \tag{104}$$

The amplitude $\pi(x,t)$ satisfies the following equation (see Eq. (63) with $J = 1$):

$$\left(\mathbf{l}_2^{(+)} \cdot \mathbf{r}_2^{(+)}\right)_0 \left\{ \frac{d\pi}{d\sigma} + |\varphi_x|_0 \left(\nabla \lambda_2^{(+)} \cdot \mathbf{r}_2^{(+)}\right)_0 \pi^2 \right\} = c^0\, \pi, \tag{105}$$

where $|\varphi_x|_0 = 1$ (see (104)) and

$$c^0 = \left[\nabla \left(\mathbf{l}_2^{(+)} \cdot \mathbf{B}\right) \cdot \mathbf{r}_2^{(+)} \right]_0. \tag{106}$$

Taking into account that

$$\mathbf{l}_2^{(+)} \cdot \mathbf{B} = \frac{\lambda_2^{(+)} \mathscr{S}}{2\alpha_n \tau^a} \left(\frac{n}{\tau^n} - \frac{\kappa a}{\rho} \right) - \frac{j_1^n \mathscr{S}}{2\rho \alpha_n \tau^n \tau^a} - \frac{\mathscr{V}_1}{\tau^a}, \tag{107}$$

we have

$$\nabla \left(\mathbf{l}_2^{(+)} \cdot \mathbf{B}\right) = \left(\frac{\partial \left(\mathbf{l}_2^{(+)} \cdot \mathbf{B}\right)}{\partial n}, \frac{\partial \left(\mathbf{l}_2^{(+)} \cdot \mathbf{B}\right)}{\partial j_1^n}, \frac{\partial \left(\mathbf{l}_2^{(+)} \cdot \mathbf{B}\right)}{\partial a}, \frac{\partial \left(\mathbf{l}_2^{(+)} \cdot \mathbf{B}\right)}{\partial \mathscr{V}_1} \right), \tag{108}$$

where

$$\frac{\partial\left(\mathbf{l}_2^{(+)}\cdot\mathbf{B}\right)}{\partial n}=\left[\frac{\mathscr{S}-4\rho\tau^n\tau^a\left(\lambda_2^{(+)}\right)^2}{4\tau^aG\lambda_2^{(+)}}\left(\frac{n}{\tau^n}-\frac{\kappa a}{\rho}\right)+\frac{j_1^n}{G}\right]\frac{\partial\alpha_\nu}{\partial n}+\frac{\lambda_2^{(+)}\mathscr{S}}{2\alpha_n^2\tau^n\tau^a},\tag{109}$$

$$\frac{\partial\left(\mathbf{l}_2^{(+)}\cdot\mathbf{B}\right)}{\partial j_1^n}=-\frac{\mathscr{S}}{2\rho\alpha_n\tau^n\tau^a},\tag{110}$$

$$\begin{aligned}\frac{\partial\left(\mathbf{l}_2^{(+)}\cdot\mathbf{B}\right)}{\partial a}=&\left(\frac{n}{\tau^n}-\frac{\kappa a}{\rho}\right)\left(\frac{\alpha_\nu\mathscr{S}}{4\alpha_n\tau^aG\lambda_2^{(+)}}+\frac{j_1^n-\rho\tau^n\lambda_2^{(+)}}{\alpha_nG}\alpha_\nu-\frac{\lambda_2^{(+)}\mathscr{S}}{\alpha_n}\right.\\&\left.-\frac{j_1^n\mathscr{S}}{2\rho\alpha_n^2\tau^a\tau^n}\right)\frac{\partial\alpha_n}{\partial a}-\frac{\lambda_2^{(+)}\kappa\mathscr{S}}{2\rho\alpha_n\tau^a},\end{aligned}\tag{111}$$

$$\frac{\partial\left(\mathbf{l}_2^{(+)}\cdot\mathbf{B}\right)}{\partial\mathscr{V}_1}=-\frac{1}{\tau^a}.\tag{112}$$

Furthermore, from expression (106) we obtain

$$\begin{aligned}c=&\frac{2\alpha_n(\tau^n)^2\lambda_2^{(+)}}{\mathscr{L}}\left\{\left[\frac{\mathscr{S}-4\rho\tau^n\tau^a\lambda_2^{(+)2}}{4\tau^aG\lambda_2^{(+)}}\left(\frac{n}{\tau^n}-\frac{\kappa a}{\rho}\right)+\frac{j_1^n}{G}\frac{\partial\alpha_\nu}{\partial n}\right]\right.\\&\left.+\frac{\lambda_2^{(+)}\mathscr{L}}{2(\alpha_n)^2\,\tau^n\tau^a}\right\}-\frac{\mathscr{Q}\mathscr{S}}{2\rho\tau^n\mathscr{L}\,(\tau^n)^2}+\frac{\lambda_2^{(+)}\tau^a\mathscr{R}}{\mathscr{L}}\left[\left(\frac{n}{\tau^n}-\frac{\kappa a}{\rho}\right)\right.\\&\times\left(\frac{\alpha_\nu\mathscr{S}}{4\alpha_n\tau^a\,G\lambda_2^{(+)}}+\frac{j_1^n-\rho\tau^n\lambda_2^{(+)}}{\alpha_nG}\alpha_\nu-\frac{\lambda_2^{(+)}\mathscr{S}}{\alpha_n}\right.\\&\left.\left.-\frac{j_1^n\mathscr{S}}{2\rho\,(\alpha_n)^2\,\tau^a\tau^n}\right)\left(\frac{\partial\alpha_n}{\partial a}\right)-\frac{\lambda_2^{(+)}\kappa\mathscr{S}}{2\rho\,\alpha_n\tau^a}\right]-\frac{1}{\tau^a}.\end{aligned}\tag{113}$$

Finally, we obtain $\pi=\frac{h(\sigma)}{\Phi(\sigma)}$, being

$$h(\sigma)=\pi^0exp\left[\int_0^\sigma-\frac{c^0}{\left(\mathbf{l}_2^{(+)}\cdot\mathbf{r}_2^{(+)}\right)_0}d\tau\right],\tag{114}$$

$$\Phi(\sigma)=1+\int_0^\sigma|\varphi_x|_0\left(\nabla\lambda_2^{(+)}\cdot\mathbf{r}_2^{(+)}\right)_0hd\tau,\tag{115}$$

where Eqs. (66) and (67) and the results (100), (101), (64), $J=1$ and $|\varphi_x|_0=1$ have been taken into consideration. Relation (115) gives

$$\Phi(\sigma)=1+\int_0^\sigma\frac{\tau^a|\varphi_x|_0h}{2G^0\mathscr{L}^0}\left[2\rho(\alpha_n^0)^2\tau^a\left(\frac{\partial\alpha_\nu}{\partial n}\right)^0+\alpha_a^0\mathscr{R}^0\left(\frac{\partial\alpha_n}{\partial a}\right)^0\right]d\tau.\tag{116}$$

In the case where there exists a critical time σ_c in which $\Phi(\sigma_c) = 0$, then $\pi \to \infty$, and this may correspond to a shock wave (Boillat 1965).

7. Conclusions

In this article we have presented, in the frame of extended irreversible thermodynamics with internal variables, a model describing an elastic extrinsic semiconductor with defects of dislocation, mass density constant and without polarization. Considering only the electronic and dislocation fields, we have derived a quasi-linear hyperbolic PDEs system. Since a thermodynamical model has an added value if possible solutions of the derived theory are found, and because the closed-form solutions of nonlinear PDEs are rare, we have investigated the propagation of weak discontinuities, as approximated solutions. To this aim we have introduced a new variable related to the surface across which the solutions or/and some of their derivatives undergo a jump, and following a Boillat's methodology for quasi-linear and hyperbolic systems of the first order, we obtained Bernoulli's equation governing the propagation of the amplitude of one of these approximated solutions in the one-dimesional case.

Acknowledgments

The authors thank Alessio Famà for valuable discussions. Also, they acknowledge the support of "National Group of Mathematical Physics, GNFM-INdAM".

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Paper contributed to the international workshop entitled "New approaches to study complex systems",
which was held in Messina, Italy (27–28 november 2017), under the patronage of the *Accademia Peloritana dei Pericolanti*
Manuscript received 21 September 2018; published online 20 December 2019